SAT Math Notes

By Steve Baba, Ph.D.

For SAT reading see my site:
www.FreeVocabulary.com
for a free list of 5000 SAT words with brief definitions.

Integers
Positive & negative whole numbers and ZERO.
…-3, -2, -1, 0, 1, 2, 3 …

Negative Numbers
Left of zero on **number line.**

← Smaller   Larger →

-4 -2 0 2 4

-2 is GREATER than –4
-1/4 is GREATER than –1/2

Order of Operations
PEMDAS (Please excuse my dear aunt Sally)

Parenthesis
Exponents
Multiplication/Division
  left to right
Addition/Subtraction
  left to right

$3x^2 \neq (3x)^2 = (3x)(3x) = 9x^2$

Because a+b = b+a and a • b = b • a, don't worry about order of addition or multiplication, but because

$a - b \neq b - a$, and
$a \div b \neq b \div a$

watch subtraction and division order in tricky word problems.

Odd/Even Operations
There are rules:
Odd number + Even number = Odd number ALWAYS.
Odd + Odd = Even
Even + Even = Even

But it's easier to remember by using any even or odd number
3 + 2 = 5 (odd number)
3 + 1 = 4 (even number)
2 + 2 = 4 (even number)

SAME IDEA, but not same results for multiplication:
3 • 2 = 6 (even number)
3 • 1 = 3 (odd number)
2 • 2 = 4 (even number)

SAT often combines several of the above rules:
(odd+odd+even) • odd
Use any even and any odd number to determine if result is always even or odd:
(3 + 3 + 2) • 3 = 24 (even)

Multiplying Positive and Negative Numbers
a • b • c • d
All Positive→Always Positive

All Negative is not always negative since two or any EVEN number of negative numbers CANCEL each other's negativity out. If ALL a, b, c, and d are negative, the product is positive.
-1 • -1 • -1 • -1 is POSITIVE

ONE Negative number or any other **ODD** number of negatives → **Negative**

Dividing is the same as multiplication.

The SAT often has these positive/negative questions backwards. If the result of a • b• c • d is negative then? (one OR three of a, b, c, d is negative)

Prime Numbers
A number divisible by ONLY itself and 1.

Prime numbers:
2 (the only EVEN prime number) 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, …..

1 is NOT a prime number

Prime Factors (Trees)
Factor 100:
 2 • 50
 2 • 2 • 25
2 • 2 • 5 • 5

All Factor Trees give the same prime factors, but NOT all factors.

100 can also be factored as:
 10 • 10
2 • 5 • 2 • 5
giving the same prime factors as above, but missed the nonprime factors 25 and 50. Both trees missed 4 and 20.

Find ALL (nonprime) factors by multiplying prime factors.
2 • 2 = 4 and
2 • 2 • 5 = 20 and
5 • 5 = 25 and
5 • 5 • 2 = 50

Or use "brute force" and divide 100 by 2,3,4,5,6,7,8,9,then 10.
(11 and higher is covered by checking 9 and lower)

Least Common Multiples (LCM)
LCM of 10 and 12:
10 • 12 = 120, a multiple (good enough for adding fractions) but not necessarily the least.

List multiples of each:
10, 20, 30, 40, 50, **60**, 70
12, 24, 36, 48, **60**
60 is Least Common Multiple.

On multiple-choice questions, LCM can be found by working backwards from answers:

a) 120 b) 80 c) 60 d) 36 e)10

by dividing each answer by 10 and 12 and choosing the least.

Greatest Common Factor (of 75 and 100)

Find ALL (including nonprime) factors of both.
75: 3, 5, 15, **25**
100: 2, 4, 5, 10, 20, **25,** 50

OR find the prime factors they have in common and multiply:
5 • 5 (both 75 and 100 have TWO 5's in factor tree)

OR on multiple choice questions work backwards from answers.

a) 75 b) 50 c) 30 d) 25 e) 5

Only 25 and 5 are factors of 75 and 100, and 25 is larger.

Between vs. Including
And other tricking wordings of between or including (inclusive, counting the first..)

Integers BETWEEN –2 and + 2 (-1,0 , 1) is not the same as integers ≥ -2 and ≤ 2 (-2, -1, 0, 1, 2), which includes –2 and 2.

