Specialist Mathematics

2012 Chief Assessor's Report

Government
of South Australia

SACE
Board of SA

SPECIALIST MATHEMATICS

2012 CHIEF ASSESSOR'S REPORT

OVERVIEW

Chief Assessors' reports give an overview of how students performed in their school and external assessments in relation to the learning requirements, assessment design criteria, and performance standards set out in the relevant subject outline. They provide information and advice regarding the assessment types, the application of the performance standards in school and external assessments, the quality of student performance, and any relevant statistical information.

SCHOOL ASSESSMENT

Assessment Type 1: Skills and Applications Tasks

Across the range of skills and applications tasks, students need to have the opportunity to provide evidence of their learning at the highest level of the performance standards. Most sets of tasks provided a balance of questions that ranged from routine questions or parts of questions to complex questions.

The proportion of complex questions in each test is likely to vary. For example, a test based on Topic 1: Trigonometric Preliminaries may consist of mostly routine questions. Some teachers identified and tagged the complex questions in each task. While not being a requirement, it does assist with the process of confirming the overall achievement level.

Overall, the sets of tasks allowed for the development of mathematical knowledge, skills and understanding as the year progressed. Most students and/or teachers presented the samples for moderation in the order in which the tasks were completed.

Almost all teachers provided a conventional marks scheme on each task. This technique assists students to recognise an appropriate approach (e.g. technology or algebraic method) and the amount of detailed mathematics or explanation required. Many teachers aggregated percentages on individual tasks to calculate the final achievement level for the assessment type. The decision reached must reflect achievement referenced to the performance standards, and not simply be a conversion from the percentage value.

It is helpful when teachers include information demonstrating the process that has been used to determine the grade awarded for the assessment type to support the moderators in their endeavour to confirm the assessment decision. A variety of cover sheets for individual student samples are available on the SACE website. These provide a quick summary of achievement within each assessment type.

Assessment Type 2: Folio

Most schools included two investigations to demonstrate achievement in this assessment type. The folio tasks were varied in their style. The better folio pieces were designed as investigations (i.e. not assignment style), which allowed students to provide evidence of their learning against the performance standards, particularly with respect to mathematical modelling and problem-solving. Most teachers assessed these tasks, using the performance standards, to provide students with feedback to support understanding of their achievement.

Investigations that are highly structured and arrive at a single solution may limit students' potential to achieve at the higher grades, as the results are predictable and limit the scope for higher-order problem-solving and interpretation of results.

Many teachers provided one folio task that included the development of a conjecture and its proof, with the other task including the development and application of a mathematical model. This approach ensured that students were provided with opportunities to achieve at the highest levels against specific features MMP1 and MMP5.

Students should present their work in a report format, with an introduction, an analysis section, and a conclusion. The introduction should explain the purpose and context of the investigation, outlining the problem to be explored and the method to be used in the investigation. The analysis section should consist of the mathematical working that leads to one or more solutions and/or any proof, with a clear explanation of the mathematical processes used, including discussion of any limitations to the investigation. The conclusion should be in the context of the original problem and provide an evaluation of the outcome of the investigation. Appendices and a bibliography should be included, as appropriate. Teachers and students should refer to pages 45 and 46 of the subject outline and to the description of the specific features at each grade in the performance standards.

From 2013, the specific feature MMP6, 'Constructive and productive contribution to group work', has been deleted. Therefore there is no requirement to assess group work in at least one investigation. However, teachers may include group work in one or more investigations, but the contribution each student makes to that collaborative work is not directly assessed.

OPERATIONAL ADVICE

It was a positive aspect of the quality assurance of school assessment that most schools followed the procedures for packaging and presenting materials. It is at times helpful to have access to a set of solutions for the skills and applications tasks so that the complexity of the tasks can be evaluated effectively. When student materials were placed in folders or plastic sleeves, the work of moderators was made substantially more demanding and this practice is discouraged.

Many learning and assessment plans were modified during the year as the teaching and learning program progressed. It is possible that these learning and assessment plans may be further refined and that an accurate addendum should be included with the submission of samples. It is sufficient to cross out the reference to working collaboratively (i.e. MMP6), rather than referencing this change in an addendum.

