Government
of South Australia

SACE
Board of SA

External Examination 2013

2013 SPECIALIST MATHEMATICS

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ATTACH SACE REGISTRATION NUMBER LABEL TO THIS BOX

Graphics calculator ☐

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Friday 15 November: 9 a.m.

Time: 3 hours

Pages: 45
Questions: 15

Examination material: one 45-page question booklet
one SACE registration number label

Approved dictionaries, notes, calculators, and computer software may be used.

Instructions to Students

1. You will have 10 minutes to read the paper. You must not write in your question booklet or use a calculator during this reading time but you may make notes on the scribbling paper provided.

2. This paper consists of three sections:

   Section A (Questions 1 to 9) 75 marks
   Answer ***all*** questions in Section A.

   Section B (Questions 10 to 13) 60 marks
   Answer ***all*** questions in Section B.

   Section C (Questions 14 and 15) 15 marks
   Answer ***one*** question from Section C.

3. Write your answers in the spaces provided in this question booklet. There is no need to fill all the space provided. You may write on pages 38, 43, and 44 if you need more space, making sure to label each answer clearly.

4. Appropriate steps of logic and correct answers are required for full marks.

5. Show all working in this booklet. (You are strongly advised ***not*** to use scribbling paper. Work that you consider incorrect should be crossed out with a single line.)

6. Use only black or blue pens for all work other than graphs and diagrams, for which you may use a sharp dark pencil.

7. State all answers correct to three significant figures, unless otherwise stated or as appropriate.

8. Diagrams, where given, are not necessarily drawn to scale.

9. The list of mathematical formulae is on page 45. You may remove the page from this booklet before the examination begins.

10. Complete the box on the top right-hand side of this page with information about the electronic technology you are using in this examination.

11. Attach your SACE registration number label to the box at the top of this page.

SECTION A (Questions 1 to 9)

(75 marks)

*Answer **all** questions in this section.*

QUESTION 1 (9 marks)

(a) On the axes in Figure 1, sketch the curve with parametric equations

$$\begin{cases} x = 2\cos t \\ y = \sin 2t \end{cases} \text{ where } 0 \le t \le 2\pi.$$

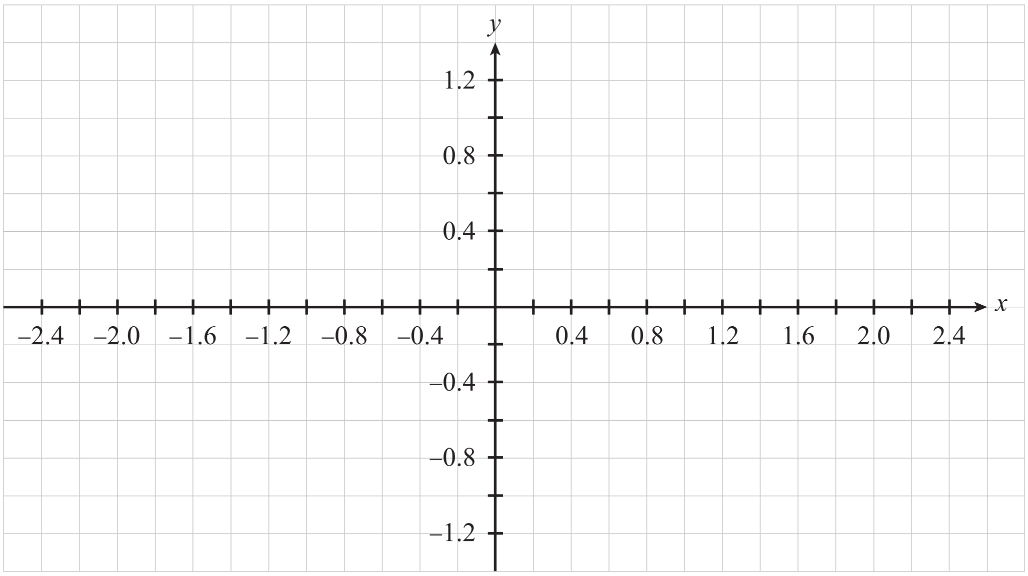

Figure 1 (3 marks)

(b) (i) Find the exact coordinates of the point P that lies on the curve where $t = \frac{\pi}{3}$.

(1 mark)

(ii) Find the exact value of $\frac{dy}{dx}$ for $t = \frac{\pi}{3}$.

(3 marks)

(c) Clearly represent the results found in part (b) on the curve you drew in part (a).

(2 marks)

QUESTION 2 (8 marks)

(a) Show algebraically that $\int_{0}^{\frac{\pi}{2}} \sin 2x \, dx = 1$.

(3 marks)

(b) Find $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sin 2x \, dx$.

(1 mark)

(c) Let $f(x) = \sin 2x$.

Show that $f\left(\frac{\pi}{4} + h\right) = f\left(\frac{\pi}{4} - h\right)$ for all real numbers h.

(2 marks)

(d) Figure 2 shows the graph of $y = \sin 2x$, where $0 \le x \le \frac{\pi}{2}$.

Use your results for parts (a), (b), and/or (c), or otherwise, to divide the area between the curve and the x-axis into four equal areas by drawing three vertical lines.

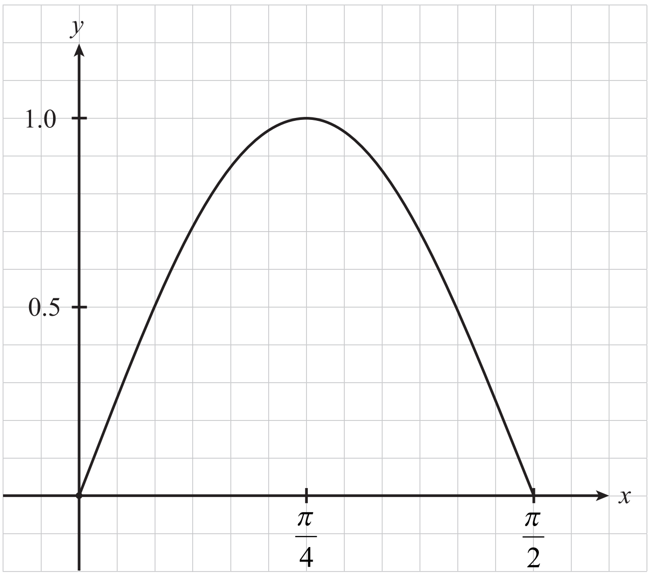

Figure 2

(2 marks)

QUESTION 3 (8 marks)

In Figure 3, AB is a diameter of a circle with centre O and radius r.
The chord CD is produced to X so that AX is perpendicular to CX.
$\angle BAD$ and $\angle ABC$ measure α and β respectively.

