2013 Further Mathematics GA 3: Examination 1

SPECIFIC INFORMATION

The tables below indicate the percentage of students who chose each option. The correct answer is indicated by shading.

The statistics in this report may be subject to rounding resulting in a total less than 100 per cent.

Section A

Core: Data analysis

Question	% A	% B	% C	% D	% E	% No Answer
1	1	3	93	3	1	0
2	1	86	8	4	1	0
3	1	2	16	79	1	0
4	15	30	45	5	4	0
5	2	77	12	8	2	0
6	9	76	10	4	2	0
7	11	7	23	9	49	1
8	5	4	5	6	79	0
9	47	24	11	7	10	0
10	2	53	10	3	32	0
11	69	8	9	7	6	1
12	2	7	80	1	9	0
13	6	11	22	51	11	0

The Core section was very well done, with the exception of Questions 4 and 9.

Question 4

From the table, the percentage of 'tall' mothers with 'short' daughters is given by

$$\frac{\text{number of tall mothers with short daughters}}{\text{total number of tall mothers}} \times 100\% = \frac{3}{3+11+8} \times 100\% = 13.63\ldots\%$$

This answer rounds to 14% (option C).

The other common, but **incorrect**, response was 4% (option B). This response was obtained because the wrong base was used in calculating the percentage, as shown below.

$$\frac{\text{number of tall mothers with short daughters}}{\text{total number of mothers}} \times 100\% = \frac{3}{82} \times 100\% = 3.65\ldots\%$$

This answer rounds to 4% (option B).

Question 9
The key to answering Question 9 was recognising that the value of any statistic that involves the mean in its calculation will change if any one data value is changed. This means that the intercept of the least squares regression line (option B), the correlation coefficient, r (option C) and the standard deviation of *age* (option E) can be eliminated because they all involve the use of the mean value of the variable *age* in their calculation. The range of *age* (option D) will change because it depends directly on the maximum value in its calculation. However, the slope of the three median line (option A) will not change because the two medians used to determine its value are not affected by a change in the maximum values of the variable *age*.

Module 1: Number patterns

Question	% A	% B	% C	% D	% E	% No Answer
1	6	1	89	4	0	0
2	9	33	5	12	41	1
3	22	2	2	3	70	1
4	7	20	7	62	2	1
5	48	10	12	9	21	1
6	1	2	58	4	34	0
7	16	61	17	3	3	1
8	1	5	15	74	4	1
9	18	33	24	16	7	1

The questions in Module 1: Number patterns were generally well answered, with the exception of Questions 2 and 9.

Question 2
The correct response to this question was option E – a geometric sequence with a common ratio of one. This answer follows directly from the rule for the *n*th term of a geometric sequence, $t_n = ar^{n-1}$, which, if $r = 1$, reduces to $t_n = a$, a rule that generates a sequence whose terms do not change from term to term. None of the other given alternatives satisfied this condition.

Question 9
Two equivalent formulas given on the formula sheet could have been used to answer this question, namely

$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ or an equivalent, but not often used, $S_n = \frac{n}{2}\left[a + l\right]$.

Use of the second formula offers a quick route to the solution as follows

$$100 = \frac{10}{2}\left[a + 2\right] \text{ or } a = 18$$

Module 2: Geometry and trigonometry

Question	% A	% B	% C	% D	% E	% No Answer
1	2	5	87	5	1	0
2	5	8	4	81	2	0
3	12	7	9	5	66	0
4	22	2	6	11	59	0
5	15	6	52	16	11	1
6	23	8	11	55	3	1
7	8	12	58	18	3	1
8	12	39	25	13	10	1
9	9	47	17	18	7	1

The questions in Module 2: Geometry and trigonometry were generally well answered, with the exception of Questions 4 and 8.

Question 4
One solution strategy to this question is

The cost of the cupcakes is proportional to their volume, so that, if k is the linear scale factor,

$$\frac{\text{cost}_{\text{large}}}{\text{cost}_{\text{small}}} = \frac{V_{\text{large}}}{V_{\text{small}}} = k^3$$

The linear scale factor, k, can be determined by comparing the widths of the two cakes, to give $k = \frac{6}{4} = \frac{3}{2}$, so that

$$\text{cost}_{\text{small}} = \frac{\$5.40}{(3/2)^3} = \$1.60 \text{ (option A).}$$

Question 8
To correctly answer this question, students needed to first construct a diagram and then perform a relatively routine computation.

Module 3: Graphs and relations

Question	% A	% B	% C	% D	% E	% No Answer
1	12	69	13	2	4	0
2	12	3	8	8	68	1
3	3	3	87	4	3	0
4	8	7	4	78	2	1
5	0	1	34	57	7	0
6	10	17	3	17	52	1
7	46	14	20	6	12	1
8	7	35	19	32	5	1
9	7	12	52	20	8	1

Module 3: Graphs and relations was generally well done, with the exception of Question 8.

Question 8
The key to answering this question was to recognise that, for the objective function to have to have its minimum value at both vertices M and N, all points on the line between M and N must also minimise the objective function.

Thus, the family of lines representing the objective function $Z = ax + by$ must be parallel to the constraint passing through both M and N. This constraint has a slope of –1. For the objective function to have a slope of –1, a must equal b, leading to option B.