Fractions, Adding/Subtracting
Common denominator (bottom) needed.

$$\frac{1}{4} + \frac{2}{3} = \frac{3}{12} + \frac{8}{12} = \frac{11}{12}$$

OR can be done on calculator (one divided by 4…), but if answers are in fractions, it's easier to stay with fractions.

Fractions, Multiplying
NO common denominator needed. Multiply across.

$$\frac{1}{4} \bullet \frac{2}{3} = \frac{2}{12} = \frac{1}{6}$$

$$\frac{1}{2} \bullet \frac{2}{3} \bullet \frac{3}{4} \bullet \frac{4}{5} = \frac{24}{120} = \frac{1}{5}$$

Look for opportunities to cancel (cross out):

$$\frac{1}{\cancel{2}} \bullet \frac{\cancel{2}}{\cancel{3}} \bullet \frac{\cancel{3}}{\cancel{4}} \bullet \frac{\cancel{4}}{5} = \frac{1}{5}$$

Fractions, Dividing
No common denominator needed. FLIP second or bottom fraction then MULTIPLY.

$$\frac{1}{4} \div \frac{2}{3} = \frac{1}{4} \bullet \frac{3}{2} = \frac{3}{8}$$

↑ flipped ↑

Mixed numbers (3 ½) must be converted to proper fractions (7/2) before operations. (3=6/2 add to ½)

Fractions, Squaring, Cubing
Same as multiplying. Multiply by self.

$(\frac{1}{2})^2 = \frac{1}{2} \bullet \frac{1}{2} = \frac{1}{4}$

Note that ¼ is LESS than ½, while for numbers greater than 1 the square is larger.

$(\frac{1}{2})^3$ is

$\frac{1}{2} \bullet \frac{1}{2} \bullet \frac{1}{2} = \frac{1}{8}$

Average: Arithmetic Mean

$\frac{\text{Sum of Terms}}{\text{Number of Terms}}$

Average 5, 5, 10, 20:

$\frac{5+5+10+20}{4} = 10$

Mode: Most frequently occurring number.

Mode of 5, 5, 10 and 20 is 5.

Median: Number in middle when numbers ordered from smallest to largest.

Median of 10, 11, 17, 19 and 20 is 17.

Median of an EVEN number of terms. Since there is no single middle number, the median is half way between the two middle numbers or the average of the two middle numbers.

Median of 10, 13, 19 and 20?

The two middle numbers are 13 and 19. Halfway between or the average is 16.

Weighted Average
A class of 3 students has an average grade of 70. The other class of 5 students has an average of 80. What is the average for the school? (It's NOT 75.)

Assume ALL 3 students in first class got exactly 70. Assume ALL 5 five in second class got exactly 80. Compute usual average:

$\frac{\text{Sum of Terms}}{\text{Number of Terms}} =$

$\frac{70+70+70+80+80+80+80+80}{8} =$

$\frac{3 \bullet 70 + 5 \bullet 80}{8} = 76.25$

Difficult weighted average questions use variables (a, b) for the number of students:

$\frac{a \bullet 70 + b \bullet 80}{a + b}$

May (sometimes) / Must (always) be true

X is a positive integer.

$X^2 > X$ MAY be true if X=2. But MUST be true is FALSE, since X could equal 1.

One false example (a counter example) proves a MUST (be true) FALSE.

One true example proves a MAY (be true) TRUE.

Inequalities (X > 6)
Like equalities (X = 6) anything done to one side of the equation, do to the other side,
EXCEPT when multiplying or DIVIDING by a NEGATIVE, switch inequality sign.
(8 > 6)
Multiply both sides by –1 is NOT: (-8 > - 6), but is (-8 < -6).

Percent - Part from Whole
What (part) is 15% of 60 (whole)?
15% = 15/100 or .15
y% = y/100 or move decimal point two spaces to convert:

Part = Percent • Whole
X = 15/100 • 60
OR
X = .15 • 60
X = 9

Percents are the same as fraction questions: What (part) is 3/20 of 60 (whole)?
Part = Fraction • Whole

Percent - Part from Whole, but very large or small percentages
What (part) is .15% of 60 (whole)? ↑
Note the decimal point

.15% = .15/100 or .0015

Part = Percent • Whole
X = .15/100 • 60
OR
X = .0015 • 60
X = .09

What is 300% of 60?

