Part 3 Geometry

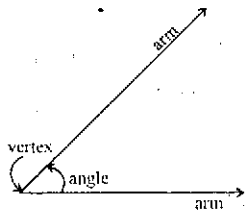
Chapter 1

LINES AND ANGLES

Angle:

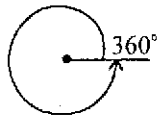
An angle is formed by the intersection of two line segments, which may be rays or lines.

In the diagram, an angle is shown by two lines (the arms) meeting at a point. The meeting point or point of intersection is called the vertex.

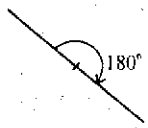


The unit of measure is the degree.

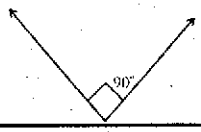
One full turn is 360 degrees (360°)



A half $\left(\frac{1}{2}\right)$ turn is 180 degrees (180°)



A $\frac{1}{4}$ turn is 90 degrees (90°).



Note:

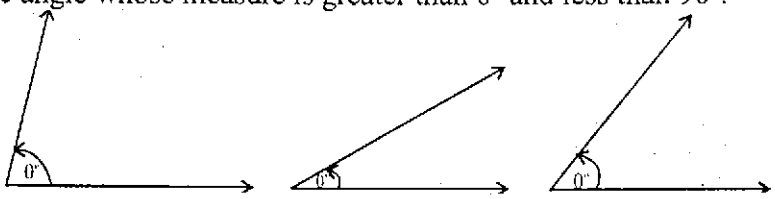
1. A half turn (180°) is also called straight angle.
2. A $\frac{1}{4}$ turn (90°) is also called a right angle.

Classification of Angles:

Angles are classified according to their degree measures.

Acute Angle:

An acute angle is the angle whose measure is greater than 0° and less than 90°.



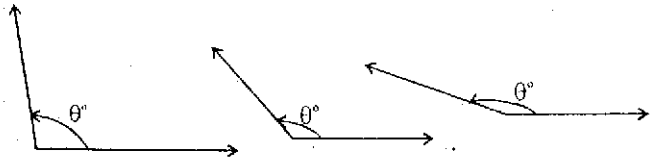
$0 < \theta < 90°$

In all above figures θ lies between 0 and 90°.

Obtuse Angle:

An angle whose measure is greater than 90° and less than 180° is called obtuse angle.

Examples:



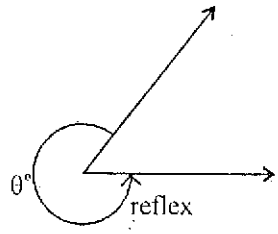
$90° < \theta < 180°$

In all above figures θ lies between 90° and 180°.

Reflex Angle:

A reflex angle is between 180° and 360°.

Example:

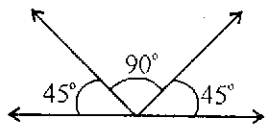


$180^\circ < \theta < 360^\circ$

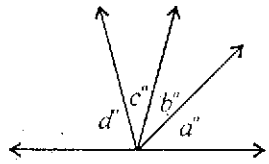
A reflex angle lies between 180° and 360° degrees.

Calculating Angles:

Angles on a straight line add up to 180°.



$45^\circ + 90^\circ + 45^\circ = 180^\circ$

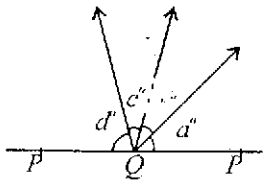


$a^\circ + b^\circ + c^\circ + d^\circ = 180^\circ$

A straight angle is 180°. So angles on a straight line add up to 180°.

Example 1:

What is the average of *a, b, c* and *d* in the following figure



Solution:

In the given figure since $\angle PQR$ is a straight angle. Because the angles on a straight line add up to 180°, therefore

$$a + b + c + d = 180^\circ$$

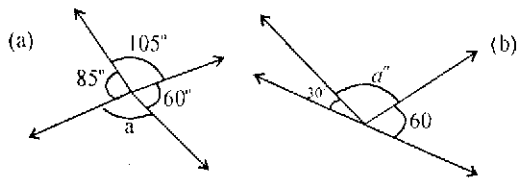
Average is

$$\frac{180^\circ}{4} = 45^\circ$$

Angles in a full turn add upto 360°.

Example 2:

Find the angle ***a*** in these diagrams.



Solution:

a) $a = 360^\circ - (60^\circ + 105^\circ + 85^\circ)$

$= 360^\circ - 250^\circ$

$= 110°$

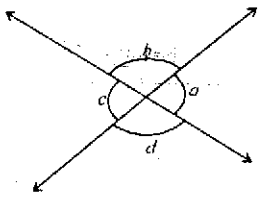
b) $a = 180° - (60° + 30°)$

$= 180° - 90°$

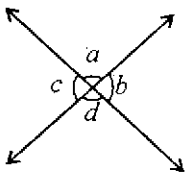
$= 90°$

Vertical Angles:

When two straight lines intersect, they make four angles. The two opposite angles are called vertical angles. In this diagram angles *a, c* and *b, d* are vertical angles.



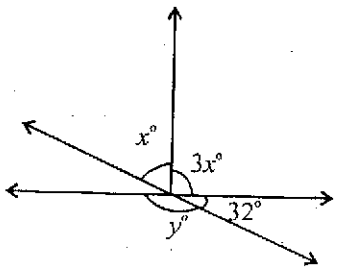
Vertically opposite angles are equal



$\angle a = \angle d$; $\angle c = \angle b$

Example 3:

Find the value of pronumerals in the following diagram, giving reasons:



Solution:

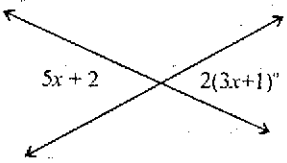
Because angles on a straight line add up to 180°, and vertically opposite angles are equal:

$\therefore \quad 3x + x + 32 = 180° \Rightarrow 4x = 148 \Rightarrow x = 37°$

Again $\quad y° = 3x° + x° \Rightarrow y = 4(37°) \Rightarrow y = 148°$

Example 4:

What is the value of x in the following diagram?



Solution:

Since the vertically opposite angles are equal:

$5x + 11 = 2(3x + 1)$

$5x + 11 = 6x + 2$

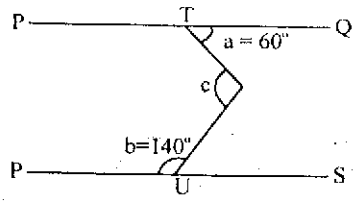
$\Rightarrow \quad x = 9$

Parallel Lines:

Parallel lines are always the same distance apart. They never meet, even if you make them longer. Parallel lines form no angles.

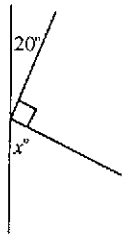
(A) 65 (B) 55
(C) 35 (D) Insufficient information

Q13. In the figure, line *PQ* is parallel to line *RS*, angle a = 60° and angle *b* = 140°. How many degrees are there in angle *c*?



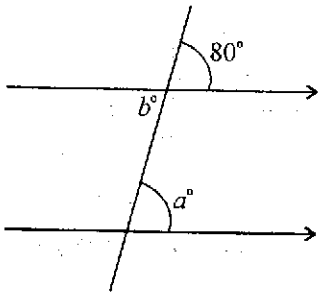
(A) 80 (B) 110
(C) 100 (D) 95

Q14. In the fig below, what is the value of *x*?



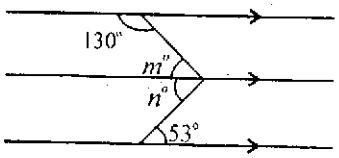
(A) 20° (B) 70°
(C) 100° (D) 110°

Q15. In the fig below, what is the value of *a*?



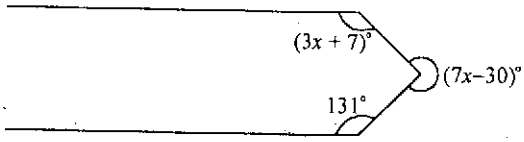
(A) 100° (B) 60°
(C) 80° (D) 40°

Q16. In the figure below, what is the value of *m* + *n*?



(A) 103° (B) 77°
(C) 130° (D) 85°

Q17. In the figure below, what is the value of *x*?



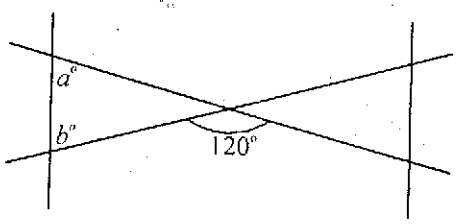
(A) 61 (B) 42

(C) 41 (D) 72

Q18. If P, Q and R are points on a line, with B between P and R. Let A and B be the mid points of PQ and QR, respectively. If $PQ : QR = 3 : 1$, what is $PQ : AB$?

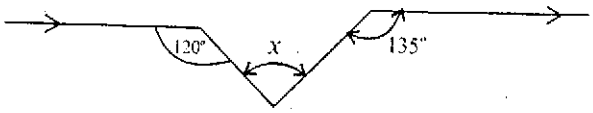
(A) 3 : 2 (B) 1 : 2
(C) 1.5 : .5 (D) 3 : 1

Q19. In the figure below, $a + b =$



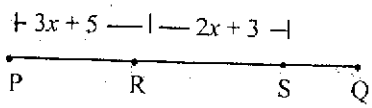
(A) 80 (B) 60
(C) 135 (D) 120

Q20. In the figure below, what is the value of x?



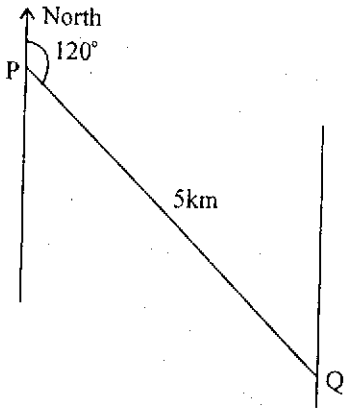
(A) 45° (B) 72°
(C) 57° (D) 65°

Q21. In the figure below, if the length of PQ is $5x + 9$, what is the length of SQ?



(A) 1 (B) 3
(C) 5 (D) 7

Q22. A ship leaves a port P and sails for 5km on a bearing of 120° to a port Q. What is the bearing of P from Q?



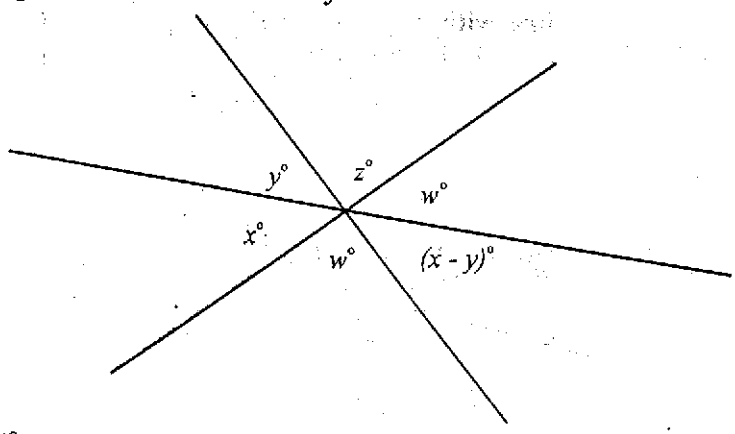
(A) 60° (B) 120°
(C) 40° (D) 300°

Q23. A hill-walker set off from P on a bearing of 225° to arrive at Q. What bearing must they take to retrace their steps?

(A) 55° (B) 135°
(C) 45° (D) 95°

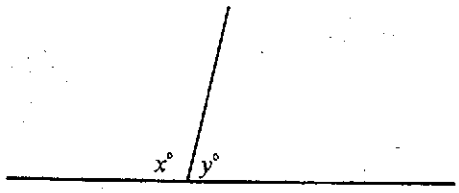
Q24. In the figure below, the straight line ABC is parallel to DE and BD is parallel to CF. $AD = BD$, $DBC = 110°$ and $FED = 45°$. What is the value of x?

Q4. In the following figure, what is the value of y?



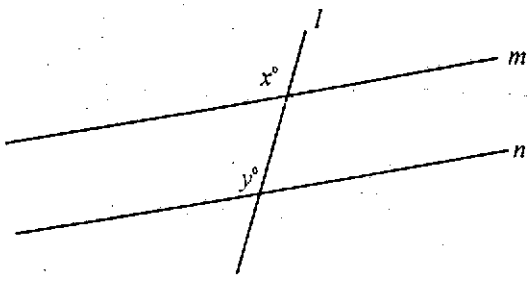
(A) 45° (B) 36°
(C) 46° (D) 35°

Q5. In the figure below, if x is 130 more than y, what is the value of y?



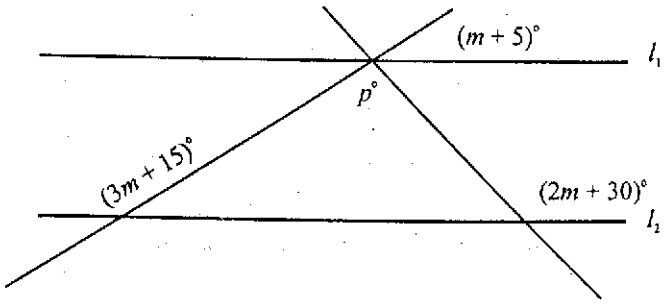
(A) 7 (B) 15
(C) 25 (D) 35

Q6. In the following figure, $m \parallel n$ and l is a transversal, then which of the following statement is (are) true?



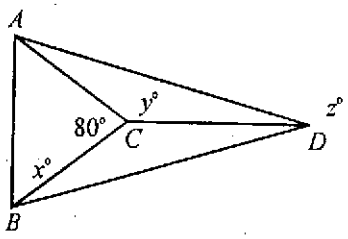
(A) $x + y = 180$ (B) $x - y = 180$
(C) $180 < x + y \leq 270$ (D) Insufficient information

Q7. In the following figure, if $l_1 \parallel l_2$, then what is the value of p?



(A) 70 (B) 45
(C) 40 (D) 65

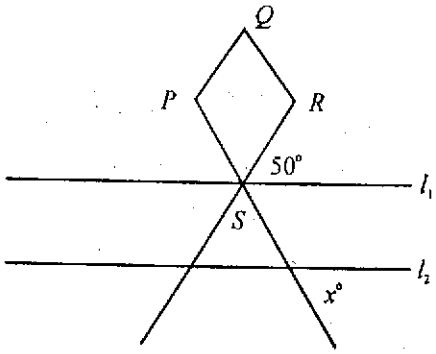
Q8.



In the above figure, AC = BC = CD and $m\angle ACB = 80°$. This information is sufficient to determine the value of which of the following?

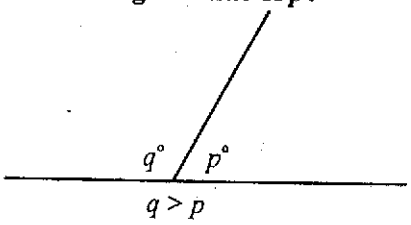
(A) x only
(B) y only
(C) x and y only
(D) y and z only

Q9. In the following figure, lines l_1 and l_2 are parallel, and line l_1 passes through S, one of the corners of square $PQRS$. What is the value of x?



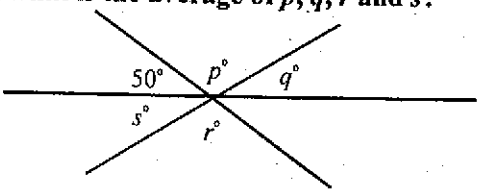
(A) 50
(B) 30
(C) 45
(D) 40

Q10. In the following figure, what is the largest value of p?



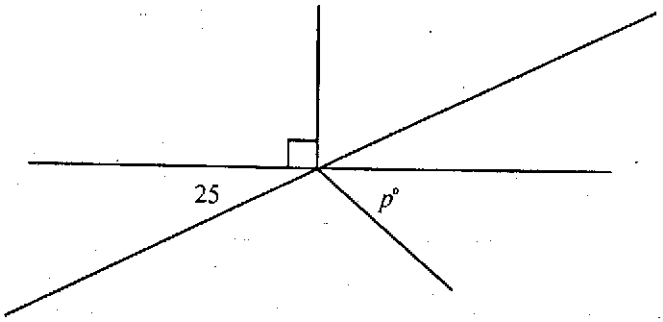
(A) 89
(B) 90.9
(C) 89.9
(D) 105

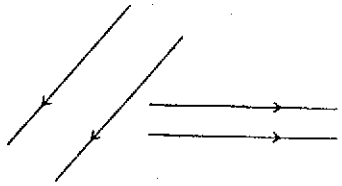
Q11. In the following figure, what is the average of p, q, r and s?



(A) 70°
(B) 60°
(C) 75°
(D) 65°

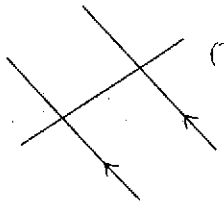
Q12. In the following diagram, what is the value of p?