It is important that supporting evidence accompanies any variations in materials for the sample to explain how the grade was determined for the assessment type. For the students selected in the sample for final moderation, any variations to their assessment or sample (e.g. lost materials) need to be identified on the Variations — Moderation Materials form and included in the teacher's pack.

When classes are combined to form one assessment group, it is strongly advised that at least some common assessment tasks are used, and that the teachers collaborate to ensure that students' work is assessed with reference to the performance standards. Moderation of the assessment group is based on a sample drawn from all the classes that form the assessment group. If an adjustment is made, it applies to all students at those grade levels for an assessment type.

EXTERNAL ASSESSMENT

Assessment Type 3: Examination

SECTION A (Questions 1 to 9)

Question 1

A straightforward introduction to the examination; this was the best-done question in the paper. Full marks were achieved by 61% of students, while less than 1% gained no marks (this included 7 students who were unable to make an attempt). Common errors were: not using the chain rule when differentiating cos 3*t*, and an inability to respond to the direction for use of surd form. The average mark for this question was 88%.

Question 2

The average for this question was 64%, with 20% of the students achieving full marks. Those who lost marks did so because they took shortcuts in the 'show' parts of the question, missed the connection between parts (a) and (b)(i), were unable to carry out the direction to 'hence evaluate' in part (b)(ii), and/or were unable to effectively use the odd function property to do part (b)(iv). Eleven students made no attempt at this question, contributing to the 5% of the students who received no marks for this question.

Question 3

This appeared to be the 'scariest' question in the paper with 115 students not making any attempt, forming almost one third of the 26% of students who garnered no marks for this straightforward geometry question. This had a concomitant penalty of 8 marks, significantly higher than corresponding questions in previous examinations. The success rate for the geometry question was the poorest in the examination, suggesting that students would benefit from further experience in this area, as the impact of a similarly poor success rate if the geometry question were in Section B would be worth 15 marks.

Unsurprisingly, given the preceding paragraph, less than 8% of the students earned full marks, while the modal score was 0 marks and the mean mark was 39%. The more successful students used the following sensible strategies:

- giving reasons for all statements, using named theorems

- using conventional angle-naming methods, such as $\angle BAT$, or marking a Greek pronumeral such as α on the diagram, as modelled in the question by the use of β
- correctly identifying *reflex* $\angle AOC = 2\beta$ as a consequence of the angle at the centre theorem
- using a construction to extend one of the tangents and using the alternate segment theorem (a small number of students used this method)
- clearly understanding and using the properties of a parallelogram and a rhombus, and using them accordingly in part (c).

Question 4

Success in this question required students to 'show clearly'.

Many correctly found $\cos\theta = \frac{9}{3\sqrt{18}}$, but then did not *show* the required result.

$\cos\theta = \frac{9}{3\sqrt{18}} \quad \therefore \theta = \frac{\pi}{4}$ gained no credit, whereas either of the following did.

$$\begin{aligned} \cos\theta &= \frac{9}{3\sqrt{18}} \\ &= \frac{1}{\sqrt{2}} \\ \therefore \theta &= \frac{\pi}{4} \end{aligned} \qquad \text{or} \qquad \begin{aligned} \cos\theta &= \frac{9}{3\sqrt{18}} \\ \therefore \quad \theta &= \cos^{-1}\left(\frac{9}{3\sqrt{18}}\right) \\ &= \frac{\pi}{4} \end{aligned}$$

That being said, this question was quite well done overall. It was the most popular question, with only 3 students not making an attempt. Less than 2% received no marks, the mean was 70%, and 25% of the students earned maximum marks.

Question 5

Maximum marks also featured highly in this straightforward question on polynomials, with 47% of students achieving all of the 5 available marks. Again the requirement to 'show' was a source of lost marks, this time with students not being careful in the resolution of negative signs in their algebra. Students need to have a greater recognition of the requirement to present enough of the argument to display a full understanding of the reasoning involved. When a required result is given, it is not sufficient to present 'opening statement $\Rightarrow$ required results'; students are expected to do some mathematics, and present the details in the argument. There were 28 students who did not attempt this question, contributing to the very small percentage of non-achievers. The mean mark was 82%.