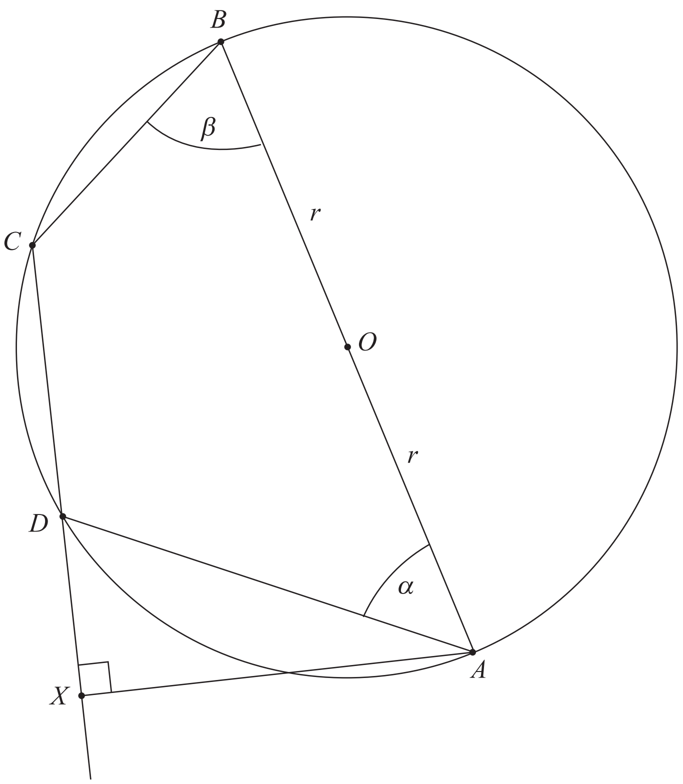

Figure 3

(a) Prove that $\angle ADX = \beta$.

(2 marks)

(b) (i) Prove that $AD = 2r\cos\alpha$. (*Hint*: Join B and D.)

(3 marks)

(ii) Prove that $DX = 2r\cos\alpha\cos\beta$.

(1 mark)

(c) If the chord DC is now produced to Y so that BY is perpendicular to DY, prove that $CY = DX$.

(2 marks)

QUESTION 4 (9 marks)

(a) A polynomial $P(x)$ has a factor $(x-k)^2$ where k is a constant.

Let $P(x)=(x-k)^2 Q(x)$.

Prove that $P'(x)$ has a factor $(x-k)$.

(3 marks)

(b) The polynomial $p(x)=x^3-5x^2+8x+b$, where b is a constant, has a factor $(x-k)^2$.

(i) Find $p'(x)$.

(1 mark)

(ii) Hence find two possible values for k.

(2 marks)

(iii) For the larger of these two values of k, find b and thus fully factorise $p(x)$.

(3 marks)

QUESTION 5 (8 marks)

Figure 4 shows the complex number $z = r\text{cis}\theta$, where $r > 1$ and $0 < \theta < \dfrac{\pi}{4}$.

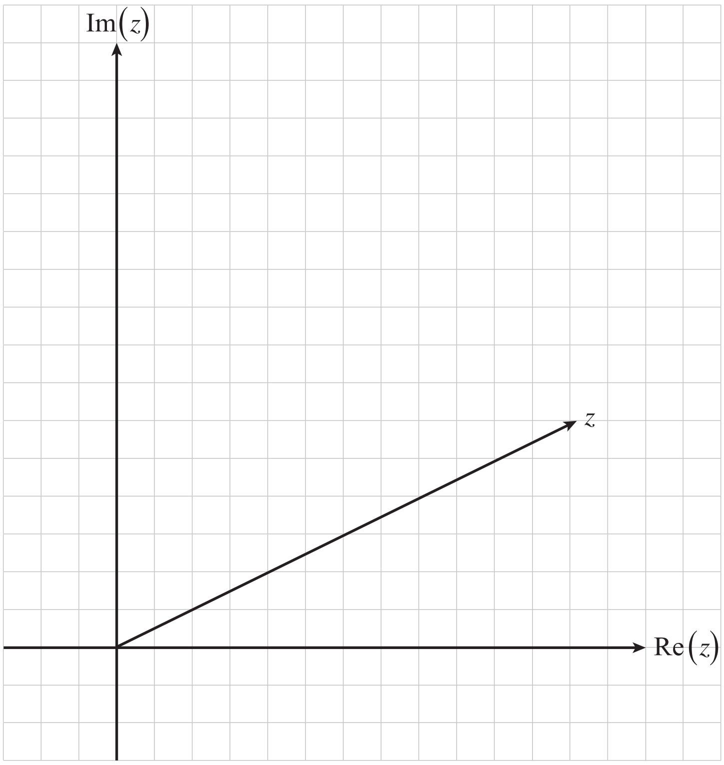

Figure 4

(a) On Figure 4 clearly show:

(i) z^2.

(1 mark)

(ii) $|z^2 - z|$.

(1 mark)

(b) Explain why:

(i) $|z^2| + |z| > |z^2 - z|$.

(1 mark)

(ii) $|z^2 - z| > |z^2| - |z|$.

(1 mark)

(c) Show that if $z = 2\text{cis}\theta$, then $2 < |z^2 - z| < 6$.

(2 marks)

(d) If $z = 2\text{cis}\dfrac{\pi}{6}$, find $|z^2 - z|$.

(2 marks)

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QUESTION 6 (8 marks)

(a) Let $\boldsymbol{a} = [1,\ 2,\ -4]$ and $\boldsymbol{b} = [5,\ -3,\ -2]$.

(i) Find $|\boldsymbol{a}|$.

(1 mark)

(ii) Find $\boldsymbol{a} \bullet \boldsymbol{b}$.

(1 mark)

(iii) Verify $|\boldsymbol{a} + \boldsymbol{b}|^2 = |\boldsymbol{a}|^2 + |\boldsymbol{b}|^2 + 2\boldsymbol{a} \bullet \boldsymbol{b}$.