Transversal:

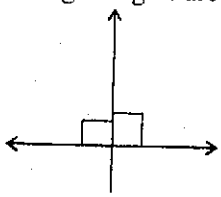
A straight line which cuts parallel lines is called a **transversal**.



(Transversal)

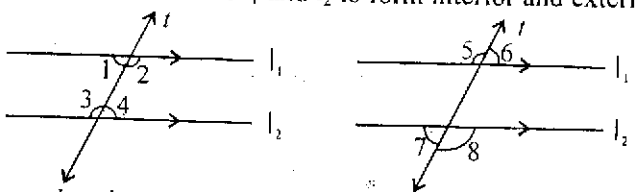
Perpendicular:

If two lines interest in such a way that they form right angles are called perpendicular



Interior Angles:

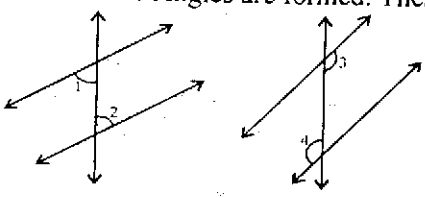
In the figure below, transversal t intersects lines ℓ_1 and ℓ_2 to form interior and exterior angle.



Interior Angles Exterior Angles

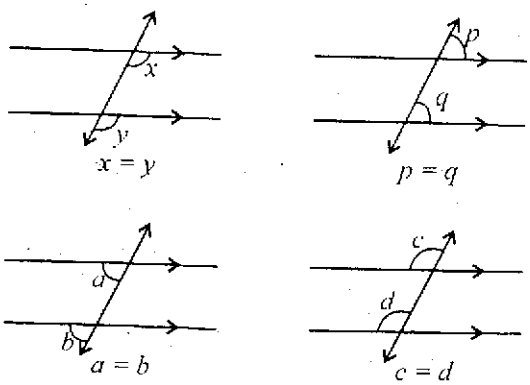
Alternate Angles:

When a transversal cuts parallel lines Alternative Angles are formed. These alternative angles are equal.

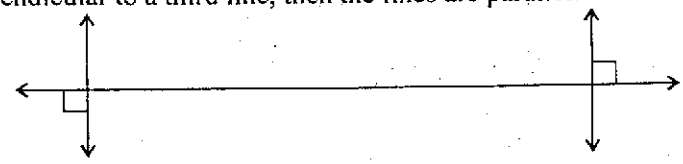


Corresponding Angles:

Corresponding angles are two angles in corresponding positions relative to the two lines and the transversal. These corresponding angles are also equal. A pair of equal corresponding angles is shown below.

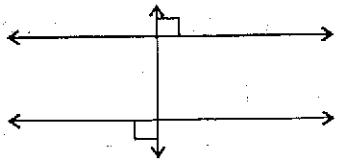


If two lines are both perpendicular to a third line, then the lines are parallel.



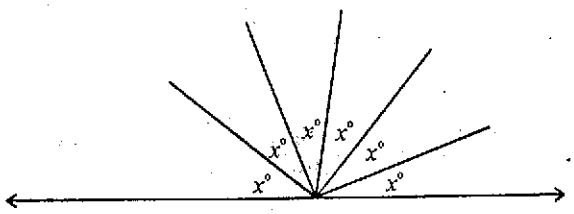
Alternatively

If a line is perpendicular to each of a pair of lines, then that pair of lines are parallel.



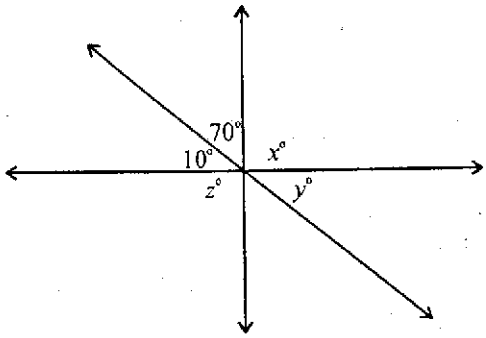
Multiple Choice Questions (MCQs)

Q1. In the following figure, what is the value of x?



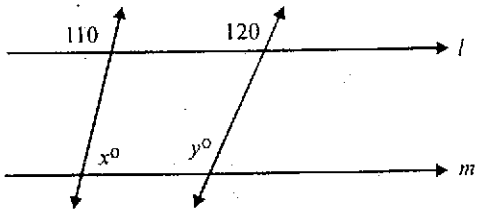
(A) 30 (B) 45
(C) 40 (D) 35

Q2. In the figure below, what is the value of $x + y + z$?



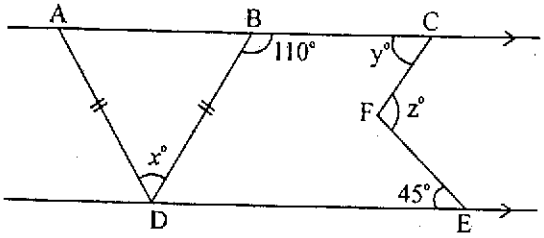
(A) 200 (B) 220
(C) 210 (D) 190

Q3. In the following figure, if $l \parallel m$



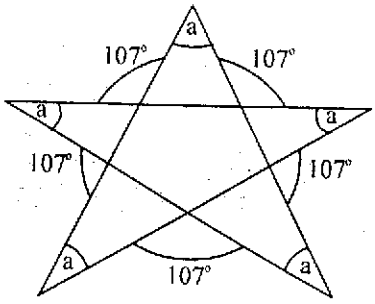
Then $x^\circ + y^\circ$ _____ 190°

(A) = (B) <
(C) > (D) ≑



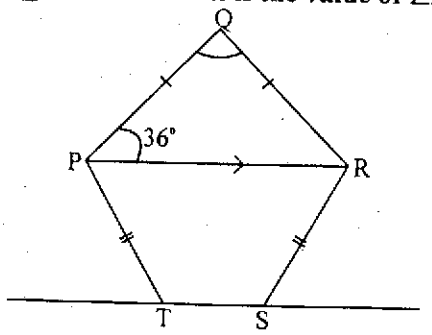
(A) 40 (B) 45
(C) 70 (D) 20

Q25. In the copy of decoration given below, what is the value of angle a?



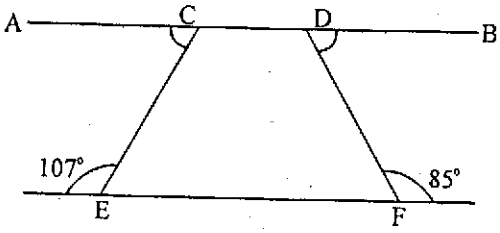
(A) 73° (B) 34°
(C) 107° (D) 45°

Q26. $PQRST$ represents a swimming pool with all its sides of equal length. A rope joins PR and is parallel to TS. Given that $\angle QPR = 36°$. What is the value of $\angle PQR$?



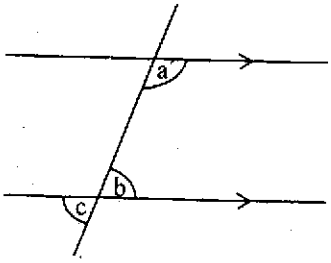
(A) 72 (B) 36
(C) 108 (D) 90°

Q27. What is the sum of $\angle ACE + \angle BDF$?



(A) 192 (B) 180
(C) 85 (D) 168

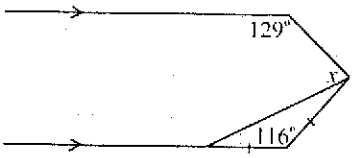
Q28. In the figure below $a + b =$



(A) $a + c$ (B) $a - b$

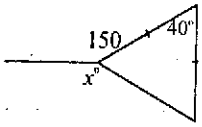
(C) $b + c$ (D) $c - a$

Q29. In the figure below, the value of x is:



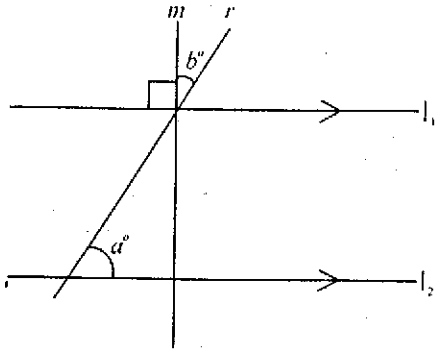
(A) 115° (B) 83°
(C) 69° (D) 43°

Q30. In the figure below, what is the value of x?



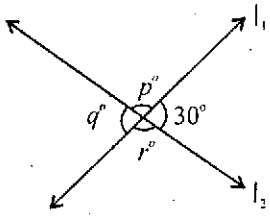
(A) 120 (B) 140
(C) 110 (D) 190

Q31. In the figure below, line ℓ_1 is parallel to line ℓ_2 and is perpendicular to line m. If $a = b$, what is the value of a?



(A) 30° (B) 90°
(C) 60° (D) 45°

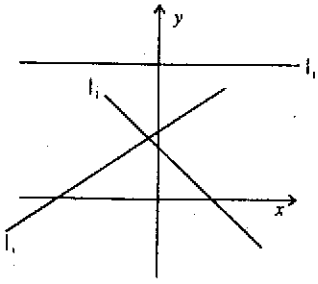
Q32. For the intersecting two lines ℓ_1 and ℓ_2 below, which of the following must be true?



I. $p > r$
II. $p = 5q$
III. $p + 30° = q + r$

(A) I only (B) I and II only
(C) III only (D) II and III only

Q33. In the figure below, lines ℓ_1, ℓ_2 and ℓ_3 have slopes a, b and c, respectively. Which of the following is a correct statement?

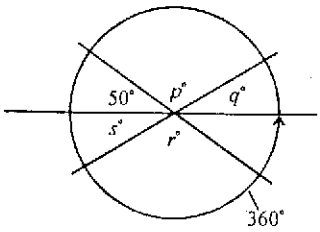


Q11. (D) $\because$ vertical angles are equal

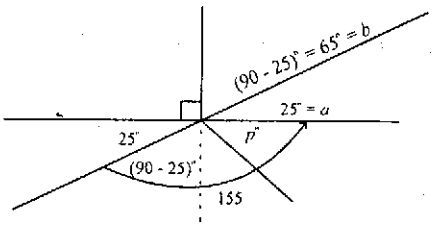
$\therefore p + q + r + s + 50 + 50 = 360$

$\Rightarrow p + q + r + s = 260$

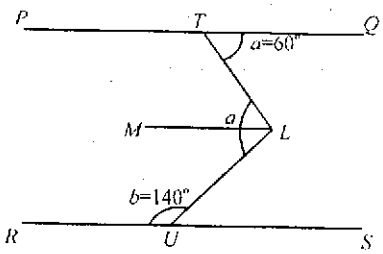
Average $\dfrac{p+q+r+s}{4} = \dfrac{260}{4} = 65$



Q12. (E) Since vertical angles are equal, here, $a = 25$, and so $p + q = 155$. We see that $b = 65$, and there is no other vertical angle. So it is impossible to determine p and q from the given information.



Q13. (C)



Through point L, draw ML parallel to RS and PQ.

$\angle c = \angle MLU + \angle MLT$

$\angle MLU = \angle LUS = 180° - 140° = 40°$

$\angle MLT = \angle LTQ = 60°$

Then $\angle c = 60° + 40° = 100°$

Q14. (B) Since $x + 90 + 20 = 180$, Therefore

$x = 70°$

Q15. (C) Since, vertical opposite angles are equal, therefore $\angle b = 80°$. Again because alternative angles are equal. Therefore, $\angle a = \angle b$

$\Rightarrow \angle a = 80°$

Q16. (D) $180 - 130 = 50°$

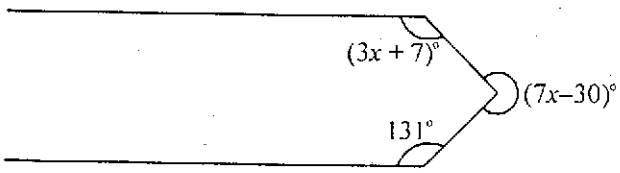
Because vertical opposite angles are equal

$\therefore \quad m = 50$

and $\quad n = 35°$

$m + n = 35° + 50° = 85°$

Q17. (B)



As $131° + (3x + 7)° = 7x - 30$

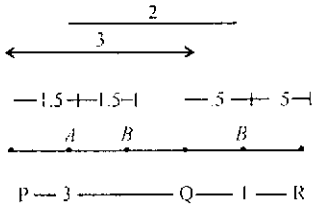
$131 + 3x + 7 = 7x - 30$

$\Rightarrow \quad 7x - 3x = 138 + 30$

$4x = 168°$

$x = 42°$

Q18. (A)

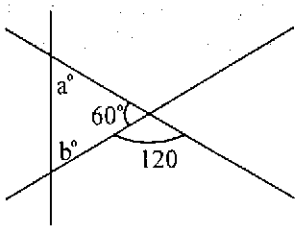


From above we see that

$$PQ : AB = 3 : 2$$

$$= \frac{3}{2} = 1.5$$

Q19. (D)



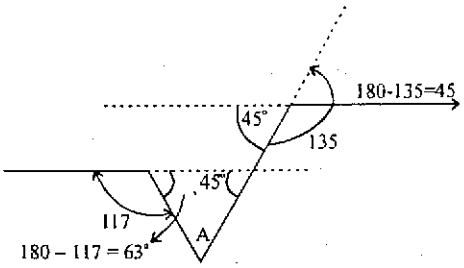
The 120 degrees angle and the angle next to it, is 60° because 180° – 120° = 60° (straight angle)

We know sum of all angles of any triangle is 180°

$$\therefore \quad a + b + 60^\circ = 180^\circ$$

$$a + b = 120^\circ$$

Q20. (B) The process of finding the value of x is illustrated by the following figure



Because the sum of the angles of a triangle always equal to 180°

$$\therefore \quad x + 45^\circ + 63^\circ = 180^\circ$$

$$\Rightarrow \quad x = 72^\circ$$

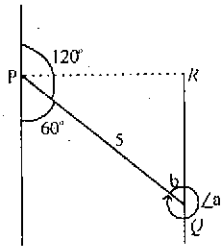
Q21. (A) Using figure we can see

$$SQ = PQ - (PR + RS)$$

$$SQ = 5x + 9 - (3x + 5 + 2x + 3)$$

$$SQ = 5x + 9 - 5x - 8$$

$$SQ = 1$$



Q22. (D) $\angle b = 180 - 120^\circ = 60^\circ$

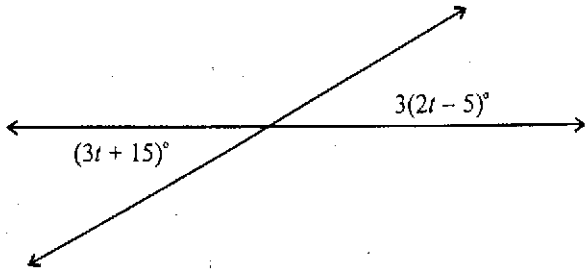
bearing of P from Q $= \angle a = 360 - \angle a$

$$= 360 - 60$$

$$= 300^\circ$$

(A) 30° (B) 40°
(C) 45° (D) 60°

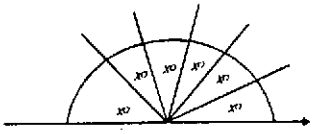
Q43. In the following figure, what is the value of t?



(A) 10 (B) 15
(C) 30 (D) 45

Explanatory Answers

Q1. (A) The sum of the given six angles make a straight angle, and the straight angle equal 180°. Thus

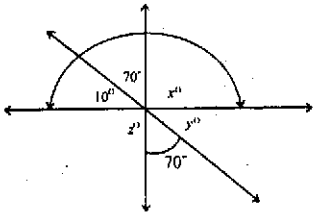


$x + x + x + x + x + x = 180^\circ$

$\Rightarrow \quad 6x = 180^\circ$

$\Rightarrow \quad x = 30^\circ$

Q2. (C) In the given figure, the arc shows a straight angle, hence



$10 + 70 + x = 180^\circ$

$x = 100$

Because opposite angles are equal, thus

$x = z^\circ = 100 \Rightarrow z = 100$

Similarly, $z + 70 + y = 180^\circ$

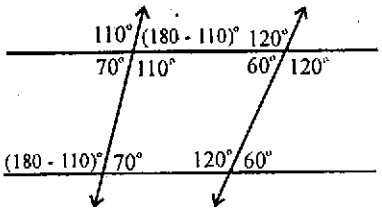
$100 + 70 + y = 180^\circ \quad (\because z = x = 100)$

$\Rightarrow \quad y = 10$

Thus,

Sum of the angles $x + y + z = 100 + 10 + 100$

$\Rightarrow \quad \boxed{x + y + z = 210}$



Q3. (A) Because when two straight lines intersect each other, the corresponding angles are equal. This fact is shown in the adjacent figure

Hence $x^\circ = 70$ and $y = 120$

$x^\circ + y^\circ = 120 + 70 \Rightarrow x^\circ + y^\circ = 190$

Q4. (B) Because vertical angles are equal, therefore

$x - y = y \Rightarrow x = 2y$

and $x = w$ also $z = w$

If we add y, z and w, then the sum of these angles is a straight angle which is equal to 180°.