300% = 300/100 or 3
Part = Percent • Whole
X = 300/100 • 60
OR
X = 3 • 60 = 180

Percent – Missing Percent
16 is what percent of 80?
(part = 16, whole = 80)

Part = Percent • Whole
16 = X/100 • 80
X = 20
OR solve for decimal
16 = D • 80
D = .2, and convert to percent by moving decimal point.
. 2 = 20%

Percent – Missing Whole (working backwards)
16 is what 20% of what?
(part = 16, percent = 20%)

Part = Percent • Whole
16 = 20/100 • X
X = 80

Percent - Increase
What is 10% more than 90?
Many alternate wordings like:
After a 10% increase from 90?

Part = Percent • Whole
X = 110/100 • 90 = 99

ADD the original 100% AND the additional 10%.
Note the "part" is more than the whole if increased.

Percent - Decrease
What is 15% less than 20?
Many alternate wordings like:
A $20 shirt on sale for 15% off (the full price) costs?

Part = Percent • Whole
X = 85/100 • 20 = 17

But the original 100% MINUS the decrease is the percent (85% = 100% - 15%)

Multiple (usually 2) percent changes

<u>A store buys cakes wholesale for $10, and adds 50% to get the fresh-cake retail price.</u> If the cake does not sell in a week, the store reduces the fresh-cake retail price by 50% and sells as week-old cakes. A week-old cake costs? (It's NOT $10)

Solve as TWO separate problems. From the first sentence (underlined), solve for the fresh-cake retail price. This is a simple percent increase problem.

Part = Percent • Whole
X = 150/100 • $10 = $15

Then reduce the $15 by 50%. The $15 is now the new whole (sometimes call new "base").

This second part is just a simple (50%) percent decrease problem.

Part = Percent • Whole
X = 50/100 • $15 = $7.5

Change the whole or base when doing multiple percent changes.

Ratios - Part to Part, no whole
The ratio of apples to oranges is 3 to 2. There are 15 apples. How many oranges?

Keep apples on top
↓ ↓
$\frac{3}{2} = \frac{15}{X}$ NOT $\frac{X}{15}$
↑ ↑
keep oranges on bottom

Cross-multiply to solve for X if answer not obvious. X = 10

You can put all apples on top or all apples on bottom, but don't mix in one equation.

Ratios – Inches to Miles
On a map 2/3 of an inch represents 10 miles. 5 inches on map is?

keep inches on top
↓ ↓
$$\frac{2/3}{10} = \frac{5}{X}, \quad X = 75$$
↑ ↑
keep miles on bottom.

Can also be solved by finding 1 inch = 15 miles and multiplying by 5 (inches).

Ratios - Part to Part, and Total
The ratio of apples to oranges is 3 to 2. There is a total of 50 apples and oranges. How many oranges?

keep apples on top
↓ ↓ ↓ ↓
$$\frac{3}{2} = \frac{15}{10} = \frac{21}{14} = \frac{30}{20}$$
↑ ↑ ↑ ↑
keep oranges on bottom

Find a ratio that adds up to 50.

On multiple choice problems work backward from answers. Only one answer works.

Can also be done with algebra:
Let 3x be number of apples.
Then 2x is number of oranges.
3x + 2x = 50, where x is the multiple of the original ratio.

Multiple Ratios
The ratio of apples to oranges is 3 to 2. The ratio of oranges to pears is 3 to 4. What is the ratio of apples to pears?
It's NOT 3 to 4.

Do one ratio at a time:

Assume 18 apples. Any number works, but pick a multiple of 3 that will divide evenly to avoid fractions.

keep apples on top
↓ ↓
$$\frac{3}{2} = \frac{18}{X}, \quad \text{Solve for } X = 12$$
↑ ↑
keep oranges on bottom

With 18 apples there are 12 oranges.

Now oranges on top
↓ ↓
$$\frac{3}{4} = \frac{12}{Y} \quad \text{Solve for } Y = 16$$
↑ ↑
keep pears on bottom.