Module 4: Business-related mathematics

Question	% A	% B	% C	% D	% E	% No Answer
1	2	92	1	1	3	0
2	59	7	7	16	10	0
3	5	40	16	28	10	0
4	9	12	20	49	9	1
5	12	19	20	10	38	1
6	3	46	30	18	2	1
7	8	11	55	13	12	1
8	4	29	12	35	19	1
9	29	15	21	9	24	1

Questions 5, 6, 8 and 9 were challenging for many students.

Question 5
To answer this question correctly students needed to realise that, by definition, a simple perpetuity is designed to last indefinitely. This is achieved by paying out no more than the interest it earns. This ensures that a fixed amount can be paid from the perpetuity investment without any reduction in the amount of money originally invested, in this case $100 000 (option E).

Question 6
One solution strategy is

$\textit{worker's salary two years ago} \times 1.03^2 = \$46\,500$

Solving this equation gives

$\textit{worker's salary two years ago} = \dfrac{\$46\,500}{1.03^2} = \$43\,830.709\ldots$ (option C)

Question 8
Students needed to use the reducing balance method to determine the amount a car depreciates in the fourth year.

This can be determined as follows

$\$25\,000 \times 0.80^3 - \$25\,000 \times 0.80^4 = \$2560$ (option B)

Students who chose option D ($10 240) appeared to have misread the question and incorrectly determined the depreciated value of the car after 4 years, which is $10 240 ($\$25\,000 \times 0.80^4$).

Question 9
In this question, students needed to test the truth of five statements relating to the repayment of a reducing balance loan. Many students struggled with this question. The key to answering this question was to use a TVM to work out the interest rate applying to the loan. This knowledge, with the aid of a TVM solver, could then be used to test the truth of each of the five statements.

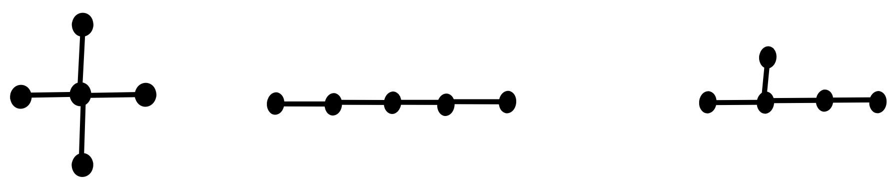
Module 5: Networks and decision mathematics

Question	% A	% B	% C	% D	% E	% No Answer
1	91	2	2	2	2	0
2	1	11	27	2	58	0
3	78	6	12	3	1	0
4	5	5	8	73	9	0
5	8	7	6	17	62	1
6	28	30	40	2	1	0
7	14	35	39	9	2	0
8	21	20	25	23	9	1
9	5	79	5	7	3	0

The questions in this module were generally well answered, with the exception of Questions 6, 7 and 8.

Question 6
Students needed to consider the network of roads connecting three towns by a graph. Students who chose option A or option B, both sub-graphs of the correct answer (option C), may have been on track towards obtaining the correct answer, but did not consider all possible road connections between the towns before making their decision.

Question 7
One way of answering this question was to construct all possible different connected graphs with five vertices and four edges. There are three.



Inspection of the graphs then shows that, of the five statements, only three are true for all graphs (option C)

- the graph is planar
- the sum of the degrees of the vertices is eight
- the graph cannot have a loop.

Question 8
One way of answering this question is as follows.

The critical path is *B-D-E* and has a completion time of 24 hours. The other path is *A-C-E* that has a completion time of 23 hours.

As activity *E* is common to paths *B-D-E* and *A-C-E*, reducing the completion time of activity E reduces the completion times to these paths to 23 and 22 hours respectively.

Further reductions can now be considered for path *A-C* (9 hours) and path *B-D* (10 hours), which have a common start and end point.

If the completion time of activities *A*, *B*, *C* and *D* is reduced by one hour each, then the time to complete path *A-C* becomes 7 hours and the time to complete path *B-D* becomes 8 hours.

Since path *B-D* takes at least 8 hours to complete, there is no point in paying for the completion time for path *A-C* to be reduced to less than 8 hours. Therefore, reduce only **one of** activity *A* or activity *C*.

This means that the completion times of the three activities *E, B, D,* plus either *A* or *C*, should be reduced by one hour each at a cost of $400 (option D).

Module 6: Matrices

Question	% A	% B	% C	% D	% E	% No Answer
1	2	91	3	1	2	0
2	1	3	11	80	4	0
3	18	63	9	5	4	1
4	7	7	23	7	55	1
5	1	3	86	7	2	0
6	41	42	7	3	6	1
7	11	10	19	43	16	1
8	5	41	19	21	13	1
9	12	17	19	12	39	1

With the exception of Question 9, Module 6: Matrices was generally very well done.

Question 9
One solution strategy is as follows.

Because the product $P = QRS$ is defined, and Q and S are square, we can deduce (in general terms) the order of these matrices as follows

Let the order of $Q = m \times m$ and the order of $S = n \times n$, where $\boldsymbol{m \neq n}$ because $Q + S$ is not defined

Since QRS is defined, the order of R must be $m \times n$

Thus, considering each option

- option A: $R - S$ is **not** defined as R and S have different orders
- option B: $Q + R$ is **not** defined as Q and R have different orders
- option C: $P^2 = P \times P$ is **not** defined because $m \neq n$
- option D: R^{-1} is **not** defined because R is not square
- option E: $P \times S$ is defined because the number of columns in P, n, equals the number of rows in S, n.