Mathematical Studies

2013 Chief Assessor's Report

Government
of South Australia

SACE
Board of SA

MATHEMATICAL STUDIES

2013 CHIEF ASSESSOR'S REPORT

OVERVIEW

Chief Assessors' reports give an overview of how students performed in their school and external assessments in relation to the learning requirements, assessment design criteria, and performance standards set out in the relevant subject outline. They provide information and advice regarding the assessment types, the application of the performance standards in school and external assessments, the quality of student performance, and any relevant statistical information.

SCHOOL ASSESSMENT

Assessment Type 1: Skills and Applications Tasks

It was very pleasing to see a general improvement in the standard of the tasks used for school assessment this year. Most tasks included an appropriate balance between the routine questions that enable students to demonstrate their learning at the C level, and those questions that differentiate students' learning at the higher levels. An appropriate balance between question types usually provides clear evidence to support the grade allocated by the teacher.

However, a significant number of sets of tasks did not include sufficient higher-order questions to enable students to provide evidence of their learning at the A level. It is advisable for each task to include some questions that extend the student beyond routine problem-solving. Examples of such questions can be found in past external examination papers and teachers could model some of their questions on these.

Teachers used a range of methods to determine the grade allocated to students. It is important to appreciate that the performance standards define the grade boundaries and — whether a spreadsheet or percentages are used to determine the grade for individual tests — the final grade should reflect overall achievement against the performance standards. Teachers are advised to reflect on the final grade awarded by looking for evidence in the skills and applications tasks that demonstrate learning against the performance standards. Moderation will not change the rank order in a class and so, if the process used to determine grades is inaccurate, it can affect the moderated grades of a significant number of students.

An A+ grade should be awarded to students who have consistently demonstrated their mathematical knowledge and skills in solving non-routine problems and reflected on their solutions to problems. It is not expected that they will be successful at every opportunity presented but there should be no weakness in any topic in a collection of tasks.

A C grade should be awarded when there is evidence of mostly correct solutions to the routine questions over a range of topics, with the general use of appropriate notation, representations, and terminology, and the development of some logical mathematical arguments.

Assessment Type 2: Folio

It was again pleasing to see a range of interesting and innovative folio tasks that enabled students to demonstrate their learning at the higher levels. These tasks included more open components that required students to select the appropriate mathematical tools and strategies in their problem-solving. In these cases evidence was usually clearly provided to support the grade allocated by the teacher.

Most schools provided one task that enabled students to develop a conjecture and then progress to a proof. The other task included mathematical modelling.

It is important for students to understand the process of developing a conjecture — usually from the recognition of a mathematical pattern from a series of results. Good communication is essential here, as the pattern could be demonstrated by summarising the results of calculations in an appropriate table and repetitive calculations could be minimised by using appropriate technology or by the inclusion of an appendix. Care needs to be taken that the task design does not direct all the students to the same mathematical result; it is preferable for the task to include opportunities that enable students to differentiate their responses.

Mathematical modelling requires an experimental approach in which real-world problems can be translated into mathematics. In constructing the model the student should explain the process that has been used to develop the model and justify the model that has been chosen. In using the model to solve the problem the student should interpret the solution in the real-world situation and recognise the limitations of the processes used. Too many tasks did not provide an opportunity for the student to develop a mathematical model, and this limited achievement to a C level in this criterion.

A significant number of tasks this year were highly directed, resulting in the presentation of responses in an extended assignment format and not in report format. The report format is an important component of the folio task. In those tasks that start with a directed component the report format may not be appropriate until the more open components of the task are reached and the students are required to make choices in their approach to problem-solving. When the report format is not used, students are rarely able to achieve at the higher levels.

Moderation seeks to confirm the grade assigned by the teacher by considering the evidence provided in the student packages. Teachers should provide information about how the grade was determined by marking or annotating the student's work. Without this assistance the moderator is relying solely on the student's communication skills to evaluate the task and if the evidence is not apparent the grade cannot readily be supported.

OPERATIONAL ADVICE

Most teachers provided with the student package a cover sheet that summarised the materials, the student performance in each task, and information about how the grade for each assessment type had been determined. Several examples of suitable cover pages are available on the Mathematical Studies minisite. It is unnecessary to tape up the individual student packages. When cardboard folders are used, the student details should be on the outside of the folder.

The lack of evidence in some of the incomplete packages is still a significant problem. The Variations — Moderation Materials form should give the reason for the missing task and, if possible, a mark or grade for the task if the work has been mislaid. If the task has not been submitted, the form should include an explanation of the circumstances and, if appropriate, the penalty applied by the teacher, to ensure that the students have been treated equitably.