Question 6

While not reaching the heights of Questions 1 and 5, Question 6 had a pleasing mean of 67%, and full marks were obtained by 10% of students. The question required intensive use of technology, and a reasonable interpretation of 'long-term behaviour'. To be successful, students needed a good technological strategy for many repeated iterations, understanding that rounding did not come in until part (c), and graphical skills to position the six final rounded iterates on the graph. A good interpretation of long-term behaviour would see that two of the six final rounded iterates were repeats of previous values, adding weight to the evidence of 4-cycle behaviour. The need for good technology strategies may well have contributed to the 6% of students who gained no marks in this question.

Question 7

This question also had 5% non-achievers, including the 31 students who made no attempt. Many students did well; in fact, the modal group achieved full marks, and the mean for the question was 69%. There were three variations on the differentiation in part (c). The majority rewrote the root as a power, and then went through the manipulations to reach the required result. Some students squared the relationship proved in part (b) and used implicit techniques on both sides of the equation, and, in an even more elegant approach, some went back to their working for part (a) and started part (c) with:

$$S^2 = (x-2)^2 + y^2$$

$$\therefore \quad 2S\frac{\mathrm{d}S}{\mathrm{d}t} = 2(x-2)\frac{\mathrm{d}x}{\mathrm{d}t} + 2\mathrm{y}\frac{\mathrm{d}y}{\mathrm{d}t}$$

thus leaving a very simple division to establish the result.

Question 8

Many students performed well in establishing the results in parts (a)(i) and (ii), and the modal score of 5 out of a possible 9 reflects not only this but also that this group, like those before them, were not clear about the structure of an inductive argument, required for part (b). There are three steps, all of which need to be done in part (b), and thus display the student's knowledge and understanding:

1. State those cases which have been seen to be true (in this case $n = 2{,}3$ were proved and $n = 1$ is trivially true).

2. Construct a proof of an inductive step, either the general step from n to $n+1$, or a particular one, but not one done in parts (a)(i) or (a)(ii).

3. Conclude the argument with a comment about its inductive nature.

All three steps need to be included in part (b) because that is where the direction to 'use an inductive argument' is given. The mean for this question was 58%, the size of the non-achieving group was 6%, but, less pleasingly, the group of students who scored maximum marks was small at 7%.

Question 9

The maximum mark was obtained by 10% of students, who avoided the common mistake of assuming that points A, B, and C were collinear, which was not given. The other parts of this question that were a concern for the majority of students were parts (c) and (d). In part (c) many of those who used the $|z-c| = r$ notation forgot the need to put $\frac{3}{2} + 2i$ in a bracket, and some squared the radius. Of those who chose the Cartesian form of the equation of a circle, many did not mention $z = x + yi$ to properly connect the z required in the question to the x and y in their answer. Others had an additional i or two in their final answer, and, of course, there were students using this form who forgot to square the radius. In part (d), $l = r\theta$ was widely referred to, but this was in vain for the majority of students, who could not extract the appropriate value of θ from the question's context. Question 9 was the second-most shunned question in the paper, with 65 students not putting pen to paper. Whether they attempted the question or not, 9% of the students scored 0 marks. The mean score for the question was 57%.

SECTION B (Questions 10 to 13)

Question 10

The mean of 60% made this the best-done question in Section B, and, considering that the modal score was 12 marks out of the available 15, it is clear that the 2012 students had an acceptable grasp of the fundamentals of 3D vector geometry. This was the most popular question in the paper; only 5 students did not attempt it, and only 1% of students scored no marks, with or without an attempt. Students who made arithmetic or procedural errors in part (b) found part (c) very difficult. The choice of diagram involved identifying that B, C, and E were collinear and then, interpreting from the vector scalar multiples, that E lay outside of CB, closer to C, thus indicating Diagram 3 as the correct choice. Some students 'proved' the collinearity of points B, C, and E using a scalar triple product involving $\overrightarrow{OB}$, $\overrightarrow{OC}$ and $\overrightarrow{OE}$. This gives a multiple of the volume of tetrahedron OBCE, and thus a zero result leads to the deduction that three out of the four points are collinear, but *not necessarily* points B, C, and E. About one-third of the students worked correctly through parts (a) and (b), but less than one-fifth of these were able to analyse their results and gain the remaining marks in part (c); thus only 6% of students achieved the maximum 15 marks.