(2 marks)

(b) Use the vector property $|\boldsymbol{c}+\boldsymbol{d}|^2=(\boldsymbol{c}+\boldsymbol{d})\cdot(\boldsymbol{c}+\boldsymbol{d})$ to prove that, for any vectors $\boldsymbol{c}$ and $\boldsymbol{d}$,

$$|\boldsymbol{c}+\boldsymbol{d}|^2=|\boldsymbol{c}|^2+|\boldsymbol{d}|^2+2\boldsymbol{c}\cdot\boldsymbol{d}.$$

(2 marks)

(c) If $|\boldsymbol{c}|=10$, $|\boldsymbol{d}|=5$, and $\boldsymbol{c}\cdot\boldsymbol{d}=25$, find:

(i) $|\boldsymbol{c}+\boldsymbol{d}|$.

(1 mark)

(ii) $|\boldsymbol{c}-\boldsymbol{d}|$.

(1 mark)

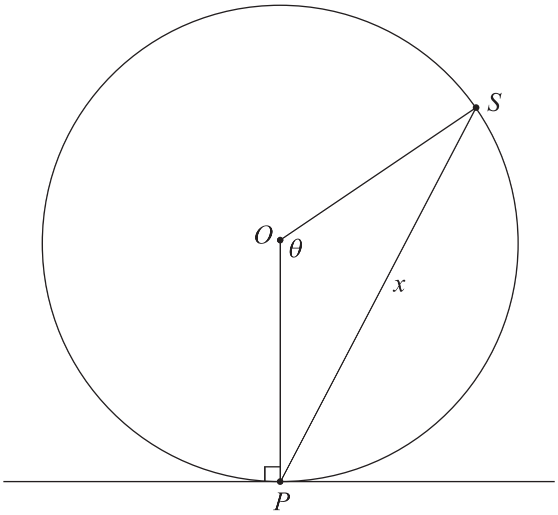
QUESTION 7 (8 marks)

Figure 5 represents a Ferris wheel rotating anticlockwise at 2 revolutions per minute. The diameter of the wheel with centre O is 16 metres.

The point P is the lowest point on the wheel. The line OP is perpendicular to the horizontal baseline through P.

A person on the Ferris wheel is at position S.

Let $SP = x$ metres and $\angle SOP = \theta$, where $0 < \theta < \pi$.



Source: © Glowonconcept/Dreamstime.com

Figure 5

(a) Use the cosine rule to show that $x^2 = 128 - 128\cos\theta$.

(1 mark)

(b) Hence show that $\dfrac{dx}{dt} = \dfrac{64\sin\theta}{x}\dfrac{d\theta}{dt}$.

(3 marks)

(c) State the exact value for $\dfrac{d\theta}{dt}$ in radians per minute.

(1 mark)

(d) Find the rate at which SP is changing when S is 8 metres above the horizontal baseline.

(3 marks)

PLEASE TURN OVER

QUESTION 8 (8 marks)

The graphic shown in Figure 6 appeared on television on the evening of 26 January 2013. The red curve represents the path of the cricket ball that George Bailey hit over the sight screen during the Australia Day cricket match between Australia and Sri Lanka.

This figure cannot be reproduced here for copyright reasons.

Figure 6

Adding a set of Cartesian axes to the graphic allows the path of the ball to be modelled by a Bézier curve. This curve is shown in blue in Figure 7.

This figure cannot be reproduced here for copyright reasons.

Figure 7

Source for Figures 6 and 7: graphic from Australia vs Sri Lanka, Twenty20 International Cricket 2013, live telecast, Channel 9, ANZ Stadium, Sydney, 26 January (adapted for Figure 7).

A Bézier curve with starting point $S(x_0,\ y_0)$, first control point $C_1(x_1,\ y_1)$, second control point $C_2(x_2,\ y_2)$, and endpoint $E(x_3,\ y_3)$ has parametric equations

$$\begin{cases} x(t) = (x_3 - 3x_2 + 3x_1 - x_0)t^3 + (3x_2 - 6x_1 + 3x_0)t^2 + (3x_1 - 3x_0)t + x_0 \\ y(t) = (y_3 - 3y_2 + 3y_1 - y_0)t^3 + (3y_2 - 6y_1 + 3y_0)t^2 + (3y_1 - 3y_0)t + y_0 \end{cases} \text{ where } 0 \le t \le 1.$$

The blue curve in Figure 7 has control points $S(8,\ -7)$, $C_1(1,\ 6)$, $C_2(-4,\ 12)$, and $E(-7,\ 3)$, and for this curve

$$y(t) = -8t^3 - 21t^2 + 39t - 7.$$

(a) Find $x(t)$ for the blue curve.

(2 marks)

(b) Find the coordinates of the highest point on the blue curve.

(4 marks)

(c) Find the x-intercept for the blue curve correct to three significant figures.

(2 marks)

QUESTION 9 (9 marks)

Consider the quadratic iteration $z_n = z_{n-1}^2$ for $n \geq 1$.

Let $z_0 = r\text{cis}\theta$, where $r > 0$, and θ is real.

(a) (i) Show that $|z_1| = r^2$.

(1 mark)

(ii) Hence find $|z_2|$.

(1 mark)

(iii) Use an inductive argument to prove that, for $n \geq 1$,

$$|z_n| = r^{2^n}.$$

(3 marks)

(b) (i) Use an inductive argument to prove that, for $n \geq 1$,

$$\arg z_n = 2^n \theta.$$

(3 marks)

(ii) If $z_0 = r\operatorname{cis}\frac{\pi}{16}$, what is the least number of iterations such that z_n is real?

(1 mark)

SECTION B (Questions 10 to 13)

(60 marks)

*Answer **all** questions in this section.*

QUESTION 10 (15 marks)

Let $A(1, 0, -3)$, $B(3, -1, -1)$, and $C(7, -3, 3)$ be three points.

(a) Show that B divides AC internally in the ratio $1:2$.

(2 marks)

Figure 8 shows the plane P_1 through B with $\overrightarrow{AC}$ as its normal and equation

$$2x - y + 2z = 5.$$

The points $D(3, 5, 2)$ and $E(1, -3, 0)$ are on this plane.