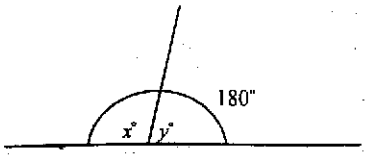
Thus, $y + z + w = 180$

$y + 2y + 2y = 180 \quad (\because z = w = x = 2y)$

$5y = 180$

$\Rightarrow \quad \boxed{y = 36}$

Q5. (C) In the given figure, the sum of the given two angles x and y is a straight angle, and straight angle equals to 180, therefore



$x + y = 180 \quad \ldots\ldots(i)$

By given condition

$$x = y + 130$$

$$\Rightarrow \quad x - y = 130 \qquad \ldots\ldots(ii)$$

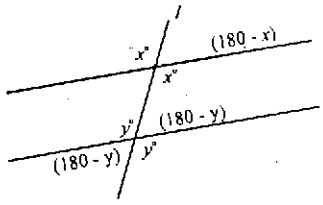
Adding (i) and (ii), we have

$$x + y = 180$$
$$x - y = 130$$
$$2x = 310$$
$$\Rightarrow \quad \boxed{x = 155}$$

Substituting the value of x, in (i), we have

$$155 + y = 180$$
$$\Rightarrow \quad y = 180 - 155$$
$$\Rightarrow \quad \boxed{y = 25}$$

Q6. (D) Since, there is not enough information, and the figure is not drawn in right scale, thus it is not possible to get the exact value of $x^o + y^o$.



Q7. (D) Since, when two straight lines intersect, the vertical angles are equal, therefore

$$x^o = m + 5$$

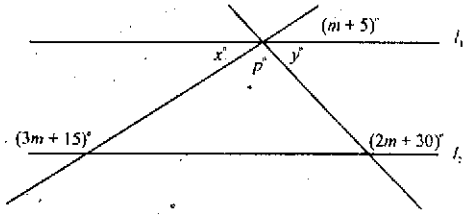
and $\quad x + (3m + 15) = 180$, but $x = m + 5$

Hence,

$$(m + 5) + (3m + 15) = 180 \Rightarrow 4m + 20 = 180 \Rightarrow 4m = 160$$

$$\Rightarrow \quad \boxed{m = 40}$$

As, $\quad x = m + 5 \Rightarrow x = 40 + 5 \Rightarrow \boxed{x = 45}$

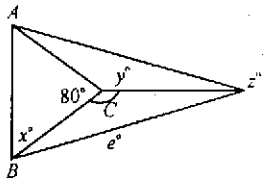


From figure, $x + p = 2m + 30$ ($\because$ Alternative angles are equal)

$45 + p = 2(40) + 30 \quad \Rightarrow 45 + p = 80 + 30 \Rightarrow 45 + p = 110$

$\Rightarrow p = 110 - 45 \quad \Rightarrow \boxed{p = 65}$

Q8. (A) Since $AC = BC$, we know that when two sides of a triangle are equal then their opposite angles are also equal. Hence $\angle A = \angle B$, and $\angle A = \angle B = x^o$. In any triangle, the sum of the three angles is equal to 180.



$\angle A + \angle B + \angle C = 180^o \Rightarrow x^o + x^o + 80^o = 180^o \Rightarrow 2x = 100$

$x = 50$

$\because$ y and e are not necessarily equal, therefore, we cannot determine y and z. The answer is x only.

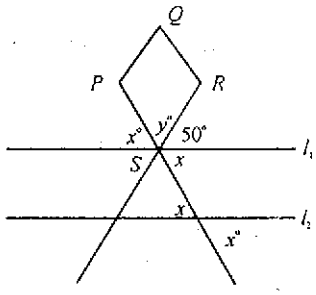
Q9. (D) $\because$ $PQRS$ is a square,

$\therefore \quad y = 90$. Then

$$x + y + 50 = 180$$
$$x + 90 + 50 = 180$$

($\because$ Alternative angles are equal)

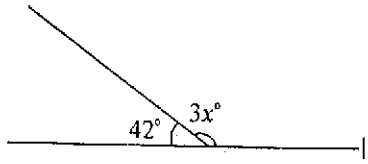
$$x + 140 = 180 \quad \Rightarrow x = 40$$



Q10. (C) $\because q > p$, then q must be greater than 90 and p less than 90. Therefore, the largest number less than 90 that can fit in the grid is 89.9.

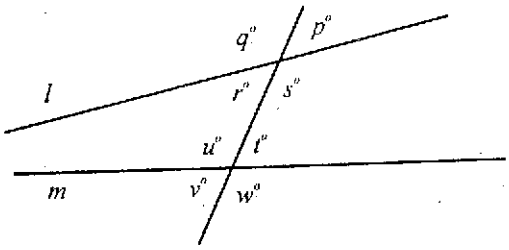
(A) $b < a < c$ (B) $a < b < c$
(C) $c < b < a$ (D) $c < a < b$

Q34. In the figure below, what is the value of x?



(A) 74° (B) 16°
(C) 46° (D) 14°

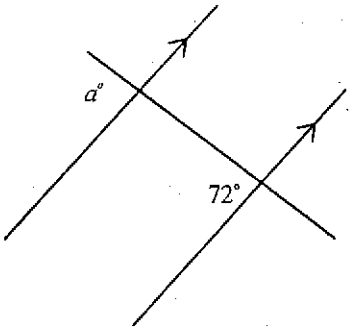
Q35. In the following diagram $l \parallel m$,



If A represents the average measure of all the eight angles, then A =

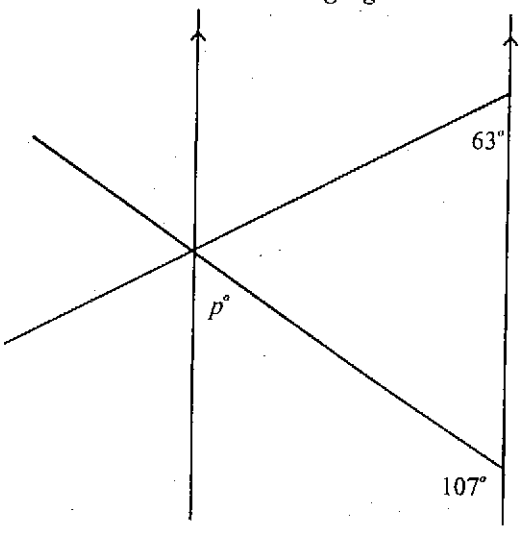
(A) 45° (B) 180°
(C) 90° (D) 360°

Q36. In the following figure, what is the value of $a°$?



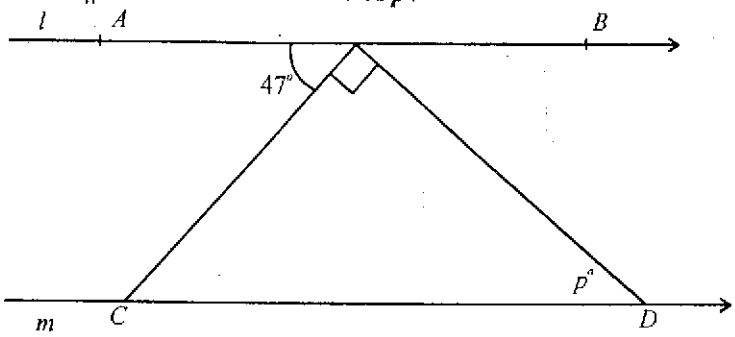
(A) 108° (B) 72°
(C) 36° (D) 16°

Q37. What is the value of pronumerals, in the following figure?



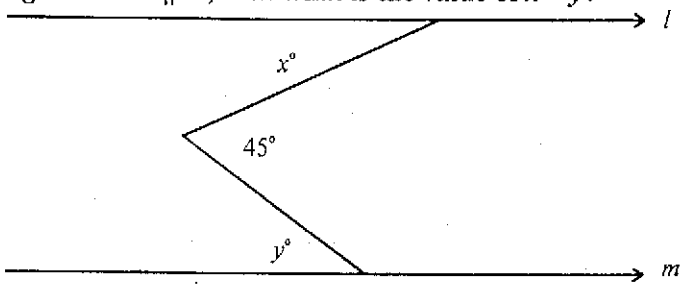
(A) 107° (B) 73°
(C) 170° (D) 44°

Q38. In the figure below $l \parallel m$. What is the value of p?



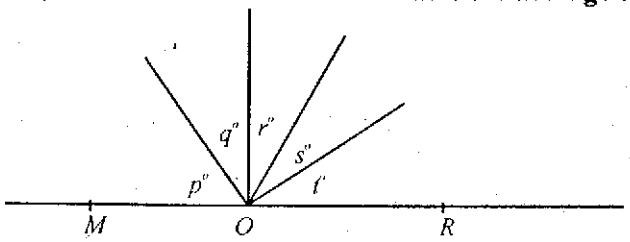
(A) 53° (B) 43°
(C) 47° (D) 30°

Q39. In the following figure lines $l \parallel$ m, then what is the value of $x + y$?



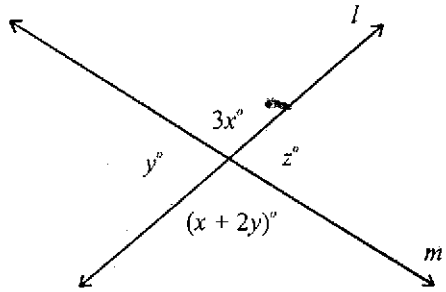
(A) 45° (B) 35°
(C) 135° (D) 180°

Q40. In the figure below M, O and N are all on line n. What is the average of p, q, r, s and t?



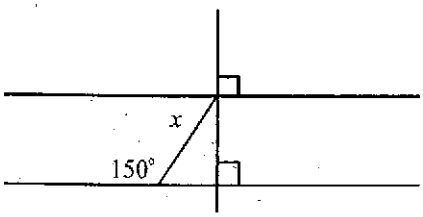
(A) 90 (B) 30
(C) 36 (D) 21

Q41. In the following figure, what is the value of x?



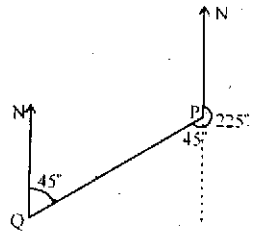
(A) 90° (B) 45°
(C) 85° (D) 180°

Q42. In the following figure, what is the value of x?



Q23. **(C)** The north lines are parallel and so $\angle NQP = 45^\circ$

So to retrace their steps, the hill-walker must take a bearing of 45° from Q



Q24. **(A)** $\angle BAD = \angle ABD$

$= 180^\circ - 110^\circ = 70^\circ$

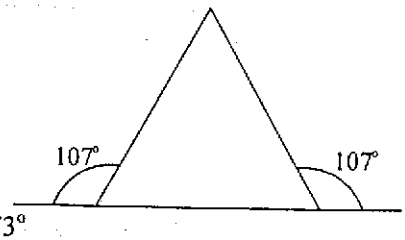
$\angle ADB = 180^\circ - 2\angle BAD$

$= 180^\circ - 140^\circ$

$= 40^\circ$

$\therefore \quad x = 40^\circ$

Q25. **(B)** Take one side of the copy of this decoration



from above $\angle b = 180 - 107 = 73^\circ$

In above triangle two angles are 73°, the third angle is

$\angle a = 180 - 73 - 73$

$= 34^\circ$

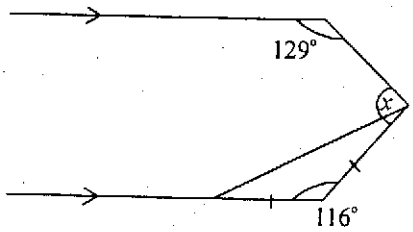
Q26. **(C)** Because In ΔPQR, the value of $\angle P$ and $\angle R$ is 36. Therefore the third angle $\angle PQR$ must equal to 108° $(180 - 36 - 36)$

Q27. **(D)** In the given figure $\angle CDF = 85^\circ$ and $\angle DCE = 107^\circ$ because the vertically opposite angles are equal. Now $\angle BDF = 95$ $(180 - 85)$ and $\angle DCE = 73$, $(180 - 107)$

Sum of the angles $\angle BDF + \angle ACE = 168^\circ$.

Q28. **(A)** b and c are vertical angle and therefore equal angles. $a + b = a + c$

Q29. **(B)**



$116^\circ + y + y = 180^\circ$

$\Rightarrow \quad 2y = 180^\circ - 116 = 64$

$y = 32^\circ$

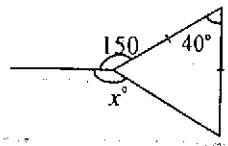
As $\quad x + y + 116 + 129 = 360^\circ$

$x + 32 + 116 + 129 = 360^\circ$

$x + 277 = 360^\circ$

$\Rightarrow \quad x = 360^\circ - 277 = 83^\circ$

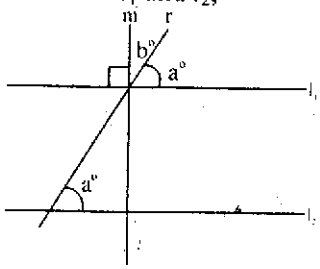
Q30. **(B)**



In triangle

$$40^\circ + y + y = 180^\circ$$
$$2y + 40 = 180^\circ$$
$$\Rightarrow \quad 2y = 180 - 40 = 140$$
$$y = 70^\circ$$
$$70^\circ + 150 + x = 360^\circ$$
$$x + 220 = 360^\circ$$
$$\Rightarrow \quad x = 360^\circ - 220 = 140^\circ$$

Q31. **(D)** Since line m is perpendicular to line l_1 and l_2,



$$a^\circ + b^\circ = 90^\circ$$
$$a^\circ + a^\circ = 90^\circ$$
$$2a^\circ = 90 \Rightarrow \boxed{a^\circ = 45}$$

Q32. **(D)** Since vertically opposite angles are equal. Therefore, $\angle q = \angle 30^\circ$ and sum of the straight angles is 180°. Thus

$$a^\circ + 30 = 180 \Rightarrow a = 150$$
$$\Rightarrow a = 5(q) \text{ and } p + 30^\circ = q + r$$

Q33. **(A)**

Q34. **(A)** $42 + 3x = 180^\circ \Rightarrow 3x = 138$

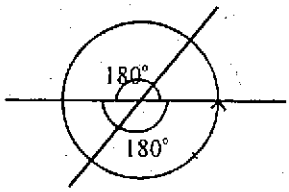
$$\Rightarrow \quad x = 46^\circ$$

Q35. **(C)** Sum of the angles $p + q + r + s = 360$

Similarly $t + u + v + w = 360$

Sum of the measure of above 8 angles $= 360 + 360 = 720$

Average $= \dfrac{720}{8} = 90^\circ$

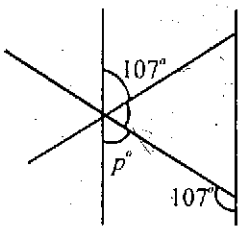


Q36. **(B)** Since corresponding angles of parallel lines are equal, therefore,

$$a = 72$$

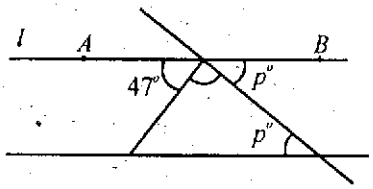
Q37. **(B)** $p^\circ = 73$

$$107^\circ + p^\circ = 180^\circ$$
$$p^\circ = 180^\circ - 107^\circ = 73$$



Q38. **(B)** The angles in the given figure, is decomposed as shown in the adjacent figure

Thus $47 + 90 + p^\circ = 180^\circ$



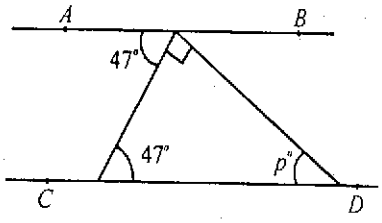
$137 + p^\circ = 180^\circ$

$p^\circ = 180 - 137$

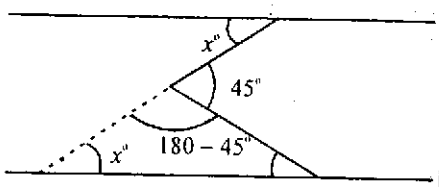
$\boxed{p^\circ = 43}$

Second Method: Because in any triangle, the sum of the three angles is equal to 180°, thus $47^\circ + 90^\circ + p^\circ = 180$

$\Rightarrow \boxed{p^\circ = 43}$

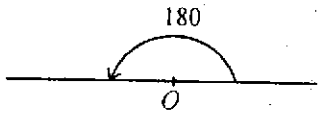


Q39. **(A)** Extend the line which makes angle x° with the upper line towards the down-ward line. Sine l and m are parallel, the measure in the bottom line in the triangle equals. In any triangle



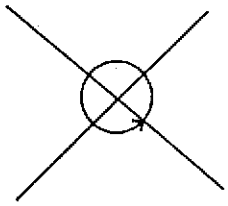
$$x + y + (180 - 45) = 180^\circ$$
$$x + y + 135 = 180^\circ$$
$$x + y = 180 - 135$$
$$x + y = 45$$

Q40. **(C)** In the figure, the angle MOR is straight angle. Thus, sum of the angles p, q, r, s and t is 180 and their average is



$$\frac{180}{5} = 36.$$

Q41. **(B)** Because, when the straight lines intersect each their then vertical angles are equal. Thus, in the given figure



$y^\circ = z^\circ$

Similarly $3x^\circ = (x + 2y)^\circ$

$\Rightarrow$ $3x - x = 2y \Rightarrow 2x = 2y \Rightarrow \boxed{x = y}$

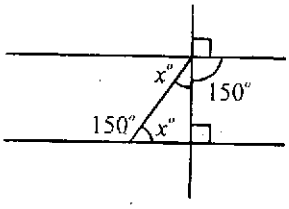
$\because$ $x = y = z \Rightarrow x = z$

Hence the four angles, are equal.