With 18 apples, there are 16 pears or 18/16 or 9/8.

Direct Proportion

Speed (X)	Miles in 30 min (Y)
30	15
60	30
90	45

In general
y = kx , k is a constant
k = ½ in this example
Y = ½ X
Miles in 30 min = ½ Speed

Can also be solved as ratio problem without finding k. At 40 MPH, what is distance in 30 minutes?

Keep speed on top
↓ ↓
$$\frac{30}{15} = \frac{40}{X}, \quad X = 20$$
↑ ↑
keep distance on bottom

Inverse Proportion

Speed (X)		Minutes to Travel 60 Miles (Y)	k
30	•	120	= 3600
60	•	60	= 3600
90	•	40	= 3600

In general
xy = k, k is a constant
as x increases, y decreases keeping k constant.
Rearranging:
y = k/x and x = k/y
k = 3600 in this example

Common Inverse Proportions:

If x doubles, y must half to keep k constant.
If x triples, y must be 1/3 to keep k constant.
If x goes up z times, y must be 1/z to keep k constant.

Most inverse proportions can be done without calculating k, using the above common inverse proportions.

Rates (MPH), Distance

Rate • Time = Distance
20 MPH • 2 Hours = 40 miles

Average MPH, Rate

Fast, 40 MPH in morning driving to school. Slow, 20 MPH in afternoon traffic. What is average MPH?

Do NOT average 20 and 40 for 30.

Assume the school is 40 miles away. 80 miles round trip. One hour in morning. Two hours in afternoon.

80 miles/3 hours=26 2/3 MPH

FOIL multiplication
First, outer, inner, last

(a + b) (c + d) =
first outer inner last
ac + ad + bc + bd

FOIL (a+b) (a+b)
first outer inner last
a^2 + ab + ba + b^2 =

$$a^2 + 2ab + b^2$$

FOIL (a-b) (a-b)
first outer inner last
a^2 - ab - ba + b^2 =

$$a^2 - 2ab + b^2$$

FOIL (a+b) (a-b)
first outer inner last
a^2 – ab + ba – b^2 =

$$a^2 - b^2$$
Difference of Two Squares

Multiplying by Zero
0 times anything is 0.
If a • b = 0 then a and/or b (one or both) is zero. This is used in factoring
If (x-3)(x-5) = 0,
(x-3) and/or (x-5) = 0,
x = 3 or x = 5

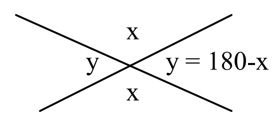
Factoring Polynomials
FOIL backwards
↓ zero here
$x^2 + 3x + 2 = 0$

Guess first terms that multiply to x^2:
(x +__) (x + __) = 0
Guess last terms that multiply to 2:
(x + 2) (x + 1) = 0
Test to see if outer + inner multiplications add to 3x:
1x + 2x = 3x.
It does, but if not try guessing other first or last terms.

(x + 2) (x + 1) = 0
x = -2 or x = -1

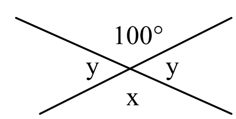
On multiple choice questions: you can work backwards from the answers without using FOIL:
a) 3 b) 2 c)1 d) 0 e) -1
by trying each in the original
$x^2 + 3x + 2 = 0$

Opposite Angles
are equal. x = x and y = y



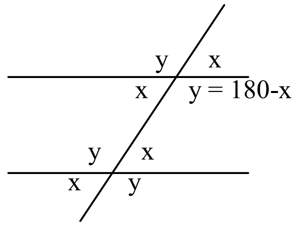
On one side of a line the angles (x+y) add up to 180° (half a 360° circle).

Given one angle is 100°:



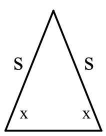
y must equal 80° to add up to 180° along a line. X must equal 100° because it's opposite of 100° AND also because x + y on one side of a line must equal 180°.

Parallel Lines:



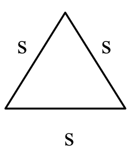
Visualize placing parallel lines on top of each other. All Xs and Ys are equal.
Given any one angle, all others can be found.