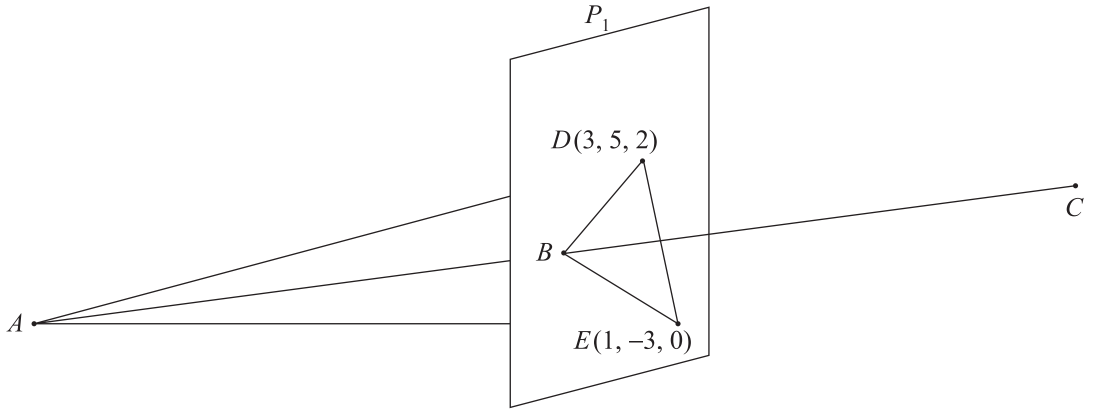


Figure 8

(b) (i) If $\overrightarrow{BD} \times \overrightarrow{BE} = k\overrightarrow{AB}$, find the value of k.

(2 marks)

(ii) Find the area of ΔBDE.

(2 marks)

(c) (i) Find the equation of the plane P_2 that contains the point $C(7, -3, 3)$ and is parallel to the plane P_1.

(1 mark)

(ii) Show that the line through points A and D has parametric equations

$$x = 1 + 2t$$
$$y = 5t$$
$$z = -3 + 5t.$$

(1 mark)

(iii) Show that the line through points A and D intersects P_2 at point $F(7, 15, 12)$.

(2 marks)

The line AE continues on to meet the plane P_2 at point $G(1, -9, 6)$ as shown in Figure 9.

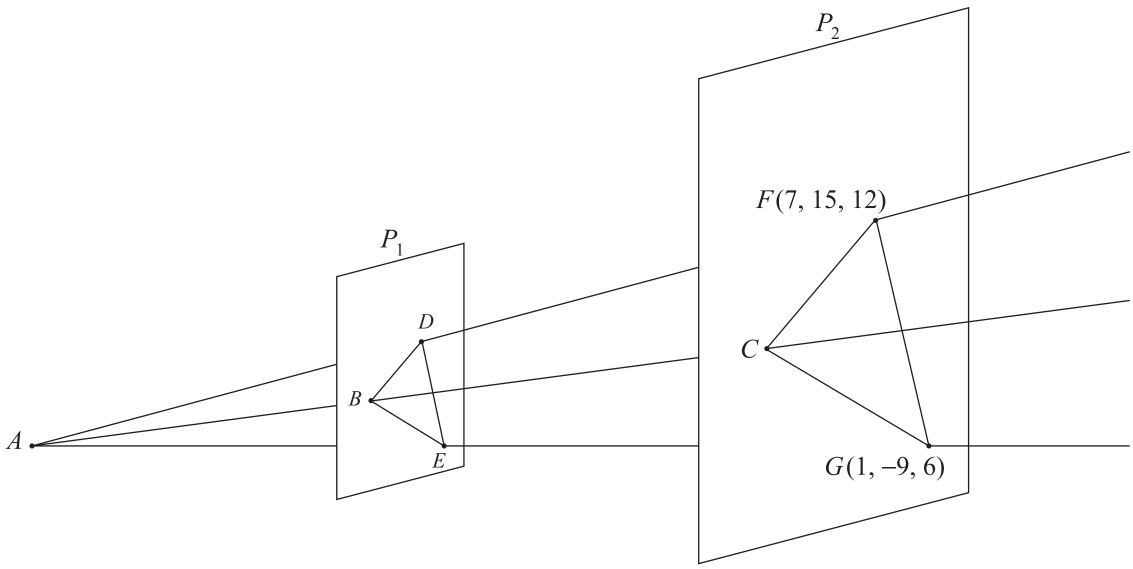


Figure 9

(d) (i) Find the area of ΔCFG.

(1 mark)

(ii) Find the distance between the planes P_1 and P_2.

(2 marks)

Figure 10 shows that the lines AB, AD, and AE continue on to meet the plane P_3, which is parallel to P_1 and P_2.

The equation of the plane P_3 is $2x - y + 2z = \lambda$, where λ is a constant.

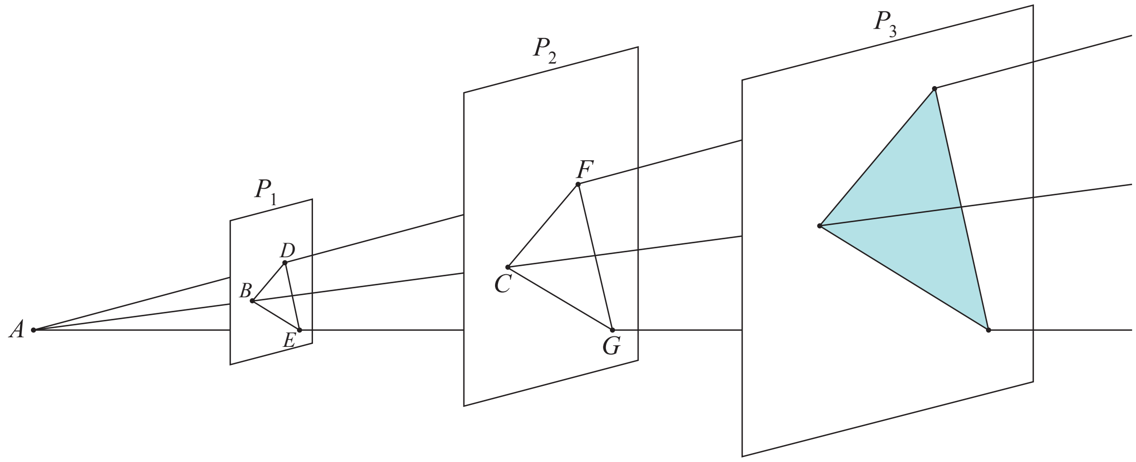


Figure 10

(e) Find the value of λ if the area of the shaded triangle that is projected onto P_3 is 16 times the area of ΔBDE.

(2 marks)

QUESTION 11 (14 marks)

In a routine hospital procedure a chemical is injected into the bloodstream at a constant rate of k grams per hour. The amount of chemical present in the bloodstream after t hours is represented by $y(t)$ grams.