As the sum of angles $= 180^\circ$

Value of each angle $= \frac{180}{4} = \boxed{45^\circ}$

Q42. **(A)** Since a line is perpendicular to each pair of lines, thus the pair of lines are parallel, and when a line intersect pair of lines corresponding angles are equal.



First Method: Calculation of upper line

$x + 150 = 180$

$\Rightarrow$ $\boxed{x = 30}$

Second Method: Calculation of lower line

$150 + x^\circ = 180^\circ$

$\Rightarrow$ $\boxed{x = 30}$

Third Method: Calculation of triangle

$\because$ Sum of the three angles in a triangle equals 180°

$\therefore$ $x^\circ + 90 + (150 - 90) = 180^\circ$

$x^\circ + 90 + 60 = 180^\circ$

$x^\circ + 150 = 180^\circ$

$\Rightarrow$ $\boxed{x = 30^\circ}$

Q43. **(A)** When two lines intersect each other corresponding angles are equal, thus

$$3t + 15 = 3(2t - 5)$$
$$3t + 15 = 6t - 15$$
$$30 = 3t$$
$$t = 10$$

****

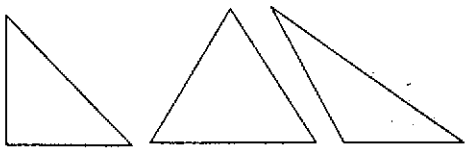
Chapter 2

TRIANGLES

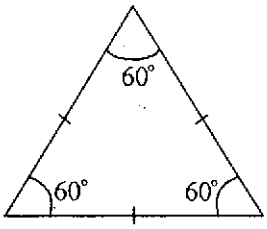
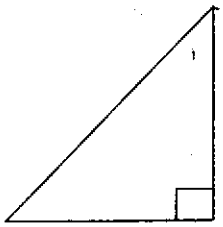
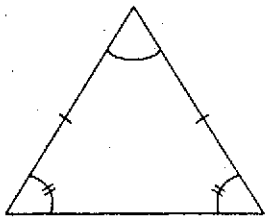
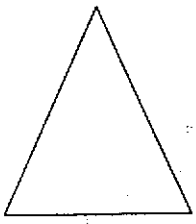
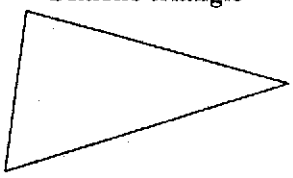
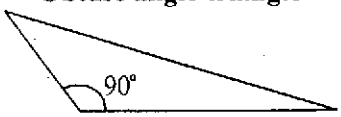
Triangle:

A three-sided polygon is called a triangle.

Examples:

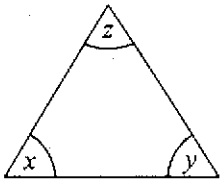


Types of Triangle:

Due to side	Due to Angle
Equilateral triangle  An equilateral triangle has 3 equal sides.	**Right angle triangle**  A right angle triangle has one angle has 90°.
Isosceles triangle  An isosceles triangle has 2 equal sides.	**Acute angle triangle**  An acute angle triangle all 3 angles measurement are less than 90°.
Scalene triangle  A scalene triangle has all 3 sides of different lengths.	**Obtuse angle triangle**  An obtuse angle triangle has one angle greater than 90°.

Angle's Sum of Triangle:

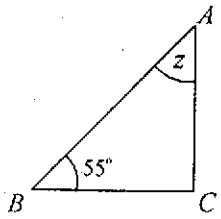
In any triangle, the sum of the measures of the three angles is 180°.



$$x + y + z = 180^\circ$$

Example 1:

In the figure below, what is the value of z?



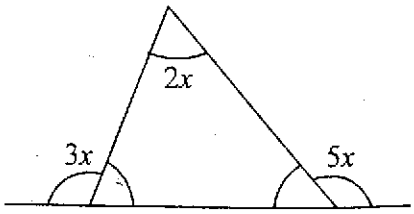
Solution:

Because, the angle of a triangle add up to 180°. Therefore

$$55^\circ + 90^\circ + Z = 180^\circ$$
$$\Rightarrow \quad Z = 180^\circ - 145^\circ$$
$$\Rightarrow \quad Z = 35^\circ$$

Example 2:

Calculate the value of x in the following figure:



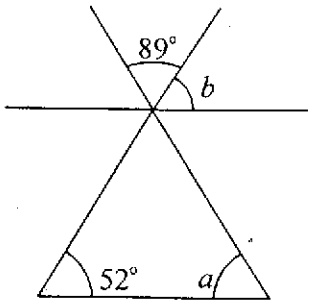
Solution:

Because the sum of the straight angles is 180°, therefore the missing angles of the triangle are $(180^\circ - 3x)$, $(180^\circ - 5x)$ and 2x.

$$(180^\circ - 5x) + (180^\circ - 3x) + 2x = 180^\circ$$
$$360^\circ - 6x = 180^\circ$$
$$-6x = 180^\circ - 360^\circ$$
$$-6x = -180^\circ$$
$$\boxed{x = 30^\circ}$$

Example 3:

Calculate the value of



Solution:

We know when two lines intersect each other then opposite angles are equal, therefore, the third angle of the triangle will be 89°. Hence

$$\angle 52^\circ + \angle a + \angle 89^\circ = 180^\circ$$
$$\angle a = 180^\circ - (52^\circ + 89^\circ)$$
$$\boxed{\angle a = 39^\circ}$$

Now, because corresponding angles are equal, here ∠52 and ∠b are pair of corresponding angies. Therefore

$$\angle b = \angle 52$$
$$\boxed{\angle b = 52^\circ}$$

Properties of Isosceles Triangle:

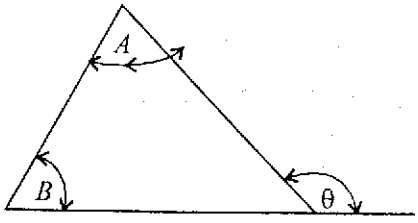
1. If two sides of a triangle are congruent, then the angles opposite to these sides are congruent.
2. If the three angles of a triangle are congruent, then the three sides are also congruent.
3. If two angles of a triangle are congruent, then the sides opposite these angles are also congruent.
4. If three sides of a triangle are congruent, then the three angles are also congruent.

Angle Properties of Triangle:

1. In every triangle the greatest angle is opposite to the longest side.
2. In every triangle the sum of the lengths of any two sides is always greater than the length of the third side.
3. In every triangle the shortest side is opposite to the smallest angle.
4. When the side of a triangle is produced the exterior angle so formed which is equal to the sum of the opposite interior angles.

Example:

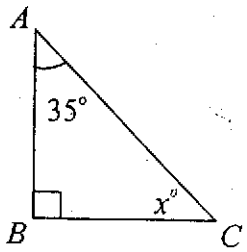
In the figure below



$$\angle\theta = \angle A + \angle B$$

5. In any right triangle, the sum of the measures of the two acute angles is 90°.

Example:



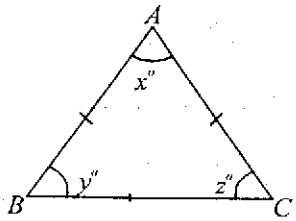
Find the value of x.

Solution:

Since, the sum of the measures of the two acute angles is 90°, therefore

$$x + 35^\circ = 90^\circ$$
$$\boxed{x = 90^\circ - 35^\circ = 55^\circ}$$

6. An equilateral triangle has three equal sides, and three equal angles of 60°.



Example:

The above triangle is an equilateral triangle. Therefore,

$$x^\circ = y = z = 60^\circ$$

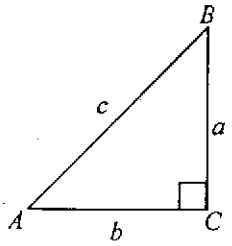
Right Triangle:

1. **Pythagoras' Theorem:**

It states that, in any right-angled triangle, the square on the hypotenuse is equal to the sum of the squares of the other two sides.

Using the letters in the diagram, the theorem is written as

$$c^2 = a^2 + b^2$$



This relation may be written as

$$a^2 = c^2 - b^2 \quad \text{or} \quad b^2 = c^2 - a^2$$

2. **Pythagorean Triples:**

Pythagorean triples are sets of numbers that satisfy Pythagorean theorem.

Let x be any positive number, then there is a right triangle whose sides are $3x$, $4x$ and $5x$.

It mean, any multiples of this set such as $6x$, $8x$, $10x$ or $9x$, $12x$, $15x$ form a Pythagorean triple. The most common Pythagorean triples are:

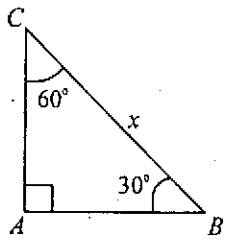
3, 4, 5
5, 12, 13
7, 24, 25

3. **The 30°–60°–90° Triangle:**

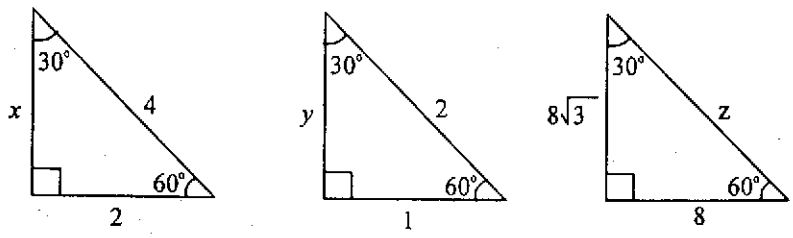
Let x be the hypotenuse of triangle ABC. Then

1) The leg opposite the 30° angle is $\frac{1}{2}(x)$.

2) The leg opposite the 60° angle is $\frac{1}{2}(x)(\sqrt{3})$.



Examples:



$x = 2\sqrt{3}$ $\quad$ $y = \sqrt{3}$ $\quad$ $z = 16$

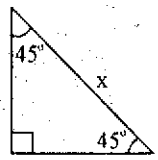
Note:

In an equilateral triangle, an altitude forms a 30°–60°–90° triangle and is equal to

$$\frac{1}{2}(\text{hyp}).\sqrt{3}$$

4. The 45°–45°–90° Triangle:

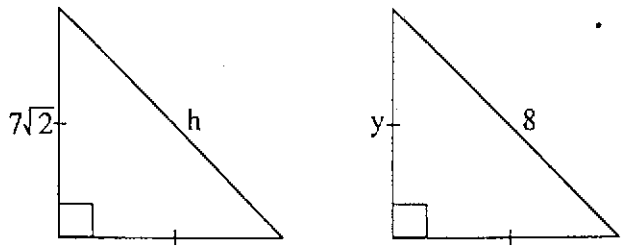
Let x be the hypotenuse of an isosceles right triangle, then



1) Each leg is $\frac{1}{2}(x).\sqrt{2}$

2) Hypotenuse = leg.$\sqrt{2}$

Examples:



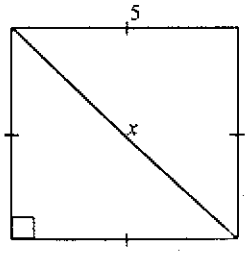
$h = (7\sqrt{2}).(\sqrt{2}) = 14 \quad y = \frac{1}{2}.8.\sqrt{2} = 4\sqrt{2}$

Note:

In a square, the diagonal forms a 45°–45°–90° triangle. Thus, in a square

Diagonal = side.$\sqrt{2}$

Example:



In a square, diagonal $= (\text{Side}).\sqrt{2}$

$x = 5.\sqrt{2}$

$x = 5\sqrt{2}$

Example:

What is the area of the square whose diagonal is 12?

Solution:

Let S be the side of the square, then

$$\text{Diagonal} = (\text{Side}).\sqrt{2}$$

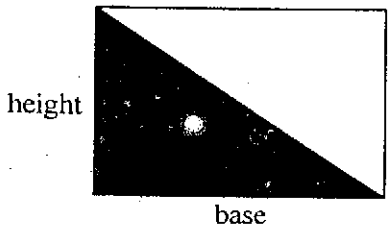
$$12 = S.\sqrt{2}$$

$$\Rightarrow \quad S = \frac{12}{\sqrt{2}}$$

$$\text{Area of square} = S^2 = \left(\frac{12}{\sqrt{2}}\right)^2 = \frac{144}{2} = 72$$

Area of Triangle:

To calculate the area of a triangle, first look at the following rectangle

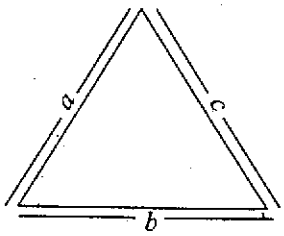


Area of rectangle = base × height

Area of triangle = $\frac{1}{2}$ × base × height

Perimeter of a triangle:

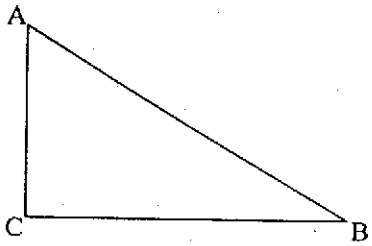
Perimeter of a triangle = sum of lengths of sides



Perimeter of a triangle = $a + b + c$

Triangle Inequality:

In ΔABC, given below



$AB > BC > AC$ and

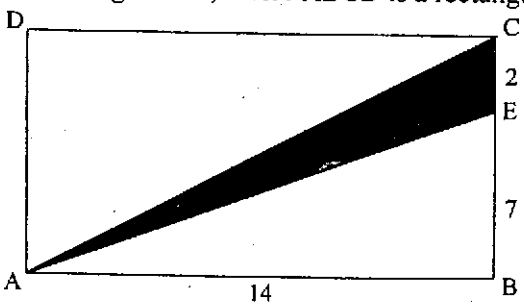
$\angle C > \angle A > \angle B$

These inequalities suggest the following theorems.

1. The perpendicular segment from a point to a line is the shortest distance from the point to the line.
2. **Triangle Inequality Theorem:**
   The sum of the lengths of two sides of a triangle is greater than the length of the third side.

Example:

What is the area and perimeter of the triangle AEC, where ABCD is a rectangular?



Solution:

The area of Rectangle ABCD is

$= 9 \times 14 = 126$

Now area of triangle ABE

$= \frac{1}{2}(14)(7) = 49$

and area of triangle ADC

$= \frac{1}{2}(9)(14) = 63$

Total area of the triangles ABE and ADC

$= 49 + 63 = 112$

Area of ΔAEC = (Area of the rectangle) – (Sun of the area of the triangle)

$= 126 - 112$

$= 14$

Perimeter of ΔAEC

In triangle ABE

$(AE)^2 = (14)^2 + (7)^2$

$= 196 + 49 = 245 \Rightarrow AE = 7\sqrt{5} = 16$

In triangle ADC

$(AC)^2 = (AD)^2 + (BC)^2$

$= (9)^2 + (14)^2 = 81 + 196 = 277 \Rightarrow AC = 17$

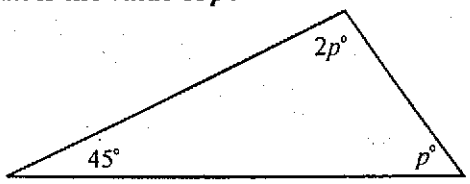
Perimeter of ΔAEC

$= AE + EC + CA$

$= 16 + 17 + 2 = 35$

Multiple Choice Questions (MCQs)

Q1. In the following triangle, what is the value of p?



(A) 35 (B) 45

(C) 55 (D) 40

Q2. The area of an equilateral triangle whose altitude is 10, is:

(A) $8\sqrt{3}$ (B) $2\sqrt{3}$

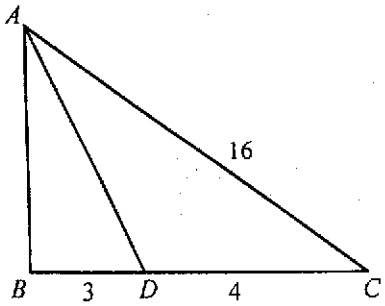
(C) $96\sqrt{3}$ (D) $4\sqrt{3}$

Q3. The two sides of a right triangle are 3 and 5. Then the length of the third side is:

(A) $\sqrt{34}$ (B) $\sqrt{22}$

(C) $2\sqrt{3}$ (D) $3\sqrt{2}$

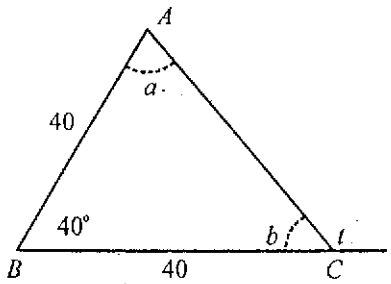
Q4. In the following triangle, AD =



(A) $3\sqrt{2}$ (B) $6\sqrt{6}$

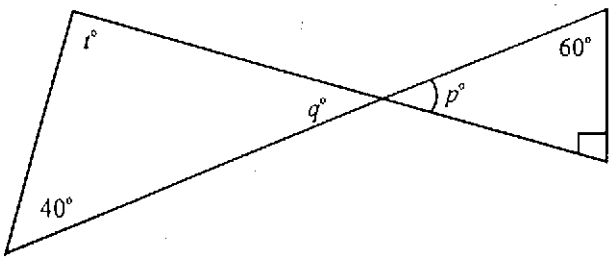
(C) $6\sqrt{3}$ (D) $3\sqrt{7}$

Q5. In the following figure, $t =$



(A) 110 (B) 115
(C) 70 (D) 140

Q6. What is the value of t, in the following diagram?



