Mathematical Studies

2012 Chief Assessor's Report

Government
of South Australia

SACE
Board of SA

MATHEMATICAL STUDIES

2012 CHIEF ASSESSOR'S REPORT

OVERVIEW

Chief Assessors' reports give an overview of how students performed in the school and external assessments in relation to the learning requirements, assessment design criteria, and performance standards set out in the relevant subject outline. They provide information and advice regarding the assessment types, the application of the performance standards in school and external assessments, the quality of student performance, and any relevant statistical information.

SCHOOL ASSESSMENT

Assessment Type 1: Skills and Applications Tasks

Most sets of skills and applications tasks had an appropriate balance of questions, from those that tested the routine skills to those that required a higher level of mathematical knowledge, skills and applications, interpretation of results, and testing of conjectures with proof. Some tasks did not provide a suitable range of questions to enable students to provide evidence at the highest achievement levels. Appropriate examples of questions that enable the students to provide evidence across a range of achievement levels, including to the highest level, can be found in the past external examination papers; teachers could model some of their questions on these. Conversely, when tasks consisted of predominantly complex questions, it was difficult for some students to demonstrate their learning as they were not able to engage with many of the questions. When there was an appropriate balance between routine and complex questions the achievements of students could be more accurately matched to the performance standards.

Most teachers used percentages when assigning the final grade but they should be aware that this can be deceptive, depending on the difficulty of the tasks. Reference to traditional grade boundaries, without due consideration of task difficulty, can lead to an allocation of grades that do not relate to achievement against the performance standards. When assigning a grade for a student it may be appropriate to minimise the impact of one skills and applications task if the student's performance was significantly different from his or her performance in the other tasks. Extenuating circumstances can affect performance in one task. Similarly an A+ grade does not require perfection in all skills and applications tasks, when a time limit may not enable the student to review his or her responses. However, A+ achievement does require the student to demonstrate no weakness in any topic within a collection of tasks.

A C– grade should be awarded when there is evidence of mostly correct solutions to routine questions in a range of topics, with the use of generally appropriate notation, representations, and terminology, and the development of some logical mathematical arguments.

Assessment Type 2: Folio

It was pleasing to see a range of interesting and innovative folio tasks this year; these must serve to enrich the learning experience for the students. Most schools included in their learning and assessment plan two folio tasks, at least one of which focused on the development of a mathematical model, as this is not usually effectively assessed in skills and applications tasks. The folio tasks were usually assessed directly against the performance standards, with the final grade reflecting the student's performance in the two tasks. An overall A+ grade for the assessment type would be expected to reflect a high achievement in both folio tasks.

When designing a task it is useful to include a more directed component that enables students to achieve a C grade, with successively more open components that allow students to demonstrate their achievement at the higher levels. Highly directed tasks rarely provide sufficient flexibility for the responses to differentiate between the achievement of the students. In particular, such tasks significantly limit the opportunity for the student to demonstrate achievement at higher levels.

Some teachers are using materials that were on the SACE website but have since been removed. Generally, these tasks did not give students opportunities to reach the higher grades when assessed against the performance standards, often as a result of their directed nature. The support materials are continually being updated and teachers should make sure that the tasks they select from this resource are current and appropriate for inclusion in the folio in its current format.

The report format is an important component of the folio task. There should be an introduction that outlines the problem to be explored and the method used in the investigation, an analysis section that shows the detail of the investigation and includes a discussion of the reasonableness and possible limitations of the results, and a conclusion that summarises the results of the investigation. Reports that are written as a response to an extended assignment limit the opportunity to differentiate between the achievement of students and rarely enable students to achieve at the higher levels.

OPERATIONAL ADVICE

Moderation seeks to confirm the grade assigned by the teacher by considering the evidence provided in the student packages. This year most of the tasks assigned by teachers enabled the students to effectively demonstrate their achievement against the performance standards, with the result that significantly fewer adjustments were made. A significant number of upward adjustments were made, to bring the school assessment into line with the assessment standards across the subject. It was sometimes necessary to adjust results downwards, often as a result of poor task design that did not enable the students to demonstrate their achievement at the higher levels. Teachers should be aware that in the moderation process the rank order that has been assigned by the teacher is preserved. When an adjustment is made, all the students at that grade level are adjusted in the same manner.

A significant problem was the lack of evidence provided in some of the incomplete student packages. The Variations — Moderation Materials form should give the reason for missing tasks and, if possible, a mark or grade for the task if it has been mislaid. If a task has not been submitted the form should include an explanation of the circumstances and, if appropriate, the penalty applied by the teacher, to ensure that the students are treated equitably.