As the chemical is injected, it is breaking down in the bloodstream. The combined effect on $y(t)$ can be modelled by the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}t} = k - \alpha y \text{, where } k \text{ and } \alpha \text{ are positive constants.}$$

(a) Solve the differential equation with initial condition $y(0) = 2$ and show that

$$y(t) = \frac{k}{\alpha} - \left(\frac{k}{\alpha} - 2\right)e^{-\alpha t}.$$

(5 marks)

(b) As $t \to \infty$, the amount of chemical present in the body approaches an equilibrium. Find an expression for this amount.

(1 mark)

(c) Figure 11 illustrates the slope field for $k = 13.5$ grams per hour, $\alpha = 1.5$, and $y(0) = 2$.

(i) Draw the solution curve on the slope field in Figure 11.

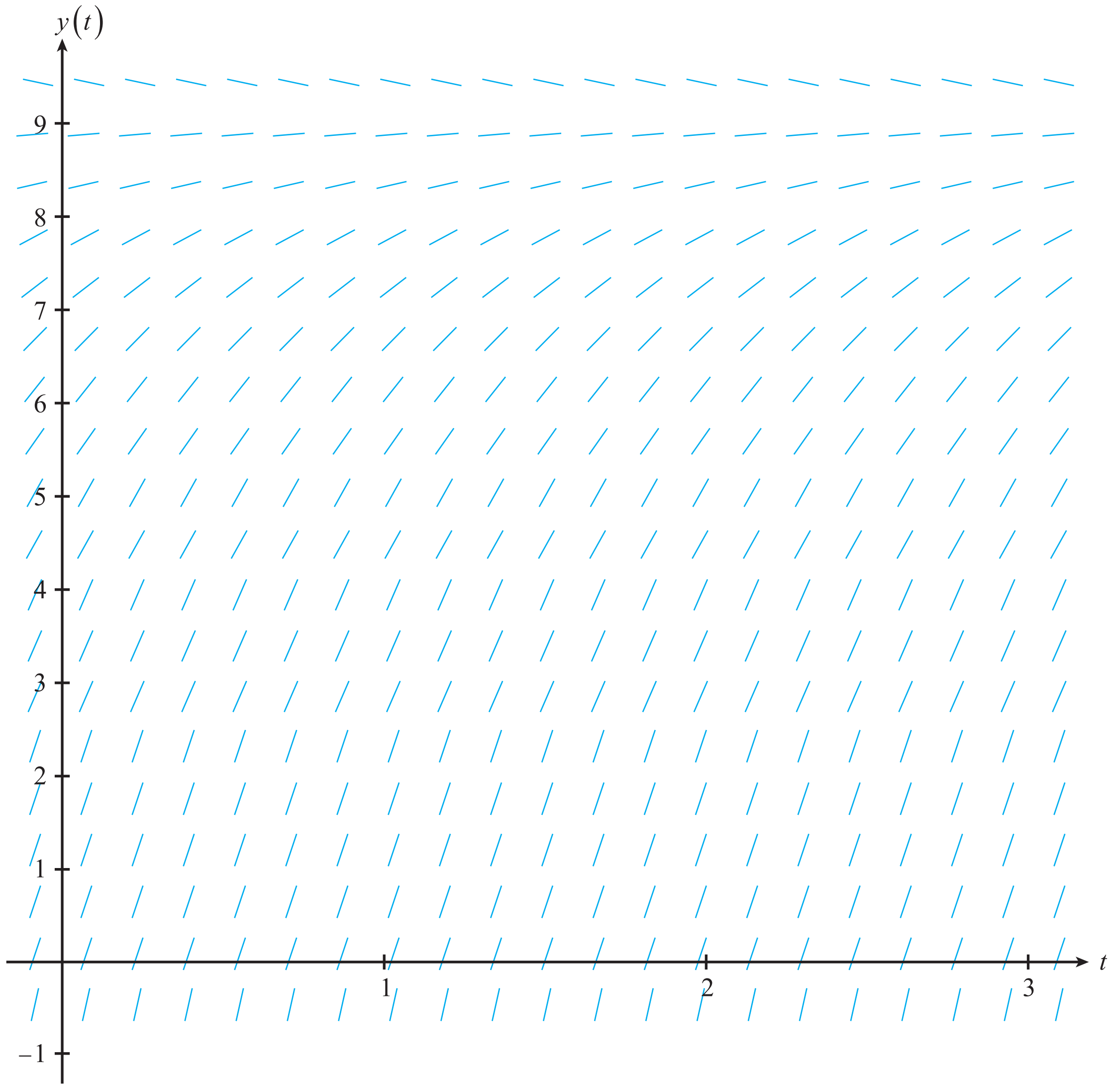


Figure 11

(3 marks)

(ii) Use your answer to part (b), or otherwise, to find the equilibrium amount in this situation.

Mark it on Figure 11.

(2 marks)

(d) (i) Show that $\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} = \alpha(2\alpha - k)e^{-\alpha t}$.

(2 marks)

(ii) If $k = 2\alpha$, find $\frac{\mathrm{d}y}{\mathrm{d}t}$.

(1 mark)

QUESTION 12 (15 marks)

(a) Consider the curve with parametric equations $x = \cos\theta$ and $y = \sin\theta$, where θ is a parameter.

(i) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

(2 marks)

(ii) Hence show that the tangent to the curve when $\theta = \frac{\pi}{6}$ has the equation

$$y = -\sqrt{3}x + 2.$$

(2 marks)

(b) (i) Write the following equation exactly in the form $r\text{cis}\theta$, where $r > 0$,

$$u = \frac{\sqrt{3}}{2} + \frac{i}{2}.$$

(2 marks)

(ii) Use De Moivre's Theorem to find u^3.

(1 mark)

(iii) Solve $z^3 = i$.

(2 marks)

(c) Show that the solutions to $z^3 = -8i$ are $2\text{cis}\frac{\pi}{2}$, $2\text{cis}\frac{-5\pi}{6}$, and $2\text{cis}\frac{-\pi}{6}$.

(1 mark)

(d) Plot the solutions of $z^3 = i$ and $z^3 = -8i$ on the Argand diagram in Figure 12. Label them z_1, z_2, z_3, z_4, z_5, and z_6, with z_1 in the first quadrant and arguments increasing anticlockwise from the positive real axis.