When classes have been combined for moderation purposes, it is important for common assessment tasks to be used if possible, and for teachers to use a common marks scheme for skills and applications tasks and collaborate to confirm their assessment standards for folio tasks. This process is helpful when the rank order and the results for each assessment type are determined.

EXTERNAL ASSESSMENT

Assessment Type 3: Examination

Question 1

This question gave students a chance to demonstrate some fundamental calculus skills. They did so most successfully, with more than half earning full marks and another quarter losing only 1 mark. The marks that were lost were commonly in part (b) with the integration of $(ax + b)$^n or the omission of the constant of integration. Negative and fractional indices were the aspects of part (a) that troubled some students. The failure to use brackets in answering part (b)(ii) made many answers technically incorrect. This omission was not penalised in this question this year, but students need to make more of an effort to attend to such requirements.

Question 2

This question was similar to Question 1 in intention but focused on fundamental skills with matrices. Again, more than half the students earned full marks and another quarter earned 8 or 9 out of 10. Some students struggled to perform the ‘by hand’ matrix multiplication in part (b). There were some errors in part (c) despite the formula for the inverse of a 2×2 matrix being provided on the formula sheet. An interesting and surprising response in part (d)(i) was to convert $-4n - 4$ to $4n + 4$. It was not clear whether this was because of the ‘modulus’ notation for the determinant, because the question asked for an answer in the form $an + b$, or because of an association with equation-solving.

Question 3

Parts (a) to (d)(i) provided students with a simple application of the mathematics associated with confidence intervals for p. More than 60% of students earned 6 out of 6 for this work. Part (d)(ii) focused on the connection between the threshold for the distribution of p hat being approximately normal ($np > 10$) and the calculation of a valid confidence interval, as outlined in Subtopic 1.7. Although it was pleasing that 15% of students were familiar with this important aspect of the course, and so earned the 2 marks for part (d)(ii), it was clear, and somewhat disappointing, that most students saw no connection between the value of np and a confidence interval for p.

Question 4

This question was a relatively simple example of working with a relation, using a range of calculus techniques. Students generally handled this with confidence. Two-thirds of all students earned 6 or 7 out of 7. Marks were lost in part (b) through: errors of substitution and simplification; a failure to take the negative reciprocal slope; or not providing the answer in slope/intercept form or general form. This last error was relatively infrequent, but was penalised when it occurred.

Question 5

This question presented students with an unfamiliar, and yet quite simple, question. It looked like a calculus question but actually was about generating and solving systems of linear equations. Many students were thrown by this lack of predictability, with 30% earning 0 or 1 mark. This question produced the highest percentage of marks of 0 or 1 in the examination, suggesting that these students had had a limited exposure to questions structured in less obvious ways. Most students who attempted the question had a good degree of success, with 40% earning 6 or more marks out of 8. In part (c) a mark was lost by those who solved the system of equations but did not find the equation of $f(x)$ as requested. Part (d) provided the greatest degree of challenge in this question. Although a good number of students determined that four tangents were needed, fewer could express the relationship between unknowns and equations (eight equations were needed and, as each tangent provides two equations, four tangents were needed).

Question 6

This question presented students with some relatively straightforward tasks in parts (a) to (c), but also tested their ability to work meaningfully with a rate graph and to relate signed definite integrals to distances in a kinematic context. As a result many students experienced a good degree of success, with more than two-thirds earning better than half marks but less than 20% earning full marks. The graph in part (a) was drawn well, but a number of students did not see the connection with part (b) and provided only one or two roots of this equation, despite having drawn all three on the graph. In part (d) students needed to do more than just mention displacement. They needed to interpret negative value in the context of the saw's motion.

Question 7

In general, students handled the solution of a system of row operations very successfully. Some lost a mark for not providing sufficient definition of their row operations, and the final row operation, involving a parameter, was a challenge for some. The final part, where students needed to show a substitution of $z = 1$and then solve the resultant equation for k, was the other element of challenge in this question. Overall, two-thirds of students earned more than half marks and nearly a quarter earned full marks.

Question 8

This question combined the more straightforward 'rectangle overestimate' algorithm with some more demanding calculus and some use of logarithmic laws. A number of students lost marks with the drawing and calculation with rectangles, suggesting a lack of practice in this procedure. Part (c) tested the connection between integration and differentiation. Those who saw this connection and differentiated the right-hand side generally scored well. It was pleasing that most students attempted part (d), even those who were unsuccessful in part (c). Full marks were earned only by those who were able to work with logarithms — which remains a differentiating skill in terms of student performance. This range of degrees of challenge meant that students lacking in some areas were exposed, with 46% earning less than half marks, but those who were more accurate and confident with calculus concepts were rewarded, with nearly 40% earning 7 or more out of 10 and more than 15% earning full marks.