Most teachers provided with the student package a cover sheet that summarised the materials, the student performance in each task, and information about how the grade for each assessment type was determined. Several examples of suitable cover pages are available on the SACE website. It is important that the grade assigned is the same as the one that was submitted to the SACE Board on the results sheet; some discrepancies were noticed. Moderators are advised to assume that the signed results sheet is correct.

When classes are combined for moderation purposes it is important that as many common assessment tasks as possible are used, and that the teachers use a common marks scheme for skills and applications tasks and collaborate to confirm their assessment standards for folio tasks. This process is helpful when the rank order and results for each assessment type are determined.

EXTERNAL ASSESSMENT

Assessment Type 4: Examination

Question 1

This question gave nearly all students a chance to start the examination with a display of their core skills. More than half the students got full marks for this question, and more than 80% got 6 or more marks out of 7. In part (a)(ii) students who used the quotient rule were most successful, with students who used the product rule tending to make more procedural errors. Given that the formula for the inverse of a 2×2 matrix was provided, it was surprising that in part (b) students made errors such as failing to transpose elements within the matrix.

Question 2

This question gave students more opportunities to demonstrate their skills in applying a routine process. It was positive to see half of all students earn full marks and a quarter of students earn 4 marks out of 5. Some students lost marks through making numerous algebraic errors, the most common of which was the erroneous product $a\times0=a$. In part (b) some students did not correctly indicate that the solutions were of the ‘not equals to’ variety. Overall, the most common error was to solve the quadratic equation in part (b) in such a way as to find only one solution, missing the $a=0$ solution. In spite of these issues for some students, the cohort in general showed confidence in this aspect of the course.

Question 3

For most students this question assessed an aspect of the content with which they were familiar and confident. Most students earned 5 or 6 marks out of 6 and handled all parts of the question well; one common error made by these students was to round their working values in part (b) to two decimal places, rather than to round their final result to two decimal places, therefore causing rounding errors. Part (d) was handled well, with students making a range of correct comments. A few incorrectly equated the lower estimate with the definite integral, but many handled the inequality or limit nature of the relationship well.

In comparison with the success of most students, a significant minority found this question quite problematic; 30% of students earned 2 or fewer marks out of 6. Their responses made it clear that these students were unfamiliar with the use of rectangle sums to approximate the area under a smooth curve.

Question 4

This was another opportunity for students to demonstrate their skills in working with matrices. Nearly all the students did so successfully, with 95% earning 3 or more marks, a quarter earning 5 out of 6, and half earning full marks. A portion of those who lost marks did so in part (c) because they did not specifically identify the 'mismatch' between the 'column number' (the length of the rows) in matrix A and the 'row number' (length of the columns) in matrix B. Some students provided a response that seemed to have been copied from their notes without the necessary application to the question at hand. The other error that caused many students to lose marks was the failure to provide what was asked for in part (d); students wrote down dimensions rather than a matrix as required.

Question 5

This question gave students an opportunity to develop and use a relatively straightforward mathematical model for the cost of breakfast items. Most students used their model to obtain results with a good degree of success; 93% of students earned 2 or more marks, and 55% earned 5 out of 6. Only 10% developed their model with clearly defined variables as requested, and hence earned full marks.

Question 6

This question involved a context in which students could demonstrate some routine skills with the normal distribution, which they did well, as well as use the *z*-score formula to find an unknown parameter, which they did less well. As a result, nearly half of all students earned 4 or fewer marks out of 8. Those confident with the algorithm required in part (d) performed well, with nearly a quarter of students earning full marks. Marks were also lost in answers to part (e), which called for comments from students. A range of responses were accepted, provided that they referred to the investments, rather than to the normal distributions that modelled them.

Question 7

The relationship between the graph of a function and the graph of its derivative continues to be one of the differentiating concepts in the calculus topic. Most students struggled to earn the marks for their graph in part (b), and were only slightly more successful in deducing information about the sign of the first and second derivative from the graph of the function. For this question 40% of students earned 0 or 1 mark and two-thirds of students earned 3 or fewer marks out of 5. As a result, students who were confident with this style of question were able to distinguish themselves well, with a quarter of students earning full marks.

Question 8

This question asked students to calculate, graphically represent, and interpret confidence intervals within the context of battery life. The calculation was done well, with most, but not all, students using technology to efficiently earn these marks. The representation was handled well, given that it was something new to most students. End points were marked accurately at most times. Some responses were flawed when students either used a bell curve to join the end points, or used nothing to join the end points and indicate an interval. A number of students did not indicate the sample mean as requested, but this was not penalised this year. The interpretation provided a greater degree of differentiation, with many students struggling to show a complete grasp of the mathematics at hand. The interpretation in part (d)(ii) was handled better than the one called for in part (b)(ii). In part (d)(ii) students generally concluded that the mean dollar performance of budget batteries was better because the interval for the mean was above the mean dollar performance for alpha batteries. Some students reached the same conclusion, but justified it by saying that the interval did not contain the mean dollar performance for alpha batteries. This reasoning is sufficient to conclude that there is a difference, but not sufficient to support the conclusion that the mean dollar performance of the budget batteries is better.