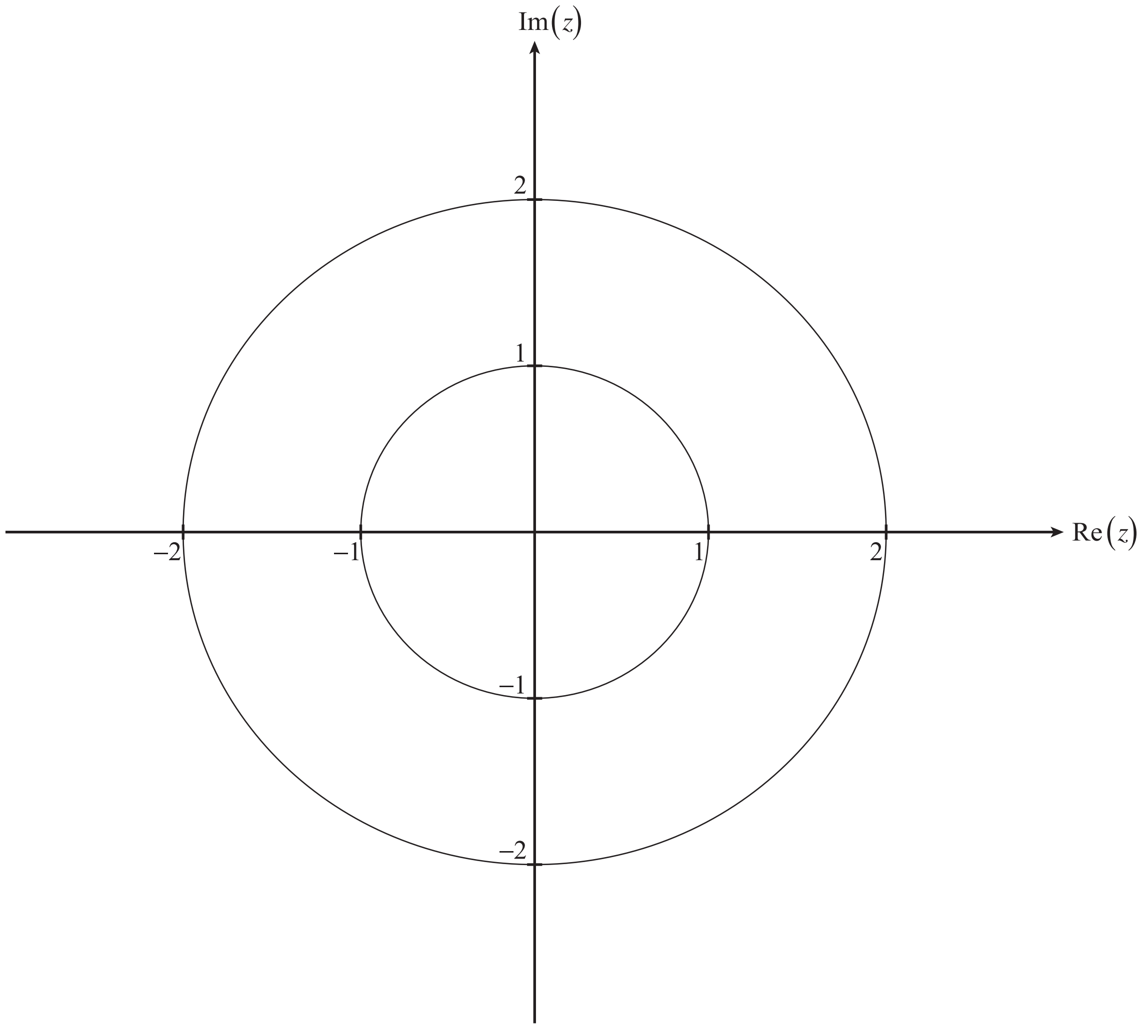


Figure 12

(2 marks)

(e) (i) Using your results for parts (a) to (d), or otherwise, show that:

$$z_1 = \frac{z_2 + z_6}{2}.$$

(1 mark)

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(ii) Show that the line joining z_2 and z_6 is a tangent to the inner circle at z_1.

(2 marks)

QUESTION 13 (16 marks)

The weight W, in grams, of a native wild bee hive has been modelled by entomologists as having a rate of change given by

$$\frac{\mathrm{d}W}{\mathrm{d}t}=\frac{1}{2}\frac{W(1000-W)}{1000}$$

where t is time in days.

Source: © Dole/Dreamstime.com

(a) The equations for Euler's method are

$$t_{n+1}=t_n+h$$
$$W_{n+1}=W_n+hW'(t_n)$$

For the differential equation given above, use Euler's method to complete the following table given $t_0=0$, $W_0=10$, and $h=0.1$.

You may complete all cells in the table if it helps you, but you do not need to complete the shaded cells. In the last column, state values of W_{n+1} correct to four significant figures.

n	h	t_n	W_n	W_{n+1} (four sig. figs)
0	0.1	0	10	10.50
1	0.1	0.1	10.50	11.01
2	0.1	0.2	11.01	11.56
3	0.1			
4	0.1			
5	0.1			
6	0.1			
7	0.1			
8	0.1			
9	0.1			
10	0.1			

(3 marks)

(b) From the table in part (a), state the value of $W(1)$ correct to four significant figures.

(1 mark)

(c) State how you would use Euler's method to produce a better estimate for $W(1)$ than that found in part (b).

(1 mark)

(d) (i) Show that $\dfrac{1000}{W(1000-W)} = \dfrac{1}{W} + \dfrac{1}{1000-W}$.

(1 mark)

(ii) Given the differential equation $\dfrac{dW}{dt} = \dfrac{1}{2}\dfrac{W(1000-W)}{1000}$, use integration and the initial condition $W(0)=10$ to show that

$$W = \frac{1000}{1+99e^{-\frac{1}{2}t}}.$$

(5 marks)

(iii) Graph the function $W = \dfrac{1000}{1+99e^{-\frac{1}{2}t}}$ on the axes in Figure 13.