Isosceles Triangles



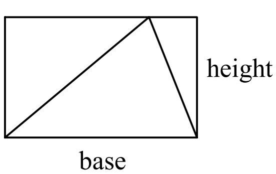
Two equal angles (x) ↔
Two equal sides (s)
opposite the equal angles

Equilateral Triangles
Are always 60°- 60°- 60°

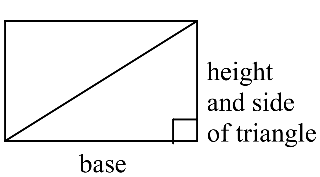


Three equal sides (s) ↔
Three equal angles.
All 60° because every triangle is 180°, and 180°/3 = 60°.

Area of a triangle
½ base • height:
which is half the area of a rectangle (base • height) or (length • width)

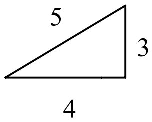


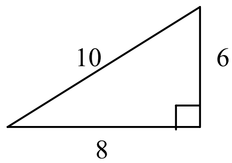
Side of triangle is NOT the height unless it's a right (90°) triangle:



Similar Triangles
Have same angles, but one is larger or smaller than other.
All sides are proportional.
Use ratio to solve.

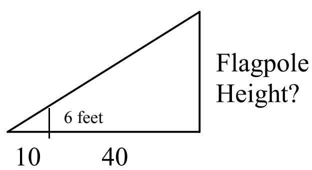
This 3-4-5 Triangle is half the size of the larger 6-8-10 similar triangle.





Similar triangles have the same three angles.

A similar triangle inside a larger similar triangle:



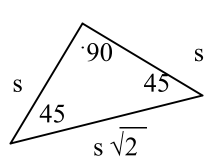
6/10 = Flagpole Height/50
Solve for Flagpole Height= 30

Pythagorean Theorem
For right (90°) triangles only.

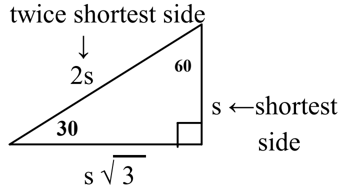
3-4-5 triangle shown above:
$(\text{leg})^2 + (\text{leg})^2 = (\text{Hypotenuse})^2$
$(3)^2 + (4)^2 = (5)^2$
$9 + 16 = 25$

6-8-10 triangle shown above:
$(\text{leg})^2 + (\text{leg})^2 = (\text{Hypotenuse})^2$
$(6)^2 + (8)^2 = (10)^2$
$36 + 64 = 100$

45° - 45° - 90° triangles
(an Isosceles Triangle)
Two equal angles ↔
Two equal legs (sides)



30° - 60° - 90° triangles



Congruent
Same shape (angles) AND same size (lengths).

Contrast with **similar** shapes with have the same shape (angles) but not same size (lengths). One similar triangle can be larger than other.

Polygons: Interior Angles
(number of sides – 2) • 180°

Triangles (3 sides) = 180°
Rectangles (4 • 90°) = 360°
Same for square or ANY 4 sided figure.
Pentagon (5 sides) = 540°
180° for each additional side
N–gon (n sides) = (n-2) • 180°

Absolute Value
Make positive if negative

$|x| = x$ if positive, -x if x is originally negative

$|5| = 5$ and $|-5| = 5$

Absolute value is used for "within" problems:

Adam (a = Adam's age) does not date women (w = date's age) more than two years older or younger than himself.

$|a\text{-}w| \leq 2$
which is the same as
$|w\text{-}a| \leq 2$
Plug in numbers for ages to test:
$|17\text{-}15| \leq 2$ same as
$|15\text{-}17| \leq 2$

Probability =

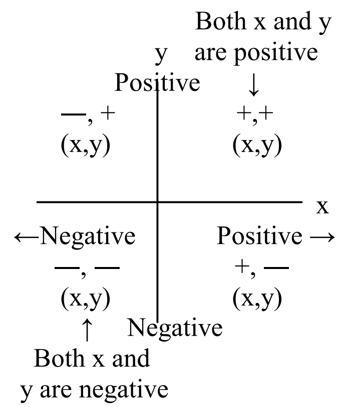
$$\frac{\text{Number of OK Outcomes}}{\text{Total Number of Outcomes}}$$