(A) 100° (B) 60°
(C) 30° (D) 110°

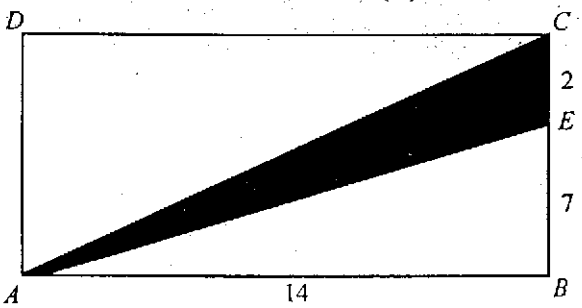


Fig. 1

Q7. What is the area of the triangle AEC, in the above figure 1?

(A) 12 (B) 49
(C) 14 (D) 21

Q8. What is the perimeter of ΔCEA, in the figure 1?

(A) 16 (B) 25
(C) 17 (D) $2 + 7\sqrt{5} + \sqrt{277}$

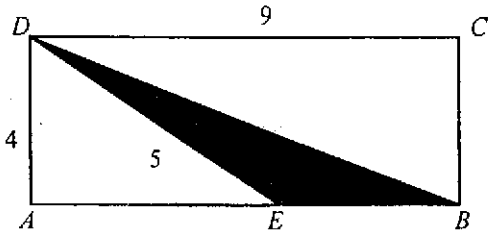


Fig. 2

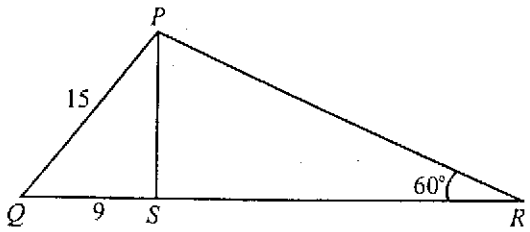
Q9. In figure 2, what is the area of ΔBED?

(A) 16 (B) 14
(C) 12 (D) 6

Q10. In figure 2, what is the perimeter of ΔBED?

(A) $3+\sqrt{93}$ (B) 11
(C) $11+\sqrt{97}$ (D) 81

Questions 11-12 refer to the following figure:



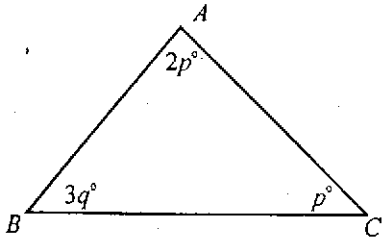
Q11. What is the area of ΔPQR?

(A) $3+4\sqrt{3}$ (B) $18(3+4\sqrt{3})$
(C) 54 (D) $84\sqrt{3}$

Q12. What is the perimeter of ΔPQR?

(A) 54 (B) 72
(C) $4+\sqrt{3}$ (D) $12(4+\sqrt{3})$

Q13. In the following figure, which of the following expresses a true relationship between p and q?



(A) $p = 180 + q$ (B) $q = 30 + p$
(C) $p = 90 + q$ (D) $p = 60 - q$

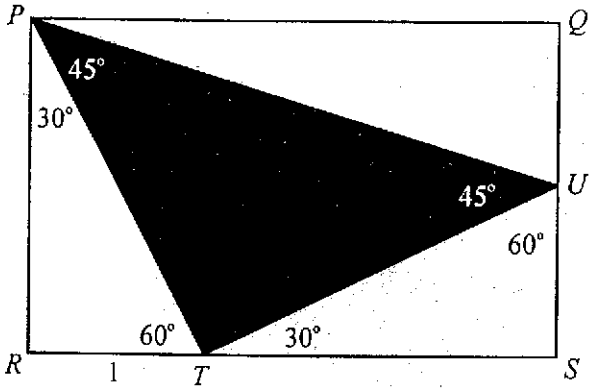


Fig. 3

Questions 14-15 refer to the above figure 3:

Q14. What is the perimeter of shaded triangle PTU?

(A) $2(2+\sqrt{2})$ (B) $4+\sqrt{2}$
(C) $2+\sqrt{2}$ (D) 4

Q15. What is the area of the shaded triangle?

(A) 2 (B) 4
(C) $2\sqrt{2}$ (D) $4\sqrt{2}$

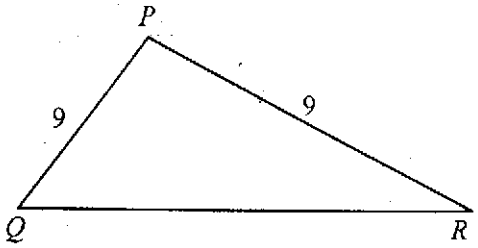
Q16. If the length of the two sides of a triangle are 4 and 6, then the length of the third side is:

(A) less than 11 (B) greater than 11
(C) less than or equal to 11 (D) None of these

Q17. What is the ratio of the diagonal to a side of a square?

(A) 1 : 1 (B) $\sqrt{2} : \sqrt{3}$
(C) $\sqrt{2} : 1$ (D) $\sqrt{2} : \sqrt{2}$

Q18. In the following figure, the perimeter of ΔPQR is:



(A) less than 18 (B) greater than 18
(C) equal to 18 (D) None of these

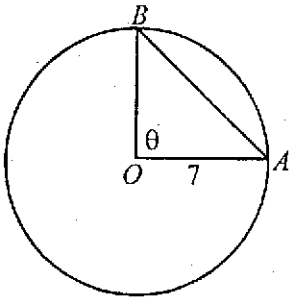


Fig. 4

Q19. In figure 4, if $\theta > 90$, then the length of AB is:

(A) less than 14 (B) greater than 14
(C) equal to 14 (D) not possible

Q20. In figure 4, if $\theta = 90$, then the perimeter of ΔAOB is:

(A) $7 + \sqrt{7}$ (B) $14 + \sqrt{2}$
(C) $14 + \sqrt{7}$ (D) $7(2 + \sqrt{2})$

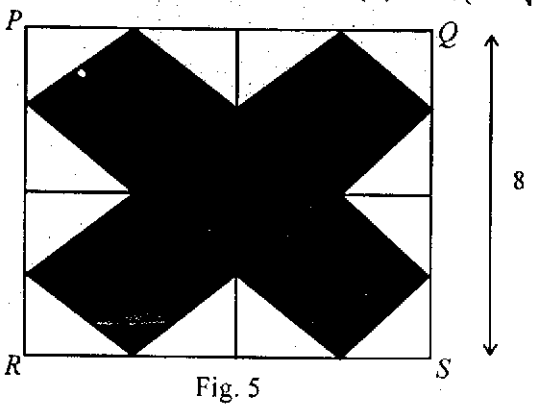


Fig. 5

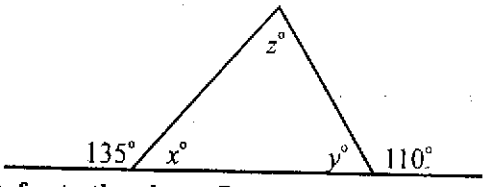
Q21. **In figure 5, the perimeter of shaded region is:**

(A) 17 (B) 24
(C) 34 (D) 21

Q22. **In figure 5, what is the area of the shaded region?**

(A) 40 (B) 24
(C) 20 (D) 12

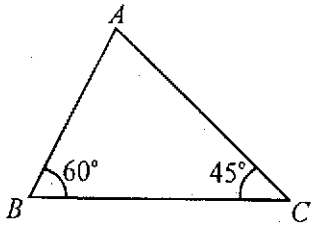
Q23.



Refer to the above figure, which of the following statement is true?

(A) $x + y > z$ (B) $x + y < z$
(C) $x + y = z$ (D) $x + y = x - z$

Q24. **In the following figure:**



which of the following statement is true?

(A) $AB > BC$ (B) $AB < BC$
(C) $AB = BC$ (D) $AC > BC$

Q25. **In a right triangle, if the difference between the measure of the two smaller angles is 30°, then what is the measure (in degrees) of the smallest angle?**

(A) 35 (B) 45
(C) 60 (D) 30

Q26. **In an isosceles triangle that is not equilateral, if all of its sides are integers and no side is longer than 25, then what is the largest perimeter?**

(A) 74 (B) 75
(C) 47 (D) 72

Q27. **What is the smallest integer, s, for which, s, $s + 3$, and $2s - 15$ can be the lengths of the sides of a triangle?**

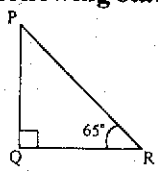
(A) 9 (B) 11
(C) 10 (D) 8

Q28. **If the perimeter of the triangle is $45 + 15\sqrt{3}$, and if the measure of the angles of the triangle are in the ratio of 1 : 2 : 3, then what is the length of the smallest side?**

(A) 10 (B) 15
(C) $3 + \sqrt{3}$ (D) $2 + \sqrt{2}$

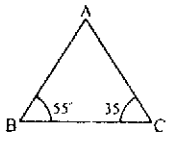
Q29. **Consider the accompanying diagram. Which of the following statements is true?**

(A) $PQ < QR$
(B) $PR < PQ$
(C) $PQ > QR$
(D) $PQ + QR < PR$



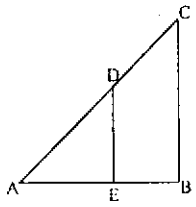
Q30. **Regarding the adjacent triangle, which of the following statements is true?**

(A) AB > AC (B) AB > BC
(C) AC > AB (D) AC > BC



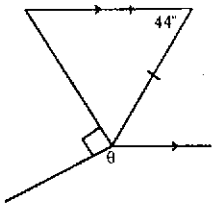
Q31. **In diagram, DE is parallel to CB, AE = BE, DE = 4, and EB = 3. What is CB?**

(A) 4
(B) 8
(C) 6
(D) 12

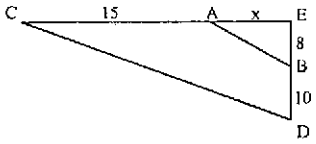


Q32. The value of θ is:

(A) 134°
(B) 148
(C) 112
(D) 206

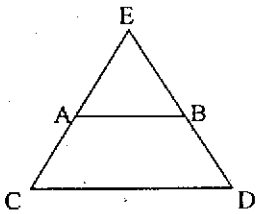


Q33. In ΔECD, $\overline{AB} \parallel \overline{CD}$. What is the value of x?



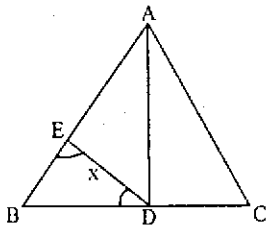
(A) 15
(B) 8
(C) 12
(D) 7

Q34. In ΔCDE, $\overline{AB} \parallel \overline{CD}$, then $\frac{EA}{AC}$ =



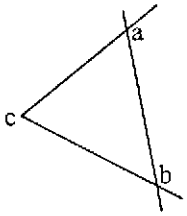
(A) $\frac{BD}{AC}$
(B) $\frac{EC}{AC}$
(C) $\frac{EB}{BD}$
(D) $\frac{AB}{CD}$

Q35. In figure given below, If AB = AC, AE = AD and ∠DAC = 20°. What is the value of x?



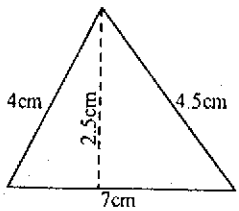
(A) 45°
(B) 10°
(C) 40°
(D) 35°

Q36. In figure below. What is the size of a + b – c



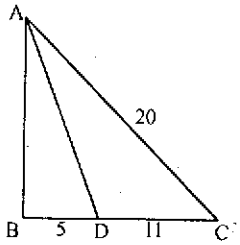
(A) 120° (B) 175°
(C) 110° (D) 180°

Q37. What is the area of the triangle?



(A) $8.75cm^2$ (B) $15.5cm^2$
(C) $17.5cm^2$ (D) $3.5cm^2$

Q38. What is the value of AD in the following triangle?



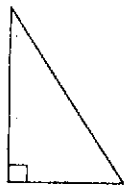
(A) 11 (B) 12
(C) $5\sqrt{2}$ (D) 13

Q39. If a triangle of base 4 has the same area as a circle of radius 4, what is the altitude of the triangle?

(A) 4π (B) 8π
(C) 2π (D) 10π

Q40. The area of the right triangle given below is $18cm^2$. The ratio of its legs is 2 : 3. What is the length of the hypotenuse?

(A) $\sqrt{39}cm^2$
(B) $3\sqrt{13}cm^2$
(C) $2\sqrt{18}cm^2$
(D) $6\sqrt{13}cm^2$



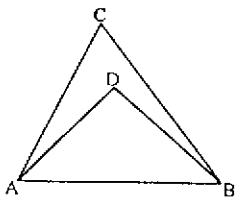
Q41. In a triangle, the ratio of the legs is 1 : 2. If the area of the triangle is $32cm^2$, what is the length of the hypotenuse?

(A) $4\sqrt{5}$ (B) $2\sqrt{3}$
(C) $3\sqrt{2}$ (D) $5\sqrt{6}$

Q42. The angles of a triangle are in the ratio 1 : 2 : 3. The largest angle in the triangle is:

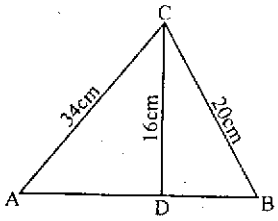
(A) 160° (B) 75°
(C) 40° (D) 90°

Q43. In triangle ABC below, angle BAC is greater than angle CBA. The bisector of angle A and angle B meet at point D. which of the following statement is (are) true?



(A) BD = AD (B) BD > AD
(C) BD ≤ AD (D) BD < AD

Q44. What is the length of AB in the figure below?

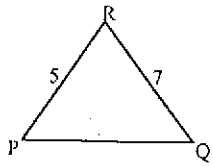


(A) 30cm (B) 12cm
(C) 24cm (D) 42cm

Q45. Two angles of a triangle are $(2a - 40)^\circ$ and $(3a + 10)^\circ$. The third angle is:

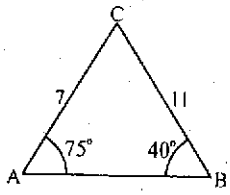
(A) $(230 + a)^\circ$ (B) $(180 + a)^\circ$
(C) $(210 - 5a)^\circ$ (D) $(220 - 5a)^\circ$

Q46. If the perimeter of ΔPQR below is 3 times the length of QR, then PQ =



(A) 3 (B) 9
(C) 7 (D) 5

Q47. Which of the following statements concerning the length of side AB is true?



(A) $AC < 7$ (B) $AB < 7$
(C) $7 < AB < 11$ (D) $AB > 11$

Q48. The three angles of a triangle are $(2a + 20)^\circ$, $(3a + 20)^\circ$ and $(a + 20)^\circ$. The value of a is

(A) 10 (B) 80
(C) 20 (D) 30

Q49. What is the area of an equilateral triangle PQR whose altitude is 6?

(A) $2\sqrt{3}$ (B) $\frac{1}{2}$
(C) 4 (D) $12\sqrt{3}$

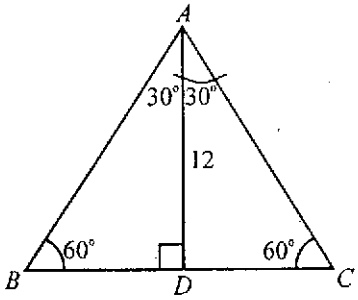
Explanatory Answers

Q1. (B) In any triangle, the sum of the angles = 180°

$\therefore \quad 45 + p + 2p = 180^\circ \Rightarrow 3p = 180 - 45$

$\Rightarrow \quad p = \frac{135}{3} = 45$

Q2. (C) To find the area, first of all we draw a equilateral triangle *ABC*, in which *AD* is altitude.