Question 9

This question tested students' knowledge of the links between a function and its derivative via their graphs. Despite this relationship having been regularly assessed in past examinations, a number of students seemed ill prepared for this question, with one-third earning 2 or fewer marks out of 8. Part (a) was handled best. In part (b) many students overlooked the words 'most rapidly' and gave the interval from *b* to *d*. The use of interval notation in these parts was very varied. Although they were not penalised this year, many students were not able express themselves using this standard aspect of mathematical communication. In part (c) students commonly were successful in locating the roots of the graph. It was least common to show the graph's asymptotic behaviour. In part (d) the common error was to mark *a*, *b*, *c*, *d*, and *e* on the axis, rather than just the critical values of the second derivative.

Question 10

This question gave students an opportunity to show some fundamental skills in the use of the binomial distribution, as well as to do some problem-solving with it. The skills in parts (a) and (b) were generally executed well, although the need to use a complement to calculate a probability in part (b) was a source of error for many. The problem-solving approach required in part (c), being a trial-and-error structure, was not one that many students were comfortable with. Those who were, were quite successful, although not all showed sufficient working to gain all 3 marks. It was necessary for students to show 'the greatest number', which meant that they needed to show that 241 was satisfactory and that 242 was not. In terms of student performance, more than 5% earned no marks, suggesting that some students have 'given up' on statistics and/or 'wordy questions', despite the simplicity of part (a). Two-thirds of students earned 3 or more marks out of 7, showing that their fundamental skills were sound. Only 20% earned 5 or more out of 7 and only 10% earned full marks, showing that problem-solving skills were a discriminating factor.

Question 11

This question was accessed successfully by students with sound fundamental skills, but contains some points where students with a greater depth of knowledge could distinguish themselves. The skills shown in parts (a), (c), (d), and (e) were generally good, although many students presented minimal evidence of their *Z*-test and risked losing marks. A more formalised algorithm would have been appropriate for many. Greater depth of knowledge was called for in parts (b) and (f). Only 2% to 3% of students clearly asserted that the null hypothesis was a statement about a population by using terms such as 'in general', 'overall', or 'on average' before stating that there was no difference in pain tolerance as a result of language use. Part (f) was better answered, but a lot of students did not see how the confidence interval for mu related to the result of a *Z*-test for mu. For the reasons given above, 64% of students earned 6 or more out of 10, but only 18% earned 9 or 10 out of 10, and only 2% of students earned full marks.

Question 12

This question involved a range of fundamental calculus skills and presented a relatively accessible opportunity for students to make a conjecture and undertake its proof. The quality of curve-sketching in part (a) was variable, with many student responses weakened by failure to indicate the vertical asymptote as requested. Parts (a) to (e) were generally done well. Some students opted to used algebraic methods in parts (b) and (c) — which seemed unwarranted, given what was asked for and the number of marks allocated. It was very pleasing that many students attempted the proof in part (f), and that fewer students attempted to ‘prove’ by checking further individual cases. Overall, 60% of students earned 6 or more out of 10 and 24% earned full marks.

Question 13

This question called for both reasoning and computation with the central limit theorem. Students attempted both tasks well, and those who had a deeper understanding and more precise problem-solving skills were able to distinguish themselves. The reasoning in parts (a) and (b) was not always well expressed. Student responses in part (b) were weakened, focusing only on a sample size of 25 and not comparing it with what was known about a sample size of 15. Many students compared it with a sample size of 30, which was most unhelpful to them in this question. Most students attempted part (d) but there were a range of errors. Often limited marks could be awarded as insufficient working-out was shown as evidence of student knowledge. As a new distribution was being used here, it would have been most beneficial for students to state the distribution they were working with, as a means of clarifying their thinking and earning marks for understanding. Very few did so, and most relied solely on ‘getting it right’. Overall, more than half of all students earned more than half marks, 30% earned 7 or better, and 12% earned 9 out of 9.

Question 14

Overall, students attempted this question well. Its gradation of complexity meant that relatively few students were able to work through to the point where they could meaningfully tackle part (e), but most students made meaning of most of the question. A lack of careful reading caused too many students to overlook part (c)(ii). In part (e) many students calculated the water stored in the first part of the day but failed to appreciate the periodic nature of the water use and incorporate the water stored later in the previous day. Overall, 56% of students earned between 3 and 7 marks out of 10, 17% earned more than 7, and only 4% earned 10 out of 10.