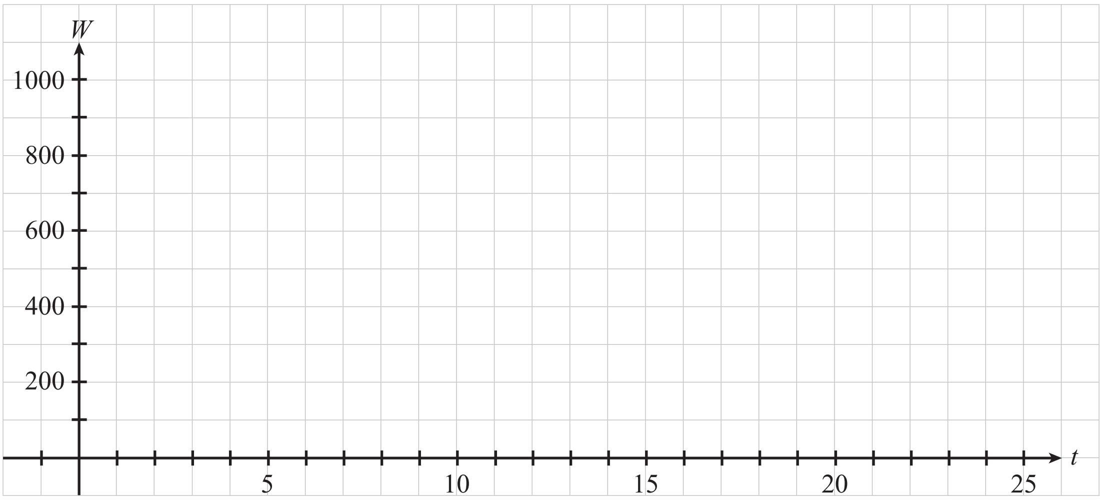


Figure 13

(3 marks)

(iv) On the graph you drew in part (d)(iii), mark the point B that corresponds to the maximum rate of change of the weight of the hive.

(1 mark)

(v) What is the value of the maximum rate of change of the weight of the hive?

(1 mark)

SECTION C (Questions 14 and 15)

(15 marks)

Answer ***one*** *question from this section,* ***either*** *Question 14* ***or*** *Question 15.*

Answer ***either*** *Question 14* ***or*** *Question 15.*

QUESTION 14 (15 marks)

A point, $P(x,\ y,\ z)$, is moving in three dimensions according to the parametric equations

$$\begin{cases} x = a\cos\omega t \\ y = a\sin\omega t \\ z = bt \end{cases}$$

where $a > 0$, b and ω are constants, and $t \geq 0$ is in radians.

(a) Show that $x^2 + y^2 = a^2$.

(1 mark)

(b) What does the equation in part (a) represent in the x–y plane?

(1 mark)

In three dimensions, the given parametric equations represent the path of the point $P(x,\ y,\ z)$ lying on a cylinder and moving along an anticlockwise spiral called a helix, as shown in Figure 14.

The velocity vector of point $P(x,\ y,\ z)$ is $\boldsymbol{v}(t) = \left[\dfrac{dx}{dt}, \dfrac{dy}{dt}, \dfrac{dz}{dt}\right]$.

(c) Find $\boldsymbol{v}(t)$ and hence find an expression for the speed of point P as it moves along the helix.

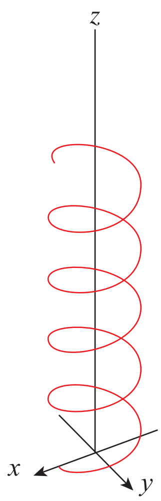


Figure 14

(3 marks)

(d) The arc length, L, along the helix from $t=0$ to $t=k$ is given by

$$L=\int_0^k \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2+\left(\frac{\mathrm{d}z}{\mathrm{d}t}\right)^2}\,\mathrm{d}t.$$

(i) For the first complete turn on the helix, from $t=0$ to $t=\frac{2\pi}{\omega}$, show that the arc length is

$$L=\frac{2\pi}{\omega}\sqrt{a^2\omega^2+b^2}.$$

(2 marks)

(ii) For the case when $a=b=\omega=1$, show that the arc length is $\sqrt{2}$ times the circumference of the circle in the x–y plane with centre $(0,\ 0,\ 0)$ and radius $r=a$.

(2 marks)

(e) Let $\boldsymbol{T}(t)=\frac{1}{\sqrt{a^2\omega^2+b^2}}[-a\omega\sin\omega t,\ a\omega\cos\omega t,\ b]$.

(i) Find $\frac{\mathrm{d}\boldsymbol{T}}{\mathrm{d}t}$.

(2 marks)

(ii) The curvature of the helix is given by $\kappa = \frac{1}{|\boldsymbol{v}(t)|}\left|\frac{\mathrm{d}\boldsymbol{T}}{\mathrm{d}t}\right|$. Hence show that

$$\kappa = \frac{a\omega^2}{a^2\omega^2 + b^2}.$$

(1 mark)

(f) Given the information in part (e)(ii) and that the curvature of the circle is $\kappa_c = \frac{1}{a}$, for what values of b is:

(i) the curvature of the circle the same as the curvature of the helix?

(1 mark)

(ii) the curvature of the helix less than the curvature of the circle?

(1 mark)

(g) What can you conclude from your answer to part (f)(ii)?

(1 mark)

You may write on this page if you need more space to finish your answers. Make sure to label each answer carefully (e.g. 'Question 9(b)(ii) continued').

Answer ***either*** *Question 14* ***or*** *Question 15.*

QUESTION 15 (15 marks)

A point moves away from its initial position $(1, 0)$ in the x–y plane so that its coordinates obey the differential system

$$\begin{cases} x' = 2x - y \\ y' = 10x - 4y. \end{cases}$$

It is known that the system has a solution of the form

$$\begin{cases} x(t) = e^{-t}\left(A\cos t + B\sin t\right) \\ y(t) = 10e^{-t}\sin t \end{cases}$$

where A and B are constants and $t \geq 0$.

(a) Use the given form of the solution to show that $x'(t) = e^{-t}\left((B-A)\cos t - (A+B)\sin t\right)$.

(2 marks)

(b) Use x, x', and the initial conditions to show that $A = 1$ and $B = 3$.