By, 30 – 60 Right Triangle Theorem,

$$BD = \frac{12}{\sqrt{3}} = \frac{4 \times \sqrt{3} \times \sqrt{3}}{\sqrt{3}} = 4\sqrt{3}$$

Now, Base= $4\sqrt{3} + 4\sqrt{3} = 8\sqrt{3}$ and altitude = 12

Thus, Area= Base × Altitude

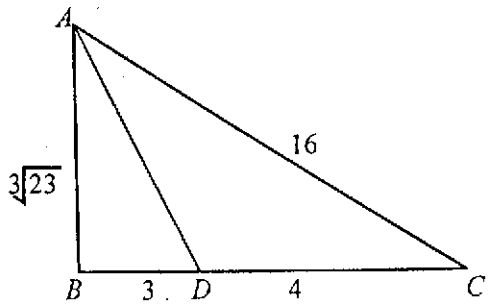
$= 8\sqrt{3} \times 12 = 96\sqrt{3}$

Q3. If the triangle is not right, then any number greater than 1 and less than 25 could be the length of the third side. Now, if the triangle is right, then there are only two possibilities:

(i) If 5 is the hypotenuse, then the legs are 4 and 3.

(ii) If 3 and 5 are two legs then hypotenuse is $\sqrt{34}$.

Q4. (B)



In ΔABC, $AC = 16$, $BC = 3 + 4 = 7$, using

Pythagorean theorem, $AC^2 = (AB)^2 + BC^2 \Rightarrow 16^2 = AB^2 + 7^2$

$\Rightarrow 256 = AB^2 + 49 \Rightarrow AB^2 = 256 - 49 = 207 \Rightarrow AB = \sqrt{207}$

$\Rightarrow AB = 3\sqrt{23}$.

Now, in ΔABD, $AD^2 = AB^2 + BD^2 \Rightarrow AD^2 = (3\sqrt{23})^2 + (3)^2$

$\Rightarrow AD^2 = 9(23) + 9 \Rightarrow AD^2 = 207 + 9 \Rightarrow AD^2 = 216$

$\Rightarrow AD = 6\sqrt{6}$

Q5. (A) Here, $\angle A + \angle B + \angle C = 180 \Rightarrow a + b + 40 = 180 \Rightarrow a + b = 140$

Because the given triangle is an isosceles, *i.e.*, $a = b$

Therefore, a and b are each 70, and

$x = 180 - 70 = 110°$

Q6. E Here $90° + 60° + p° = 180 \quad \Rightarrow p° = 30 \Rightarrow q° = 30°$

Now $q + t + 40 = 180 \quad \Rightarrow q + t = 140 \Rightarrow t = 140 - q$

$\Rightarrow t = 140 - 30 \quad \Rightarrow t = 110°$

Q7. (C) Area of the rectangle $ABCD = 14 \times 9 = 126$

To find the area of the shaded region, we subtract the two white areas of the right angled triangles ABE and ADC

Area of $\Delta ABE = \frac{1}{2}(14)(7) = 49$

Area of $\Delta ADC = \frac{1}{2}(9)(14) = 63$

Sum of the white areas = 49 + 63 = 112

Area of the shaded region = 126 – 112 = 14 square unit

Q8. (D) To calculate the value of AE in ΔABE, we use Pythagorean theorem,

$(AE)^2 = (AB)^2 + BE^2$

$\Rightarrow \quad AE^2 = (14)^2 + (7)^2 \Rightarrow AE^2 = 196 + 49$

$\Rightarrow \quad AE^2 = 245 \quad \Rightarrow AE = \sqrt{245} = 7\sqrt{5}$

Now, in ΔADC, we calculate AC

$AC^2 = AD^2 + DC^2 \quad \Rightarrow AC^2 = 9^2 + 14^2$

$\Rightarrow \quad AC^2 = 81 + 196 \quad \Rightarrow AC^2 = 277 \Rightarrow AC = \sqrt{277}$

Perimeter of $\Delta AEC = 2 + 7\sqrt{5} + \sqrt{277}$

Q9. (C) Area of the rectangle $ABCD = 4 \times 9 = 36$

Now, in ΔDAE,

$DE^2 = (4)^2 + AE^2 \Rightarrow (5)^2 = 4^2 + (AE)^2 \Rightarrow AE^2 = 25 - 16$

$\Rightarrow AE^2 = 9 \Rightarrow AE = 3$

Thus, $EB = AB - AE \Rightarrow ED = 9 - 3 = 6$ ($\because AB = DC = 9$)

Now Area of $\Delta DAE = \frac{1}{2}(3)(4) = 6$

and Area of the $\Delta BCD = \frac{1}{2}(9)(4) = 18$

Now Area of the shaded region

= Area of the rectangle $ABCD$ – (Area of ΔAED + Area of ΔBCD)

= 36 – (18 + 6) = 36 – 24 = 12 Square units

Q10. (C) In ΔDEB, $DE = 5$, $EB = 6$(from above Q), $DB = ?$

Now we find the value of DB

In ΔBDC,

$(BD)^2 = (BC)^2 + (DC)^2 \Rightarrow BD^2 = 16 + 81$

$\Rightarrow \quad BD^2 = 97 \Rightarrow BD = \sqrt{97}$

Now the perimeter of the ΔDEB

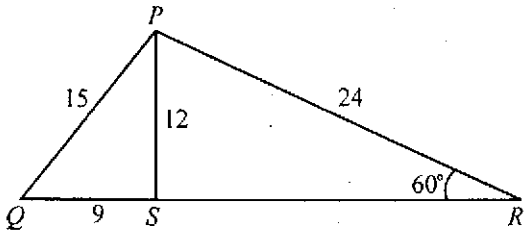
$= DE + EB + BD = 5 + 6 + \sqrt{97} = 11 + \sqrt{97}$

Q11. (B) ΔPQS is a right triangle, whose hypotenuse is 15 and its one leg is 9, using Pythagorean theorem

$PQ^2 = QS^2 + PS^2$

$\Rightarrow \quad (15)^2 = (9)^2 + PS^2 \Rightarrow = 225 - 81 = PS^2$

$\Rightarrow \quad PS^2 = 144 \Rightarrow PS = 12$



Now ΔPRS is a 30 – 60 – 90 right triangle, its shorter leg is 12. Then according to 30 – 60 Right Triangle Theorem hypotenuse PR will be 24 and leg RS will be $12\sqrt{3}$. So the area of the triangle PQR is

Area$=\frac{1}{2}$(Base) Altitude

$=\frac{1}{2}(9+12\sqrt{3})(12)$

$=18(3+4\sqrt{3})$

Q12. (D) The perimeter of the triangle PQR is the sum of its sides

So perimeter of $\Delta PQR = PQ + QR + PR$

$= 15 + (9 + 12\sqrt{3}) + 24$

$= 48 + 12\sqrt{3}$

$= 12(4 + \sqrt{3})$

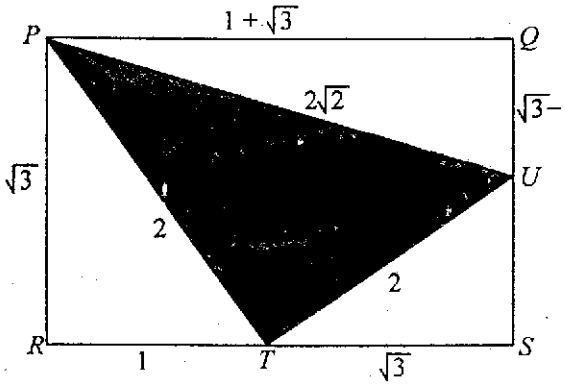
Q13. (D) In any triangle, sum of its interior angles is 180°. So

$p + 2p + 3q = 180°$

$\Rightarrow \quad 3p + 3q = 180 \quad \Rightarrow 3(p+q) = 180$

$\Rightarrow \quad p + q = 60 \quad \Rightarrow p = 60 - q$

Q14. (A) Triangles PRT and PQU both are 30 – 60 – 90 triangles. Thus both triangles will have sides 1, $\sqrt{3}$ and 2, and the inner triangle PTU is a 45 – 45 – 90 triangle and has sides 2, 2 and $\sqrt{2}$, as shown in the following figure



Also, $PQ = RS = 1 + \sqrt{3}$ and

$PR - US = QU = \sqrt{3} - 1$

Thus the perimeter of the shaded triangle is

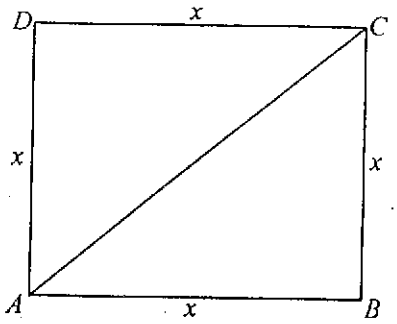
$2 + 2 + 2\sqrt{2} = 4 + 2\sqrt{2} = 2(2 + \sqrt{2})$

Q15. (A) The area of $\Delta PTU = \frac{1}{2}$(Base)(Altitude)

$=\frac{1}{2}(2)(2)$

$= 2$

Q16. (A) The sum of the two sides of a triangle always greater than the third side. Hence the third side is less than
$4 + 6 = 10 < 11$.



Q17. (C) First we draw the diagram, the diagonal of a square is the hypotenuse of each of the 45 – 90 – 45 triangle. Using

$(AC)^2 = (AB)^2 + (BC)^2$

$AC^2 = \sqrt{x^2 + x^2} = \sqrt{2x^2} = x\sqrt{2}$

Now $x\sqrt{2} : x \quad = \sqrt{2} : 1$

Q18. (B) In the given triangle, $QR < PQ + PR \quad \Rightarrow QR < 9 + 9$

$\Rightarrow QR < 18$

Therefore, the perimeter can be any number greater than 18.

Q19. (A) Since in the given figure OA and OB are radii, each is equal to 7. Thus AB could be any positive number less than 14.

Q20. (D) $(AB)^2 \quad = (OA)^2 + (OB)^2$

$= (7)^2 + (7)^2$

$(AB)^2 \quad = 49 + 49$

$AB \quad = \sqrt{98} \Rightarrow AB = 7\sqrt{2}$

Now perimeter of $AOB = AO + OB + AB$

$= 7 + 7 + 7\sqrt{2}$

$= 14 + 7\sqrt{2}$

$= 7(2 + \sqrt{2})$

Q21. (B) In the given figure, the perimeter of the shaded region consists of 12 line segments, these line segments are the hypotenuse of a 45 – 45 – 90 white triangle whose legs are 2. Then each line segment is, $\sqrt{(2)^2 + (2)^2} = \sqrt{4 + 4} = \sqrt{8} = 2\sqrt{2}$, and the perimeter of the shaded region is $24\sqrt{2} = 24$ approx.

Q22. (A) The white region consists of 12 right triangles, each of which has area of $\frac{1}{2}$ of the small square. Now the area of the small square is $2 \times 2 = 4$. The area of $\frac{1}{2}$ small square $= \frac{1}{2} \times 4 = 2$, the total area of the white half small squares is $= 12 \times 2 = 24$. Since the area of the large square is $8 \times 8 = 64$. Thus the area of shaded region is $64 - 24 = 40$.

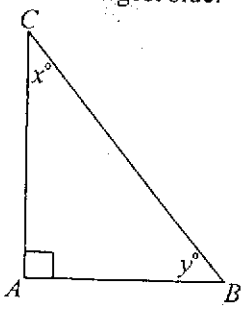
Q23. (A) Here, $x = 180 - 135 = 45$, and $y = 180 - 110 = 70$, then $x + y = 45 + 70 = 115$

Now, $x + y + z = 180 \Rightarrow 45 + 70 + z = 180 \Rightarrow z = 65$

Thus $115 > 65 \Rightarrow x + y > z$

Q24. (B) Since $60 + 45 = 105 \Rightarrow m\angle A = 75$, this shows that $\angle A$ is the largest angle and BC is the side opposite to the largest angle. Thus BC is the largest side.

Q25. (D) First of all we draw the diagram



By the given condition

$x - y = 30 \quad \ldots(i)$

Because, $\angle A + \angle B + \angle C \quad = 180$

$90 + y + x \quad = 180$

$x + y \quad = 90 \quad \ldots(ii)$

Adding (i) and (ii), we have

$x - y = 30$

$\underline{x + y = 90}$

$2x = 120 \quad \Rightarrow \boxed{x = 60^\circ}$

Put $x = 60$ in (ii), we get

$60 + y = 90 \quad \Rightarrow \boxed{y = 30^\circ}$

Q26. Since no side of the triangle can be longer than 25, thus, we suppose that both of the equal sides are 25. Then the largest possible value of the third side is 24. Its perimeter is

$$25 + 25 + 24 = 74$$

Q27. (C) In a triangle the sum of the sides of any sides must be greater than the third side. $S + (S + 3)$ to be greater than

$2S - 15$, thus $2S + 3$ must be greater than $2S - 15$; but that is always true. For $S + (2S - 15)$ to be greater than

$S + 3 \Rightarrow 3S - 15$ must be greater than $S + 3$;

but $3S - 15 > S + 3$ is true only if

$3S - S > 3 + 15$

$2S > 18 \Rightarrow S > 9$

Thus answer is 10.

Q28. (B) Since, angles are in ratio 1 : 2 : 3, thus

$\theta + 2\theta + 3\theta = 180 \Rightarrow 6\theta = 180 \Rightarrow \theta = 30$

Here, $\theta = 30$, $2\theta = 60$ and $3\theta = 90$, so the given triangle is a 30 – 60 – 90 triangle, thus its sides are x, $2x$ and $x\sqrt{3}$. Thus its perimeter is $3x + x\sqrt{3}$

but given that perimeter is $45 + 15\sqrt{3}$,

$\Rightarrow \quad 3x + x\sqrt{3} = 45 + 15\sqrt{3}$

$x(3 + \sqrt{3}) = 15(3 + \sqrt{3})$

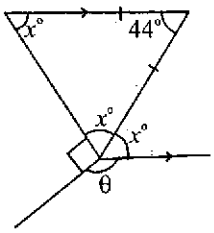
$\Rightarrow \quad \boxed{x = 15}$

Q29. **(C)** Since the measure of angle R is 65°, the measure of angle P is 25°. Since the larger side is opposite the larger angle, therefore, PQ > QR

Q30. **(C)** Since the larger side is always opposite to the larger angle. In the fig, angle A is 90° the larger side of the triangle is BC, followed by AC and then at last AB.

Q31. **(B)** Since DE is parallel CD, the triangle ADE and ACB are similar. Therefore, corresponding sides are proportional. So DE is to AB as AB to CB. Since AE = EB, $\frac{AB}{AE}$ is $\frac{1}{2}$. Therefore CB is twice or 8.

Q32. **(A)**



Since the triangle is isosceles triangle than

$x^\circ + x^\circ + 44^\circ = 180^\circ$

$\Rightarrow \quad 2x^\circ = 180^\circ - 44^\circ = 136^\circ$

and

$2x + \theta + 90^\circ = 360^\circ$

$136 + \theta + 90^\circ = 360^\circ$

$\theta + 226 = 360^\circ$

$\Rightarrow \quad \theta = 360^\circ - 226 = 134^\circ$

therefore $\theta = 134^\circ$

Q33. **(C)** Since, If a line interesting the interior of a triangle is parallel to one side, then the line divides the other two sides proportionally.

$$\frac{x}{15} = \frac{8}{10}$$

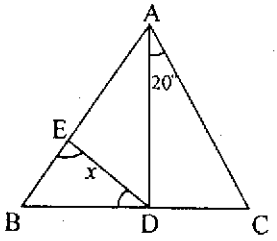
$$10x = 120$$

$\boxed{x = 12}$

Q34. **(C)** Since the line AB is parallel to line CD, therefore the line AD divides CE and ED proportionally.

$$\therefore \frac{EA}{AC} = \frac{EB}{BD}$$

Q35. **(B)**



As $AB = AC$

So, $m\angle ACB = m\angle ABC = y$

In ΔABC

$m\angle ACB + m\angle BCA + m\angle CAB = 180^\circ$

$y + y + 40 = 180^\circ$

$\Rightarrow 2y = 180^\circ - 40^\circ = 140$

$y = 70^\circ$

In ΔADE

$AD = AE$

So, $m\angle AED + m\angle ADE = z^\circ$

$m\angle AED + m\angle ADE + m\angle DAE = 180^\circ$

$z + z + 20^\circ = 180^\circ$

$\Rightarrow 2z = 180^\circ - 20^\circ = 160^\circ$

$\Rightarrow z = 80^\circ = m\angle AED$

$m\angle AED + m\angle DEB = 180^\circ$

$80^\circ + m\angle DEB = 180^\circ$

$\Rightarrow m\angle DEB = 180^\circ - 80^\circ = 100^\circ$

In ΔBED

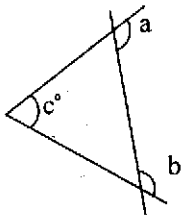
$m\angle BED + m\angle EDB + m\angle DBE = 180^\circ$