Question 15

This question contained the greatest degree of procedural complexity in the examination and assessed students’ algebraic skills most rigorously. Parts (a) and (b) were quite straightforward, with three-quarters of students earning at least 3 marks for this question. Part (c) required an interpretation of two inflection points, something that many struggled to express clearly. It was pleasing that most students attempted part (d) and earned at least 1 or 2 marks, with many earning all 4 marks. Part (e) was the section most commonly left blank in the examination, perhaps unsurprisingly, given the complexity of what was being asked. There were many ways to complete the task set. Those who solved the second derivative to zero successfully — no mean feat — were able to determine the k value easily. Those who substituted $k = 4$ into the second derivative and then solved to zero had an easier time of it initially, but had to work harder to earn the final mark. When presented with two *k* values, they needed to clearly express which one was appropriate, by means of graphs shown or equivalent. Very few of these students provided the necessary detail. Overall, 46% of students earned 6 marks or more out of 12, 19% earned 10 or more, but less than 2% earned full marks, in keeping with the complexity of parts (d) and (e).

Question 16

This unfamiliar application of matrices was received very well by students. Some who had done poorly in other parts of the examination responded very positively to this question. Such success reinforces the need for students to manage their time well and work efficiently so that they have the time to immerse themselves in later questions that require thought but offer excellent rewards for mathematical thinking. Parts (a) to (c) of this question were handled well by many students, with more than half of all students earning 7 or more marks out of 14. The problem-solving required in parts (e) and (f) meant that this question provided some differentiation between students. A quarter of students earned 12 or more marks and, pleasingly, more than 10% earned full marks.

GENERAL COMMENTS

Some general observations can be made about student performance in the examination. The following four areas diminished the performance of students this year: effective communication of mathematics, use of technology, thorough knowledge of the curriculum, and examination technique.

The communication of mathematical information is an important part of the Mathematical Studies course. Mathematics has a language and a notation that students need to become proficient in, if they are to express their thinking clearly in the examination. Examples include using brackets when expressing a product function, expressing a linear function in a widely recognised form, drawing a sign diagram, and using interval notation. This last example was a particular weakness in many students' responses. The showing of working is another aspect of communicating mathematical information. This was a weakness for some students when using technology to answer statistical questions. The structuring of a *Z*-test and the recording of a distribution in cases where one has not been defined by the question (e.g. Question 13 (d)) are important in completing a full solution and having the best chance for thinking to be rewarded.

Although most students used technology in this examination with a good degree of success, there were areas where more efficient use would have saved time and provided accurate results. Considering what is asked and the marks allocation can guide students towards the efficient use of technology, as, for example, in parts (b) and (c) of Question 12. Most graphics calculators can perform all the calculations involved in *Z*-tests. This would avoid errors and let students focus on representing their test in a logical manner. The use of technology as a graphing tool is one of its paramount applications. That being said, technology does not replace mathematical thinking. The setting of sensible view windows and the identification of vertical asymptotes are examples of where thinking needs to accompany the use of technology.

It is not always easy for students to work at their best in examination conditions. Gaining familiarity with working in comparable conditions before the final examination is obviously important, as it will help to develop good examination technique. Looking for the connection between parts of questions can be helpful, as, for example, when a graph is drawn and the roots of an equation are asked for subsequently. It is highly likely that these tasks are connected and that their answers should fit together. It is important to make sure that answers actually supply what the questions called for. Rereading the question before moving on is one way to detect any such oversights. Rereading may also reveal any overlooked tasks, some of which may, for example, involve annotating a graph and so may not have a grid box allocated to them.

An examination is an assessment of curriculum knowledge and obviously the strengths and weaknesses in this knowledge will vary greatly from student to student. Areas that were weaknesses for larger groups of the cohort were noted, however. In calculus the use of rectangles to overestimate (or underestimate) an area is a relatively simple algorithm that seemed unfamiliar to many students. The use of logarithmic laws was also an area of common weakness. In statistics there were a number of areas of weakness. Students working with the binomial distribution too often err when using a complement. Many are uncomfortable or unfamiliar with using trial and error to complete a statistical (or other) solution, and attempt convoluted and illogical use of random formulae in its place. The criterion that np must be greater than 10 in relation to the content of Subtopics 1.6 and 1.7 was definitely not widely known. As in previous years, a clear understanding that intervals and tests are about populations, and not an individual called ‘mean’, was not developed in many students. Greater practice in interpreting information in these areas may help to develop this important idea.

Mathematical Studies
Chief Assessor