A student has 15 dirty shirts and 5 clean shirts in his dorm room. Randomly picking a shirt in the dark, what is the probability of picking a clean shirt? (It's not 5/15, the ratio of clean to dirty shirts)

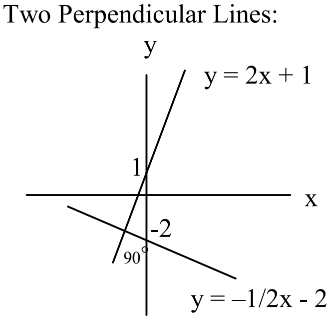
First find the total number of outcomes, which is 20 (15 dirty + 5 clean).

$$\frac{\text{OK Outcomes}}{\text{Total Outcomes}} = \frac{5}{20} = \frac{1}{4}$$

Coordinates



Lines y = mx + b
Two Perpendicular Lines:



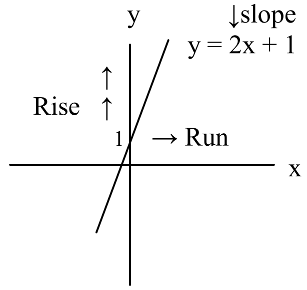
y = 2x + 1 , in general
y = mx + b
↑ ↑
slope y-intercept

When x = 0 (on the y axis), y = b (the y-intercept)

A point on a line (x and y), and either slope (m) or the y-intercept (b) can be used to find the other (m or b) using y=mx + b.

Perpendicular lines cross at 90° (right) angles and the slope of one (2 in this case or m in general) is the negative reciprocal (one over) of the other's slope (-1/2 in this case or –1/m in general).

Slope: Rise/Run
increase in y/increase in x



If the line is clearly graphed, often it's possible to easily count the rise and run between any two points for slope.

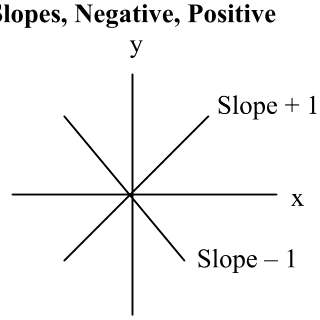
Given any two points (1,3) and (0,1) slope is rise/run or:

$$\frac{\text{The first y} - \text{the second y}}{\text{The first x} - \text{the second x}} =$$

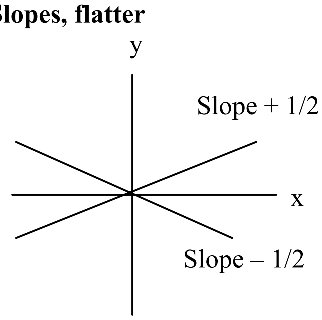
$$\frac{3-1}{1-0} = 2$$

Either point could be the "first" point or the "second," but the result is the same.

Slopes, Negative, Positive



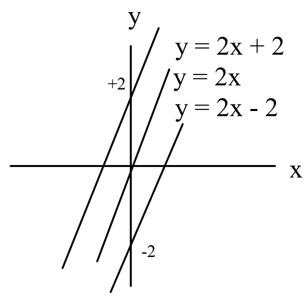
Slopes, flatter



Shifting graphs
With any function adding (subtracting) OUTSIDE the function moves the graph up (down).

Take the simplest function:
$y = 2x$, the line previously used.

Adding 2 AFTER/OUTSIDE THE FUCTION 2x moves the line up 2 to the new y-intercept of 2. Subtracting 2 moves the line down 2 to the new y-intercept of -2:

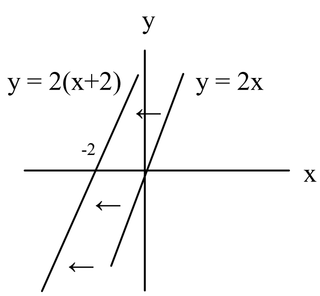


Again take the function $y = 2x$ but add or subtract before performing the function:

Original: $y = 2x$
New: $y = 2(x+2)$

One might guess (incorrectly) that adding 2 moves the line up 2 or maybe to the right 2.