$100 + x + 70^\circ = 180^\circ$

$\Rightarrow x + 170^\circ = 180^\circ$

$x = 180^\circ + 170^\circ = 10^\circ$

Q36. **(D)**



$(180^\circ - a) + (180^\circ - b) + c = 180^\circ$

$180^\circ - a + 180^\circ - b + c = 180^\circ$

$360^\circ - (a + b - c) = 180^\circ$

$a + b - c = 360^\circ - 180^\circ$

$\Rightarrow a + b - c = 180^\circ$

Q37. **(A)** Area of the triangle $= \frac{1}{2}$ Base $\times$ Altitude

$= \frac{1}{2} \times 7 \times 2.5$

$= 8.75 \text{ cm}^2$

Q38. **(D)** Using the Pythagorean theorem in ΔABC

$(per)^2 = (hyp)^2 - (base)^2$

$AB^2 = (20)^2 - (11 + 5)^2$

$AB^2 = 400 - 256 = 144$

$AB = 12$

Again using the Pythagorean theorem in ΔABD

$(hyp)^2 = (base)^2 + (per)^2$

$AD^2 = (12)^2 + (5)^2$

$= 144 + 25 = 169 \Rightarrow \boxed{AD = 13}$

Q39. **(B)** Area of the circle $= \pi r^2$

$= \pi(4)^2 = 16\pi$

Area of the triangle $= \frac{1}{2} \times 4 \times$ Altitude

$= 2 \times$ Altitude

Since area of the given triangle is equal to the area of the circle of radius 4, therefore

$16\pi = 2 \times$ Altitude

$\Rightarrow$ Altitude $= 8\pi$

Q40. **(D)** Let legs be $2x$ and $3x$, by Pythagorean theorem

$(hyp)^2 = (2x)^2 + (3x)^2$

But $\frac{1}{2}.2x.3x = 18$

$3x = 18 \Rightarrow \boxed{x = 6}$

$(hyp)^2 = (12)^2 + (18)^2$

$= 144 + 324 = 468$

$hyp = \sqrt{468} = 6\sqrt{13}$

Q41. Let legs of the triangle be x and $2x$, then

$hyp^2 = (x)^2 + (2x)^2$

But $32 = \frac{1}{2}.x.2x$

$\Rightarrow x^2 = 16 \Rightarrow x = 4$

$hyp^2 = (4)^2 + (8)^2 = 16 + 64 = 80$

$\Rightarrow hyp = \sqrt{80} = \boxed{4\sqrt{5}}$

Q42. **(D)** The sum of the angles in a triangle $= 180^\circ$

Given ratio 1 : 2 : 3

Sum of the ratios $= 1 + 2 + 3 = 6$

Largest angle $= \frac{3}{6} \times 180 = 90^\circ$

Q43. If $\angle CAB > \angle ABC$, then $\angle A > \frac{1}{2}\angle B$. Then $\angle DAB$ greater than $\angle DBA$. Therefore

$DB > DA$ (opposite sides of the angle)

Q44. **(D)** Solving ΔCAD

$(AD)^2 = (34)^2 - (16)^2$

$= 1156 - 256 = 900$

$\boxed{AD = 30}$

Now solving ΔCBD

$(BD)^2 = (20)^2 - (16)^2$

$= 400 - 256$

$(BD)^2 = 144$

$\boxed{BD = 12}$

$AB = AD + BD$

$= 30 + 12 = \boxed{42}$

Q45. **(C)** Let the angles of triangle be

$(2a - 40)^\circ, (3a + 10)^\circ, x$ then

$(2a - 40)^\circ + (3a + 10^\circ) + x = 180^\circ$

$\Rightarrow \quad x = 180^\circ - [(2a - 40) + (3a + 10)]$

$= 180^\circ - [5a - 30^\circ]$

$= 180^\circ - 5a + 30^\circ$

$x = (210 - 5a)$

Q46. **(B)** The perimeter of the triangle is the sum of the lengths of the 3 sides. Since the perimeter is equal to 3 times the length of QR ($3 \times 7 = 21$)

$5 + 7 + PQ = 21$

$PQ = 21 - 12$

$\boxed{PQ = 9}$

Q47. **(C)** $\angle C + 75^\circ + 40^\circ = 180 \Rightarrow \angle C = 65$. Here $\angle A$ is the largest angle, B is the smallest and C is in between. Therefore

$CA < AB < BC$

$7 < AB < 10$

Q48. **(C)** $(2a + 20) + (3a + 20) + (a + 20) = 180$

$6a = 180 - 60$

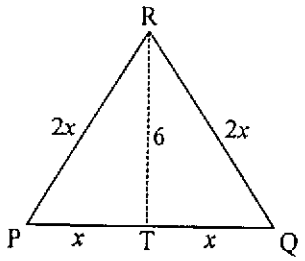
$6a = 120$

$\Rightarrow \quad \boxed{a = 20}$

Q49. **(D)**

As PQR is equilateral triangle

$\therefore \overline{PQ} = \overline{QR} = RP = 2x$



In ΔPTR

$(hyp)^2 = (base)^2 + (prep)^2$

$(2x)^2 = (x)^2 + (6)^2$

$4x^2 = x^2 + 36$

$4x^2 - x^2 = 36$

$3x^2 = 36$

$x^2 = 12$

$x = 2\sqrt{3}$

So,

$PQ = 2x = 2(2\sqrt{3})$

$= 4\sqrt{3}$

Area of PQR $= \frac{1}{2} \times$ base $\times$ Altitude

$= \frac{1}{2} \times 4\sqrt{3} \times 6$

Area of PQR $= 12\sqrt{3}$

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Chapter 3

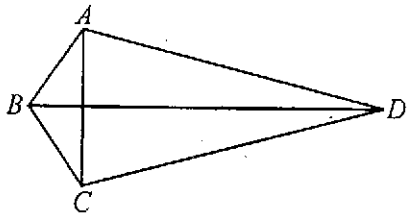
QUADRILATERALS AND POLYGONS

Quadrilateral:

A quadrilateral is a plane figure with four straight sides. The elements of a quadrilateral are its four sides and four angles.

Diagonal of Quadrilateral:

A diagonal of a quadrilateral is a line segment joining two non-consecutive vertices. In the following figure, the diagonals of the quadrilateral *ABCD* are *AC* and *BD*.



Family or Types of Quadrilateral:

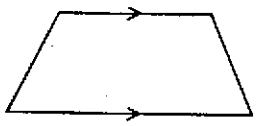
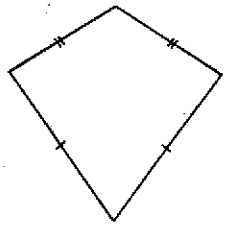
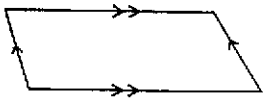
The properties of a quadrilateral are the features that are characteristic of that shape. They can include any of the following:

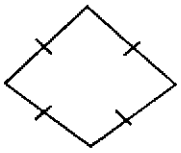
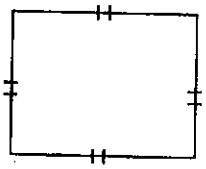
Sides: Are the side lengths equal? Are the sides parallel?

Angles: Are any angles equal? Are any angles right angles?

Diagonals: Are the diagonals equal? Do the diagonals bisect each other? Do the diagonals bisects the angles through which they pass? Do the diagonals cut at right angles?

The combination of properties is different for each quadrilateral.

Quadrilateral		Properties
Trapezium		❖ A quadrilateral with one pair of opposite sides parallel.
Kite		❖ A quadrilateral with two pairs of equal adjacent sides.
Parallelogram		❖ A quadrilateral with opposite sides parallel.
Rectangle		❖ All properties of parallelogram plus. (i) All angles are right angles. Diagonals are equal. (ii) Two axes of symmetry (perpendicular to sides)

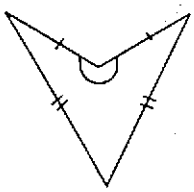
Rhombus		❖ All properties of parallelogram plus. (i) All sides are equal. (ii) Diagonals bisect at right angles. (iii) Diagonals bisect the angles through which they pass. (iv) Two axes of symmetry.
Square		❖ All properties of rectangle plus. ❖ All sides are equal ❖ Four axes of symmetry.

Convex Quadrilateral:

A quadrilateral is called convex if each of its interior angles is less than two right angles.

Re-entrant Quadrilateral:

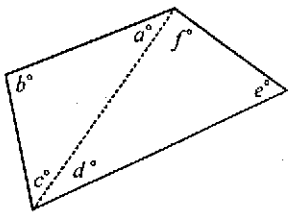
A quadrilateral is called re-entrant if one of its interior angles is reflex. A kite with one reflex interior angle is an example of a re-entrant quadrilateral.



Angle Sum of a Quadrilateral:

If we split a quadrilateral into two triangles as shown, then we can calculate the angle sum.

Angle sum $= a + b + c + d + e + f$



But $a + b + c = 180$

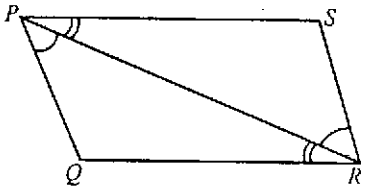
and $d + e + f = 180$

So $a + b + c + d + e + f = 180 + 180$

$= 360°$

Additional Properties of Parallelograms:

We have learnt that, in a parallelogram, it has two pairs of parallel sides. Opposite sides are equal. Opposites angles are equal. Diagonals bisect each other. Now, we shall derive some of the properties of parallelogram using the properties of parallel lines and angles. Consider a parallelogram $PQRS$ with diagonal $\overline{PR}$.



Then $\angle SPR = \angle PRQ$ ($\because$ Alternate angles)

$\angle QPR = \angle PRQ$ ($\because$ Alternate angles, $PS \parallel QR$)

$\therefore \angle SPR + \angle QPR = \angle PRQ + \angle PRS$

$\therefore \quad \angle QPS = \angle QRS$

Now, in ΔPQR

$\angle PQR + \angle QPR + \angle PRQ = 180°$

$\therefore \quad \angle PQR = 180 - (\angle QPR + \angle PRQ)$

$= 180 - (\angle PRS + \angle SPR) = \angle PSR$

$\therefore \quad \angle QPR = QRS$ and $PQR = PSR$ .....(i)

Hence, from (i), we can say that in a parallelogram "opposite angles are congruent."

Now, also in parallelogram $PQRS$,

$\angle QPS + \angle PQR + \angle QRS + \angle RSP = 360°$

But $\angle QPS = \angle QRS$ and $\angle PQR = \angle RSP$

$\therefore \quad 2(\angle QPS + \angle PQR) = 360^\circ$

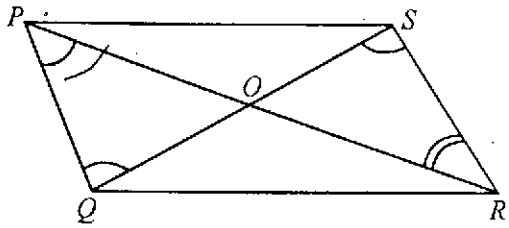
$\therefore \quad \angle QPS + \angle PQR = 180^\circ$

Similarly, $\angle QRS + \angle RSP = 180^\circ$ .....(ii)

Thus, from (ii), we can say, that, in a parallelogram,

"Pairs of adjacent angles are supplementary."

Now in triangle PQR and RSP,



$\angle PRQ = \angle SPR$

$\angle QPR = \angle PRS$

$\Rightarrow \quad PR = PR$ (common)

$\therefore \quad \Delta PQR = \Delta RSP$ ($\because$ Two angles and a side are congruence)

$\Rightarrow \quad PQ = RS$ and $PS = QR$ .....(iii)

Hence, in a parallelogram,

"Opposite sides are congruent."

In triangles PQO and RSO

$PQ = RS$

$\angle PQO = \angle RSO$

$\angle QPO = \angle SRO$

$\therefore \quad \Delta PQO \cong \Delta RSO$ (by AAS congruence)

$\Rightarrow \quad PO = RO$ and $QO = SO$ .....(iv)

Hence, in a parallelogram

"Two diagonals bisect each other."

Now in triangles PQR and RSP

$QR = PS$

$PQ = RS$

$PR = PR$ (common)

$\therefore \quad \Delta PQR \cong \Delta RSP$

Similarly $\Delta PQS \cong \Delta RSQ$

Hence, in a parallelogram

"A diagonal divides into two congruent triangles."

Test for Quadrilaterals:

We have seen, that each of the quadrilaterals has several properties, but it is not necessary to check all the properties when trying to identify a shape.

Tests for a Parallelogram:

To identify the shape as a parallelogram, satisfying any of these conditions is sufficient.

1. Both pairs of opposite sides are parallel or equal.

2. Both pairs of opposite angles are equal.
3. One pair of opposite sides is equal and parallel.
4. Diagonals bisect each other.

Tests for a Rhombus:

To identify the shape as a rhombus, satisfying any of these conditions is sufficient.

1. All sides are equal.
2. Diagonals bisect at right angles.

Common Properties in all Quadrilaterals:

The common properties in all quadrilaterals are:

1. Diagonals bisect each other.
2. Opposite angles are equal.

Polygons:

A simple closed figure formed by three or more line segments is known as a polygons. Polygons are named by the number of sides they have. The following is a list of the names given to polygons according to the number of their sides.

Number of sides	Name of the polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nanagon
10	Decagon

Vertex of a Polygon:

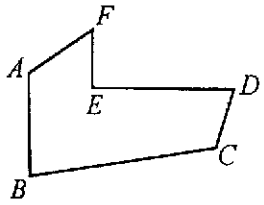
The angular points of a polygon are called its vertices, and the number of sides of a polygon is equal to the number of its vertices.

Other Names of Polygons:

Other words used to describe polygons are:

Concave:

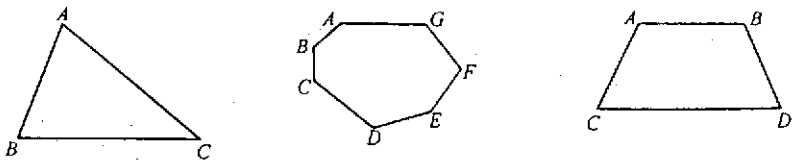
A polygon is said to be concave when one or more of its interior angles is greater than two right angles.



The above figure shows a concave polygon: the measure of interior angle *DEF* is greater than 180°.

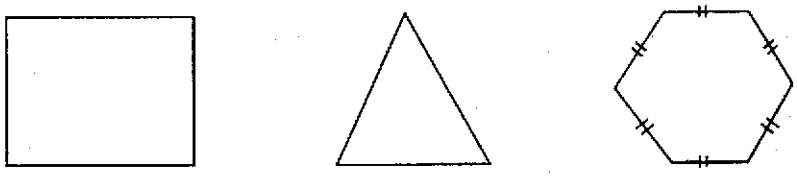
Convex Polygon:

A polygon is said to be convex when all its interior angles are less than two right angles. The following figures are examples of convex polygon.



Regular Polygon:

A polygon is said to be regular if all its sides as well as its angles are equal. The following figures are some examples of regular polygons:



Angle Sum of a Polygon:

The angle sum of a polygon with n sides is equal to

$$(n-2)\times 180^\circ$$

The size of each interior angle of a regular polygon with n sides is

$$\frac{(n-2)\times 180^\circ}{n}$$

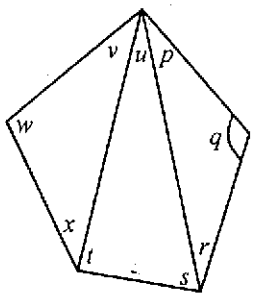
or $S = (2n-4)$ right angles.

In words

The sum of the angles in a polygon is equal to 'the number of sides less 2' multiplied by 180°.

Example 1:

What is the sum of the angles in a pentagon?



Solution:

$S = p + q + r + s + t + u + v + w + x$

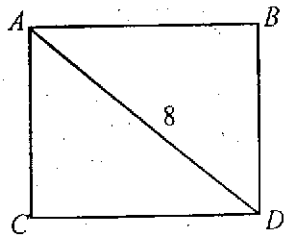
$S = (n-2)\,180^\circ \qquad n = 5$

$S = (5-2)\,180^\circ$

$S = 540^\circ$

Example 2:

What is the length of each side of a square if its diagonals is 8?



Solution:

In the adjacent diagram, diagonal AD is the hypotenuse of a 45 – 45 – 90 right triangle.

$$\text{Then} = \frac{AC}{\sqrt{2}} = \frac{8}{\sqrt{2}} = \frac{8\sqrt{2}}{2} = 4\sqrt{2}$$

Example 3: A decagon is drawn in which each angle has the same measure. What is the measure of each angle?

Solution: The sum of the measures of n sided is

$$(n-2)\,180^\circ$$

Here $n = 10$

$\therefore$ Sum of the angles of a decagon $= (10-2)\,180$

$= 8(180)$

$= 1440$

$\because$ Angles are equal measure, so each angle is

$= 1440 \div 10 = 144$

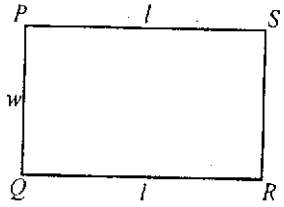
Perimeter:

The distance all around a shape is called its perimeter. Or

The perimeter of a figure is the measure of its bounding line-segments or curves.

Perimeter of a Quadrilateral:

For a quadrilateral $PQRS$



Perimeter = Sum of all the sides

$= PQ + QR + RS + SP$

$= 2(PQ + QR)$

$= 2(l + w)$

Perimeter of a Rectangle:

Similarly, the perimeter of rectangle $= 2(l + w)$, where l and w are the length and the width, respectively.