In part (b)(ii) students struggled with two aspects of the required interpretation. Many students were not confident about the comparison value within the interval, and concluded that the two means could be the same or could be different in either direction. Some students erroneously believed that some statement of likelihood could be made because the comparison value was in the upper section of the interval. The other area of weakness in student responses was the tendency to refer to the batteries in absolute terms, rather than to refer to their means or their average behaviour. Comments along the lines of ‘the running time of the batteries is the same’ misrepresent the role of the mean and overlook the entire distributional nature of the statistics course. This is a common weakness in statistics interpretations that needs to be explicitly addressed.

Question 9

This question provided a more challenging opportunity for students to obtain a derivative function by using first principles. A significant number of students seemed to be unfamiliar with the first principles algorithm, with a third of all students earning 0 or 1 mark. The more challenging function chosen meant that those familiar with first principles still had to use some algebraic skills to earn full marks, and a quarter of all students did so. Those who lost 1 or 2 marks in part (b) did so mainly because they omitted brackets and made errors with negative signs. The result that needed to be shown in part (a) was handled well by some. The work of some students was flawed because they started with the equality that was to be obtained. It is strongly recommended that students approach such tasks by starting with one side of the expression and work to obtain the other side of the expression.

Question 10

This question asked students to undertake an ‘unstructured’ *Z*-test and then to demonstrate an understanding of the significance of their results. The *Z*-test was, in general, done well, with many students earning full marks in part (a). Students most commonly lost a mark in part (a) for failing to provide a correct statement of the null hypothesis. In part (b) the common conclusion ‘there is a difference in IQs’ showed partial understanding, with once again relatively few students making it clear that this result indicated a difference in *mean* IQs. Furthermore, very few students showed an awareness that the value of *x* bar provided information about the direction of the difference, despite this being stated clearly in the subject outline. In part (c) many students chose the first statement, showing that they saw an inferential result about a population as applying to individuals, rather than to a distribution. As a result of this combination of technical proficiency and issues with interpretation, nearly 40% of students earned either 4 or 5 marks out of 8, but only 30% earned 6 or more, and just 3% earned full marks, the lowest for any question in the examination.

Question 11

This question examined students' ability to work with an implicit relation and its derivative, as well as with vertical and horizontal tangents to a graph. The differentiation was done well, but many students were then unable to work successfully with the tangents in parts (b) and (c), with 27% of students earning 2 marks or less. In parts (b) and (c) issues arose when students needed to solve an equation that had more than one solution, and then select a particular solution. Too often this selection was not made clearly. Students who used technology in answering this question were often successful, but some provided too little evidence of their procedures to be awarded full marks. As was to be expected, students generally found the vertical tangent more challenging to deal with than the horizontal one. Despite these issues, more than a quarter of students earned full marks for this question.

Question 12

Parts (a) and (b) of this question provided an opportunity for students to display some routine skills in the calculation of binomial probabilities. Although part (a) posed no problems, the complement in part (b) was handled poorly by some, with 20% of students earning 0 or 1 mark out of 8. The nested structure of part (c) challenged many students, with a number erroneously multiplying the previous result by 50. Part marks were made more difficult to award in part (c) as many students wrote down calculator inputs instead of a statement of what distribution was being used. Such notation is not a suitable alternative to working out, and cannot earn part marks. The interpretation asked for in part (d) also proved a challenge for many students. Comments about the unlikelihood of Paul's result being due to chance alone were common, but relatively few students went on to observe that, if considered as one of many, his result was reasonably likely. With the challenges in parts (c) and (d), this question gave an opportunity for students with high levels of skill and understanding to distinguish themselves, with 35% earning 6 or more marks, and 10% earning full marks.

Question 13

The material in this question allowed students to demonstrate a range of achievement levels. Three-quarters of students earned 4 or more marks out of 12, but only a third earned 8 or more and 9% earned full marks. Parts (a) and (c) were accessible to many; the only issue in part (a) was that some students overlooked it. Part (b)(i) was done best by students who used the u-substitution algorithm. Part (d) proved challenging. In part (i) a common error made by students who did not see the link with parts (b) and (c) was to treat the translated function as $f(x)$. In part (ii) students had a chance to distinguish themselves through their proficiency in the laws of logarithms and associated algebraic manipulation.