But the curve shifts left by two. $X = -2$ in the new function gives the same result as $x = 0$ in the original. $X = 0$ in the new function gives the same result as $x = 2$ in the original.



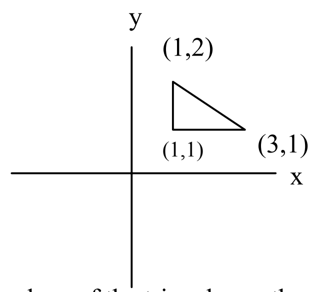
Subtracting (not adding) inside the function shifts to the right.

The SAT often tests for these counterintuitive shifts.

One can also do these point-by-point by picking a value for x, finding y, and plotting to see which way the curve shifts. Some can be done on a calculator if the formula is given.

Distance between 2 Points
(Pythagorean Theorem)

Given any two points ((1,2) and (3,1) choose a third point to make a right triangle by taking the x from one point and the y from the other point. Either (1,1) or (3,2) makes a right triangle, but (1,1) is shown below.



the legs of the triangle are the change in x and the change in y. Graphing the triangle may be skipped.

$(\text{leg})^2 + (\text{leg})^2 = (\text{Hypotenuse})^2$
$(1)^2 + (2)^2 = (h)^2$
$5 = (h)^2$

$\sqrt{5} = h$

Midpoint of a line segment.
The midpoint of (1,1) to (3,7) is half way between the Xs (halfway between or average of 1 and 3 is 2) and halfway between the Ys (halfway between or average of 1 and 7 is 4). The midpoint is (2,4).

Counting Consecutive Integers
(or consecutive tickets….)

Tickets number 9 through 15 were sold today. How many? It's NOT 15-9 or 6.

For small numbers one can count 9, 10, 11, 12, 13, 14, 15 for 7 tickets sold.
Subtract (15-9) AND add 1 to count the first ticket sold for 7.

Exponents – Multiplication

same base, **add** exponents
$a^3 \bullet a^2 =$
$(a \bullet a \bullet a) \bullet (a \bullet a) =$
$a^5 =$
a^{3+2}

Can also be solved, as a backup method or check, by letting a=2 and solving.

Exponents – Division

same base, subtract exponents
$a^4 \div a^2 =$

$$\frac{a \bullet a \bullet a \bullet a}{a \bullet a} =$$

cancel all except two top a's
$a^2 =$
a^{4-2}

Exponents – Raising Powers
Multiply exponents

$(a^3)^2 =$
$(a \bullet a \bullet a) \bullet (a \bullet a \bullet a)$
$a^6 =$
$a^{3 \bullet 2}$

$a^3 \bullet a^2 \neq (a^3)^2$
$a^5 \neq a^6$

Negative Exponents

$a^{-1} = \frac{1}{a}$ $a^{-2} = \frac{1}{a^2}$

In general $a^{-b} = 1/a^b$
(put under 1 and drop the negative)

negative exponents follow the rules for division

$a^2 \div a^4 =$

$$\frac{a \bullet a}{a \bullet a \bullet a \bullet a} =$$

cancel all except two bottom a's

$\frac{1}{a \bullet a} = \frac{1}{a^2} =$

$\mathbf{a^{2-4}} =$

$\mathbf{a^{-2}}$

Exponents (ab)^2

$(ab)^2 = (ab) \bullet (ab) = a^2b^2$

In general, the exponent can be distributed:

$(ab)^k = a^k b^k$

Exponents – Square root of both sides

$a^2 = b^4$

rewriting as:

$(a \bullet a) = (b \bullet b \bullet b \bullet b)$

it's obvious that $a = (b \bullet b)$

OR take the square root of both sides (half the exponent)

$a = b^2$

This works for cube roots or any other roots.

Fractional Exponents –

Are square/cube… roots

$a^{1/2}$ = square root of a

$a^{1/3}$ = cube root of a

$a^{1/n}$ = n'th root of a

Fractional exponents are useful for reducing:

$a^3 = b^9$

$(a^3)^{1/3} = (b^9)^{1/3}$

Using the power raised rule to multiply exponents gives:

$a = b^3$

Permutations: orderings

Jane has 3 dresses. (make the dresses A, B, and C). Wearing a different dress on three different nights, how many possibilities?