(2 marks)

Figure 15 shows the system's slope field with the solution curve drawn for

$$\begin{cases} x(t) = e^{-t}(\cos t + 3\sin t) \\ y(t) = 10e^{-t}\sin t \end{cases} \quad \text{where } t \geq 0.$$

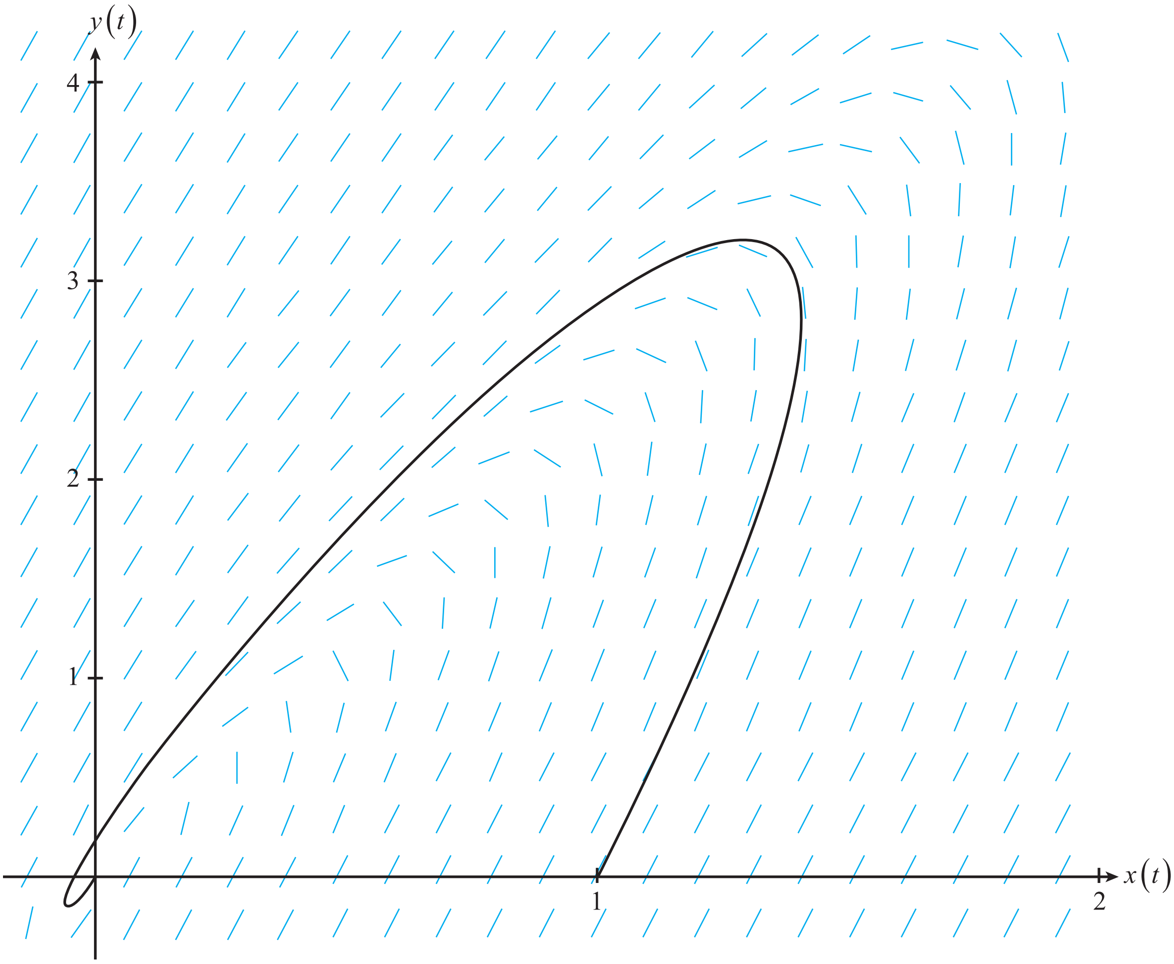


Figure 15

(c) Show that $t = \tan^{-1}\left(-\frac{1}{3}\right)$ at the y-intercept of the solution curve.

(2 marks)

(d) Find the exact value of the negative x-intercept of the solution curve.

(2 marks)

(e) (i) Use the given differential system to show that if $y' = 0$, then $y = \frac{5}{2}x$. Draw this line on Figure 15.

(2 marks)

(ii) Show that the highest and lowest points of the curve occur for $t = \frac{\pi}{4}$ and $t = \frac{5\pi}{4}$.

(3 marks)

(iii) Hence, or otherwise, find the coordinates of the highest and lowest points on the curve.

(2 marks)

You may write on this page if you need more space to finish your answers. Make sure to label each answer carefully (e.g. 'Question 9(b)(ii) continued').

You may write on this page if you need more space to finish your answers. Make sure to label each answer carefully (e.g. 'Question 9(b)(ii) continued').

You may remove this page from the booklet by tearing along the perforations so that you can refer to it while you write your answers.

LIST OF MATHEMATICAL FORMULAE FOR USE IN STAGE 2 SPECIALIST MATHEMATICS

Circular Functions

$\sin^2 A + \cos^2 A = 1$

$\tan^2 A + 1 = \sec^2 A$

$1 + \cot^2 A = \operatorname{cosec}^2 A$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin 2A = 2 \sin A \cos A$

$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A\end{aligned}$$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$2 \sin A \cos B = \sin(A+B) + \sin(A-B)$

$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$

$2 \sin A \sin B = \cos(A-B) - \cos(A+B)$

$\sin A \pm \sin B = 2 \sin \frac{1}{2}(A \pm B) \cos \frac{1}{2}(A \mp B)$

$\cos A + \cos B = 2 \cos \frac{1}{2}(A+B) \cos \frac{1}{2}(A-B)$

$\cos A - \cos B = -2 \sin \frac{1}{2}(A+B) \sin \frac{1}{2}(A-B)$

Matrices and Determinants

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $\det A = |A| = ad - bc$ and

$A^{-1} = \dfrac{1}{|A|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Derivatives

$f(x) = y$	$f'(x) = \dfrac{dy}{dx}$
x^n	nx^{n-1}
e^x	e^x
$\ln x = \log_e x$	$\dfrac{1}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$

Properties of Derivatives

$\dfrac{d}{dx}\{f(x)g(x)\} = f'(x)g(x) + f(x)g'(x)$

$\dfrac{d}{dx}\left\{\dfrac{f(x)}{g(x)}\right\} = \dfrac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$

$\dfrac{d}{dx} f(g(x)) = f'(g(x))g'(x)$

Quadratic Equations

If $ax^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Distance from a Point to a Plane

The distance from (x_1, y_1, z_1) to $Ax + By + Cz + D = 0$ is given by

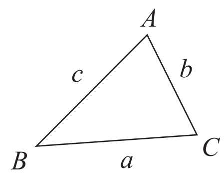
$\dfrac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$.

Mensuration

Area of sector $= \frac{1}{2} r^2 \theta$

Arc length $= r\theta$
(where θ is in radians)

In any triangle ABC:



Area of triangle $= \frac{1}{2} ab \sin C$

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc \cos A$