Question 14

This question challenged students in a couple of areas. Some of the calculus involved — for example, the derivative in part (d) — proved challenging, as did the rate function and the need to use definite integrals to find volumes in part (b). One of the challenges for a large number of students was the calculation of the capacity of a cylinder. A failure to convert the dimensions into a common unit was very common, and the conversion into capacity also caused issues for many. In contrast, the solution to the equation in part (e) was handled well, with many students effectively using technology in this part. The few who used trial and error generally provided the working out and the three-figure accuracy needed to earn the marks. These challenges meant that some students made limited headway in this question, with a third of students earning 2 marks or less out of 13. Those capable of working at a higher level of achievement stood out, with nearly a quarter of all students earning 12 or 13 marks out of 13.

Question 15

This question started with some relatively straightforward work with tangents to curves. As a result, more than half of all students earned 7 or more marks. Those who lost marks in parts (a) to (c) often did so when working with fractional indices. There were some issues with students who presented equations of tangents in non-standard forms. Some students left answers in forms such as $(y–1)/(x–1)=–1/2$, which is undefined at the point of tangency. Other unsimplified forms were also seen. It should be made clear to students that they should present straight lines in either slope-intercept form ($y=mx+c$) or general form ($ax+by=c$ or $ax+by–c=0$). The use of either of these widely accepted forms is a part of communicating mathematics within the norms of the subject area and will ensure that marks are not lost unnecessarily. The proof in part (d) was handled well by those who attempted it algebraically, with 23% of students earning full marks for this question. It was somewhat disappointing to see students still presenting a number of numerical cases when asked for a proof. It is important for students to understand that such an approach is in no way a proof and will not be rewarded with any marks, not even marks for trying.

Question 16

This question gave students opportunities to display a range of skills in working with linear equations and matrices. Part (a) was handled well, although some students failed to make it clear that the population calculated related to the following year. In part (b) a number of students did not go far enough in exploring the ‘long term’, calculating 10–20 years ahead and reporting a declining population. Students needed to provide evidence of calculating the population at least 40 years into the future to see that extinction was actually going to take place. Most students showed some ability to interpret the changed transition matrix in part (c), but many did not describe all three differences from the earlier matrix. Part (d) started well, but in part (ii) many students were either unable to derive the augmented matrix, or started row operations on the augmented matrix, ignoring the instruction given. The row operations were generally done well, with only minor errors causing some issues with follow-through. These errors were not heavily penalised, but when they followed through to non-feasible answers in part (v) students were expected to comment. Overall, students achieved at a range of levels, with nearly 40% earning better than half marks, 20% earning 13 or more marks out of 17, and nearly 10% earning 16 or 17 marks.

GENERAL COMMENTS

In a number of questions, students lost marks by failing to read the information and instructions carefully. This included students who: gave dimensions rather than a matrix in Question 4; did not ‘clearly define’ the variables in their system of equations in Question 5; did not include the sample mean in their confidence intervals in

Question 8; overlooked Question 13(a) altogether; and did part (d)(iii) of Question 16 in part (d)(ii), rather than deriving the augmented matrix as asked. Although these sorts of oversights must be expected under the pressure of an examination, students need to be reminded to read instructions carefully, and to reread these instructions when they have completed the paper, looking for missed information and ensuring that they have actually answered the question as it was asked.

Student responses seemed to indicate that a non-trivial minority of the cohort were unfamiliar with particular aspects of the course. These included the use of upper and lower rectangles to approximate the area under a curve (and hence approximate the definite integral), the use of the z-score formula to find an unknown population parameter, and the relationship between the graphs of a function and its derivative. Students should ensure that they have had adequate exposure to all aspects of the curriculum and, as much as possible, the curriculum tested in class should cover the vast majority of course material. Student exposure to past examination papers should also ensure that important aspects are not overlooked.

Another area where weaknesses in student knowledge were commonplace was in the meaningful interpretation of statistical results. Many students showed that they had no real sense of the distributional nature of statistical variables, not appreciating that these variables vary. For such students the mean *is* the distribution, and results about the mean of the population apply directly to the individuals within a population. These concepts are not easily taught or learnt, but a greater exposure to such ideas is important if meaningful learning is to take place. Students need opportunities to test their developing knowledge through the interpretation of results, and need to be focused on what constitutes correct and incorrect interpretations.

The approach of some students to questions that asked them to 'show' or 'prove' results suggested that they had not had experience with this style of question in class tests. Attempting to show something by starting with the equality that needs to be shown, or providing specific cases when asked to prove a general result, are methods that need to be explicitly avoided.

Students need to be reminded that calculator input notation is not a substitute for mathematical reasoning, such as the statement of the distribution used. Such notation is non-standard and cannot earn part marks when errors are made in statistics questions. Similarly, students need to provide the equation of straight lines in standard forms (i.e. in slope-intercept or general form).

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