Question 11

Only 2% of the students gained all 15 marks for this question on the calculus of trigonometric functions. At the other end of the scale, 6% of students gained no marks and this included the 48 students who made no attempt at all. That being said, a large majority of students successfully negotiated parts (a), (b), and (c)(i), but the remainder of the question proved difficult for all but the very capable. The mean for this question was 50%.

Question 12

Unlike Question 10, which had a modal score of 12, Question 12 had a modal score of 3. Despite this, there were some consistently good answers from the more capable students, and the mean mark was 51%. There were 46 students who made no attempt and they helped make up the 7% of the students who earned no marks at all. For those who did make an attempt, part (a)(i) was well done, but in part (a)(ii), while many obtained $A+B=210$ by setting $x(0)=210$, those who did not use the given system to write $x'(0)=-x(0)+3y(0)$ and were unable to find the necessary second equation in A and B found it very difficult to earn any more marks. Some students used the technique in part (a) to do part (b), but too many who took this route to the answer stopped too soon and did not give the required expression for $y(t)$. Those who made use of the suggested 'hence' and wrote $3y=x'+x$ had an easier route to the answer and avoided the previously mentioned pitfall. Part (c) was well done by those who completed parts (a) and (b). Marks were lost by those who failed to graph the initial conditions properly or omitted the two graphs' labels, all of which were necessary requirements. Pleasingly, in parts (e) and (f), many students grasped the concept of equilibrium and made sensible recommendations based on their results. Maximum marks were achieved by 11% of the students.

Question 13

Maximum marks was achieved by 7% of students for this question on the algebra and geometry of complex numbers. This was counterbalanced by the 7% who achieved 0 marks whether or not they attempted the question, with the modal score again being 3 marks. The mean mark for Question 13 was 52%.

In part (a)(i), most students found the modulus correctly, but far too many were unable to correctly identify the argument. Significantly, those who drew a diagram showing the number's quadrant had a much greater success rate than those who attempted to find the $r\operatorname{cis}\theta$ form without a diagram. Part (a)(ii) again had too many ignoring the 'hence' in the question, and the response to part (b) was well below expectation for such an important, basic conceptual process. Part (c) had far too many casual attempts to 'show that'. It is recommended that teachers further emphasise point 4 on the cover of the examination booklet, which says:

Appropriate steps of logic and correct answers are required for full marks.

This area should be a focus for students throughout their work in this subject.

SECTION C (Questions 14 and 15)

In Section C, 40% chose Question 14, 56% chose Question 15, and 4% did not attempt either question, while 6% earned 0 marks whether they made an attempt or not.

Question 14

The mean mark for this question was 57%. The modal mark was actually 15, for those who chose to attempt this question, so answering this question was a good decision for some students. With just 1 mark allotted to part (a), many fell short of supplying all the necessary 'steps of logic' as they attempted to verify the given relationship. The majority fared better in part (b), being able to use part (a) after separating the variables, and, pleasingly, many students made correct use of modulus signs when integrating the logarithmic terms. The majority who got this far made good starts on parts (c) and (d), but found the later stages of each part quite difficult.

Question 15

The mean mark for Question 15 was 60%, while 9% of those who attempted this question earned maximum marks. The modal mark was 10, showing that those who chose this question had a good understanding of the vector derivation of Bézier curves. Generally students worked well through parts (a) to (e) and then found the applications required in parts (f) and (g) more difficult. It seems that the implementation of Bézier curves in three dimensions, which had apparently gone smoothly to this point, fell down at this stage, as too many used $y(t)$ rather than $z(t)$ to find a maximum height. In part (g), those who developed an expression for the correct length usually made good use of the graphics calculator to evaluate the length of the 3D curve.

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