For easy problems with a small number of outcomes, the possibilities can be written:

ABC, ACB,
BAC, BCA,
CAB, CBA

OR there are 3 options for the first night (A,B, or C), 2 options for the second night (the two remaining dresses) and 1 option for the last night (the one remaining dress).
Multiply $3 \bullet 2 \bullet 1 = 6$.
(This is three factorial or 3!)

Oddball selections

A different question may have unlimited (re)selection of choices. If Jane can rewear the dresses multiple times, then she could wear the same dress three times (AAA, BBB or CCC) wear a dress twice (AAB, BBA….). Because of repeated selections, there are 3 possibilities for the first dress, AND 3 possibilities for the second dress and 3 possibilities for the third dress.
Multiply $3 \bullet 3 \bullet 3 = 27$.

Hard SAT questions may add oddball conditions such as Jane can't wear dress A on the first night. Do as above but with only two possibilities for the first night.
Multiply $2 \bullet 3 \bullet 3 = 18$.

Combinations: **Choosing unordered groups**

Again, Jane has 3 dresses, but wants to take 2 of the 3 on a trip. How many possibilities are there?

For easy problems with a small number of outcomes, possibilities can be written:

AB, AC, BA, BC, CA, CB

But before you answer six, note that AB and BA are the same combination. Likewise (AC and CA) and (BC and CB). Cross out the duplicates.

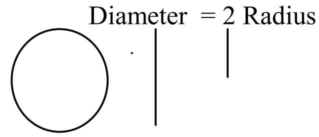
OR there are 3 options for the first dress and, 2 options for the second dress (the two remaining dresses).
Multiply $3 \bullet 2 = 6$. But there are two ordering of each combination. Divide by 2. (2!)
In general divide by the number of permutations (orderings) of the chosen (smaller) group, which is its factorial.

Sets, Double counting

5 students play chess.
4 students play football.
2 students play both chess and football. How many students? It's not 5+4 = 9, because this double counts the students who play both. It's 5+4-2 = 7.
Add sets, subtract intersection.

Circles

∏ (pi) = 3.14 approximately

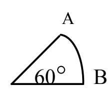


Circumference = ∏ D = ∏ 2R
length around entire circle

Remember it's 3.14 times the diameter not the radius. If you take 3.14 times the radius, drawn above outside the circle for easier comparison, you can see that you will only get halfway around the circle

Area = $\prod R^2$
Remember it's the radius squared, not the diameter squared. If you square the diameter, drawn above outside the circle for easier comparison, you get a square box larger then the circle.

Arcs and Sectors of Circles are just fractions of circles.



Sectors (wedges, slices) are fractions of the entire circle's **area.**

Arcs are fractions of the total circle's **circumference.**

But instead of saying 1/6 of a circle, questions will say 60°. A total circle is 360 °.
60°/360 ° = 1/6.

To find the length of an arc, find the circumference of the total circle and multiply by the fraction (1/6 or 60/360 in this example).

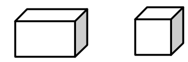
To find the area of a sector, find the area of the total circle and multiply by the fraction.

Simplifying Square Roots

$\sqrt{50} = \sqrt{25 \bullet 2} = 5\sqrt{2}$

$\sqrt{a^2 \bullet b} = a\sqrt{b}$

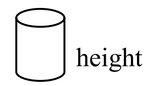
Volume =
Length • width • height



It does not matter which side is called height or width as long as you multiply all three.

For a cube all three sides are the same. Volume = $(\text{side})^3$

Cylinders:



Volume of Cylinder =
(Area of top circle) • height

The top circle and bottom circle are the same size.

Solving 2 equations:

$a + 2b = 3$
$2a + 6b = 10$

Multiply both sides of first equation by 2 and subtract from the second equation.

$2a + 6b = 10$
$2a + 4b = 6$

$2b = 4$
$b = 2$

Replace b in any equation to solve for a. Check with a and b in the other equation.

Or in first equation, isolate a:

$a = 3 - 2b$
and substitute (3-2b) for a into the second equation:
$2(3-2b) + 6b = 10$
$6-4b + 6b = 10$
$2b = 4$
$b = 2$

Bisector splits into equal parts each half the original's size.