Perimeter of a Square and Rhombus:

Since, in a square and rhombus all sides are equal, so the perimeter of a rhombus or a square $= 4l$

Where l is the length of each side.

Area:

The area of a shape is the amount of flat space taken up by the shape.

Area of the Rectangle and Square:

Area of a rectangle = length × width

$= l \times w$

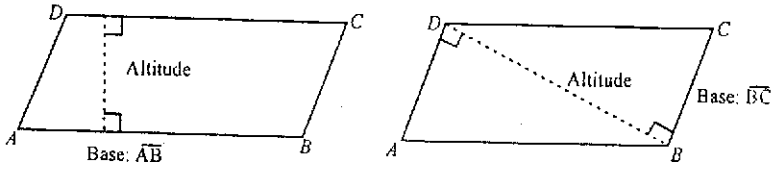
For a square

length = width *i.e.*, $w = l$

$\therefore$ Area of square $= l \times l = l^2$

Area of Parallelograms and Triangles:

Any side of a parallelogram may be called the base of the parallelogram. For each base, there is a corresponding altitude.



An altitude of a parallelogram is a perpendicular segment whose end points lie on opposite sides of a parallelogram.

In a parallelogram $ABCD$, let

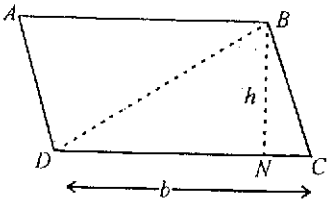
$DC = b$(base) and

distance between the parallel lines

AB and DC be h. Then

area of parallelogram = base × altitude

$= b \times h = bh$



Area of Triangle:

Since a diagonal of a parallelogram divides it into two congruent triangles, also the area of the triangles are

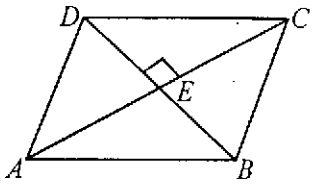
equal, therefore

Area of BDC = Area of ABD

Area of $BDC = \frac{1}{2}$ [Area of parallelogram $ABCD$]

$= \frac{1}{2} b \times c$

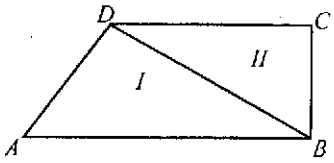
Area of Trapezium and Rhombuses:



In a rhombus, all sides are congruent and the diagonals are perpendicular to each other.

The area A of a rhombus is one-half the product of the lengths of its diagonals, d_1 and d_2, that is

$$A = \frac{1}{2} d_1 d_2$$



The formula for the area of a trapezium can be found by finding the areas of the triangles formed by drawing a diagonal.

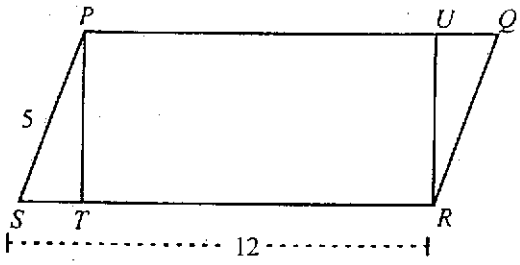
Area of $ABCD$ = Area of Triangle I + Area of Triangle II

Thus, the area A of a trapezium is one-half the product of its altitude and the sum of its bases, b and b'. That is

Area of Trapezium ABCD $= \frac{1}{2}bh + \frac{1}{2}b'h = \frac{1}{2}h(b + b')$

Multiple Choice Questions (MCQs)

Q1. In the following figure, the area of parallelogram *PQRS* is 36. What is the area of rectangle *PURT*?



(A) 60 (B) 40

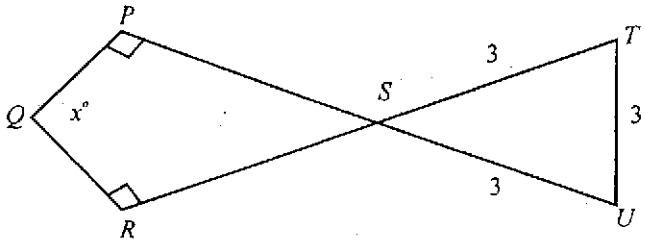
(C) 24 (D) 36

Q2. The length of a rectangle is twice its width. If the perimeter of a rectangle is the same as the perimeter of a square of size 9, what is the length of a diagonal of the rectangle?

(A) 180 (B) $3\sqrt{5}$

(C) 36 (D) $6\sqrt{5}$

Q3. In the figure below, what is the value of x?



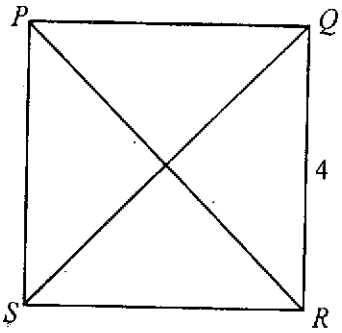
(A) 120 (B) 60

(C) 90 (D) 30

Q4. **A triangle has sides, 5 inches, 12 inches and 13 inches, respectively. A rectangle equal in area to that of the triangle has a width of 4 inches. The perimeter of the rectangle, expressed in inches, is:**

(A) 23 (B) 28
(C) 60 (D) 32

Q5. **In the following figure, square *PQRS* has divided into four triangles by drawing diagonals *PR* and *QS*. What is the sum of the perimeters of triangles?**



(A) $2+\sqrt{2}$ (B) $4+\sqrt{2}$
(C) $4(1+\sqrt{2})$ (D) $16(1+\sqrt{2})$

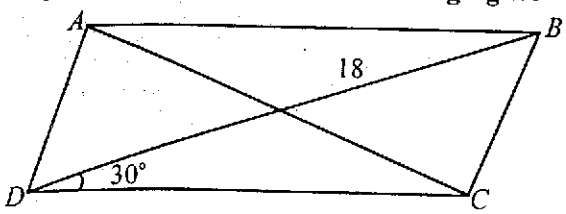
Q6. **If the length of a rectangle is 4 times its width, and if its area is 196, what is its perimeter?**

(A) 60 (B) 28
(C) 35 (D) 70

Q7. **If the angles of a hexagon are in the ratio 2 : 4 : 4 : 4 : 5 : 5, what is the degree measure of the smallest angle?**

(A) 30 (B) 60
(C) 40 (D) 70

Questions 4-5 refer to the following figure



Q8. **What is the area of the rectangle *ABCD*?**

(A) $3\sqrt{3}$ (B) $36\sqrt{3}$
(C) 81 (D) $81\sqrt{3}$

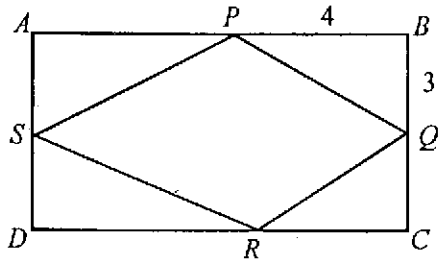
Q9. **What is the perimeter of the rectangle *ABCD*?**

(A) $81+81\sqrt{3}$ (B) $18+18\sqrt{3}$
(C) $12(1+2\sqrt{3})$ (D) $12+12\sqrt{3}$

Q10. **The length of a rectangle is 3 more than the side of a square, and the width of the rectangle is 3 less than the side of the square. If the area of the square is 58, what is the area of the rectangle?**

(A) 40 (B) 20
(C) 39 (D) 49

Question 11-12 refer to the following figure, in which *P*, *Q*, *R* and *S* are midpoints of the sides of rectangle *ABCD*.



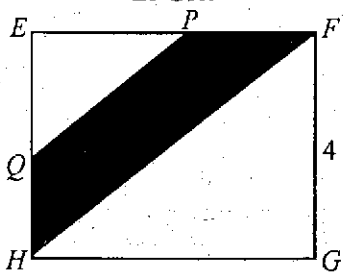
Q11. What is the perimeter of quadrilateral *PQRS*?

(A) 20 (B) 40
(C) 60 (D) 30

Q12. What is the area of the quadrilateral *PQRS*?

(A) 20 (B) 24
(C) 40 (D) 96

Question 13-14 refer to the following figure, in which P and Q are midpoints of two of the sides of square $EFGH$.



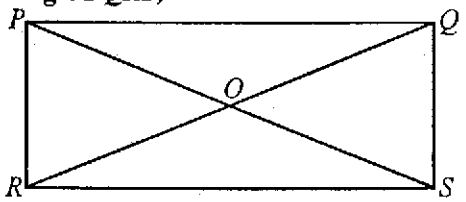
Q13. What is the perimeter of the shaded region?

(A) $2+3\sqrt{2}$ (B) 8
(C) $4+6\sqrt{2}$ (D) $3+2\sqrt{2}$

Q14. What is the area of the shaded region?

(A) 4 (B) 4.5
(C) 6 (D) 1.5

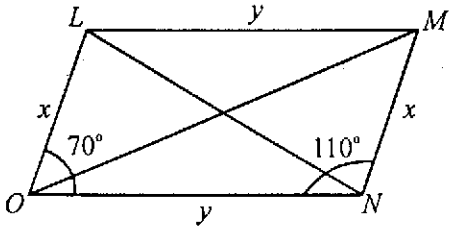
Q15. Refer to the following rectangle *PQRS*,



which of the following statements is true?

(A) Area of $\Delta POR >$ Area of ΔORS
(B) Area of $\Delta POR =$ Area of ΔORS
(C) Area of $\Delta ORS >$ Area of ΔPOR
(D) $\Delta POR \cong \Delta ORS$

Q16. Refer to the following figure,



which of the following statements is true?

(A) Diagonal $LN <$ Diagonal MO
(B) Diagonal $LN <$ Diagonal MO

(C) Diagonal MO = Diagonal LN

(D) $LA \parallel MO$

Q17. What is the perimeter of a 30-60 right triangle whose longer leg is $2s$?

(A) $4S(2 + 2\sqrt{3})$ (B) $S(4 + 2\sqrt{3})$

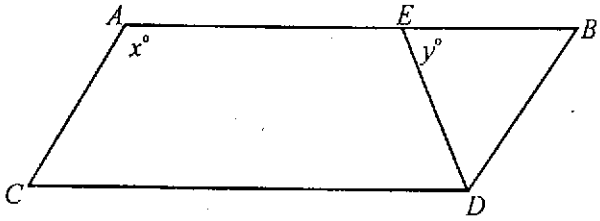
(C) $S(2 + 2\sqrt{3})$ (D) $4S + 2S\sqrt{2}$

Q18. If the area of a rectangle is 40, then its perimeter will:

(A) equal to 24 (B) less than 24

(C) greater than 24 (D) less than or equal to 22

Q19. In the following figure,

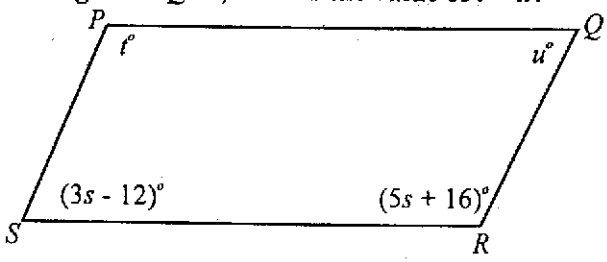


$x =$

(A) Y (B) $\frac{1}{y}$

(C) $4y$ (D) $2y$

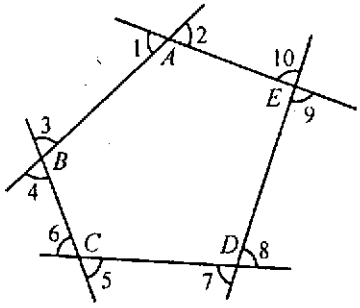
Q20. In the following parallelogram $PQRS$, what is the value of $t - u$?



(A) 72 (B) 50

(C) 91 (D) 22

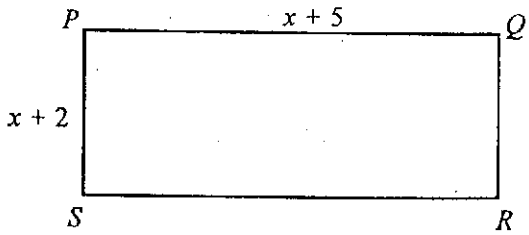
Q21. In the following pentagon $ABCDE$, what is the sum of all marked exterior angles?

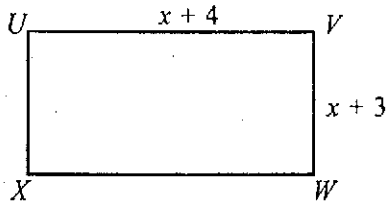


(A) 720 (B) 540

(C) 360 (D) 300

Q22. Following, two rectangles are given, the area of the rectangle $PQRS$ is 90, what is the area of the rectangle $UVWX$?

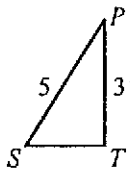




(A) 90 (B) 60
(C) 68 (D) 92

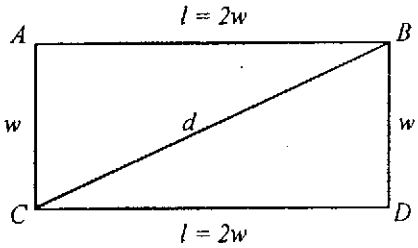
Explanatory Answers

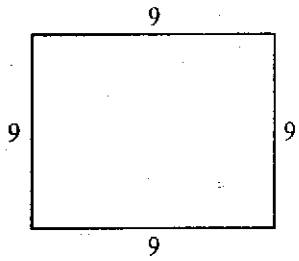
Q1. (C) Area of the parallelogram: $A = bh$, in the given figure,
$A = 36$ and $b = 12$, therefore
$36 = 12h \Rightarrow \boxed{h = 3}$
Now, in ΔPTS, $(SP)^2 = (PT)^2 + (ST)^2$
$\Rightarrow 25 = 9 + ST^2$
$\Rightarrow 16 = ST^2 \Rightarrow ST = 4$



Now $TR = SR - ST \Rightarrow TR = 12 - 4 = 8$
Now, Area of rectangle $= A = lw$
Hence, Area of rectangle $PURT = 8 \times 3 = 24$

Q2. (D) Now, Perimeter of square = Perimeter of rectangle
Since, Perimeter of square = 4(9)
= 36
Therefore, Perimeter of rectangle $= 2(l + w)$
$\Rightarrow 2(l + w) = 36 \Rightarrow l + w = 18$ ...(i)
But $l = 2w$ (Given)
Putting the value of l in (i), we have
$2w + w = 18 \Rightarrow 3w = 18$
$\Rightarrow w = 6$
Also, since $l = 2w$, therefore
$l = 2(6) = 12$
Using Pythagorean theorem, to find the length of diagonal BC
$(BC)^2 = (BD)^2 + (CD)^2 \Rightarrow d^2 = (6)^2 + (12)^2$
$\Rightarrow d^2 = 36 + 144$
$\Rightarrow d = \sqrt{180} = 6\sqrt{5}$





Q3. Since, ΔSTU is equilateral, therefore, all of its angles measure 60°. Now, at S the two angles are vertical, and since vertical angles are equal, therefore, the measure of $\angle S$ in quadrilateral PQRS is 60°.
Now, sum of the angles of PQRS = 360°
$90° + x° + 90° + 60° = 360°$
$\Rightarrow x° + 240° = 360°$
$\Rightarrow x = 360 - 240$
$\boxed{x = 120}$

Q4. The sides of the triangle are 5, 12 and 13 therefore, the given triangle is a right triangle.
Let $h = 5$ and $b = 12$, then its area

$A = \frac{1}{2}bh \Rightarrow \quad A = \frac{1}{2}(12)(5)$

$\Rightarrow \quad A = 30$

Now area of the rectangle is

Area $= w \times h$

Area $= 4 \times h$

Since the area of the rectangle equals to area of the triangle, therefore

$30 = 4 \times h \Rightarrow h = \frac{30}{4} \Rightarrow h = 7.5$ cm

The perimeter of the rectangle $= 2(w + h)$

$= 2(4 + 7.5)$

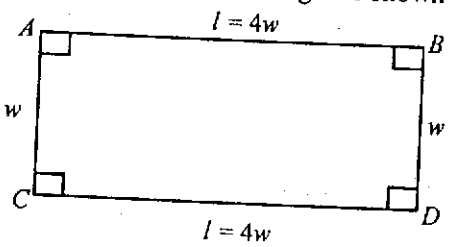
$= 23$ cm

Q5. (D) Each of the four triangle is a right triangle having hypotenuse 4. Therefore, each leg $= \frac{4}{\sqrt{2}} = \frac{2 \times \sqrt{2} \times \sqrt{2}}{\sqrt{2}} = 2\sqrt{2}$

The perimeter of each small triangle is $4 + 4\sqrt{2} = 4(1 + \sqrt{2})$

and the sum of the perimeter $= 4(4\ (1 + \sqrt{2}) = 16(1 + \sqrt{2})$

Q6. (D) According to the given condition, we draw a rectangle as shown in the figure



NowArea $= A = lw = 4w \times w = 4w^2$

and $196 = 4w^2 \Rightarrow w^2 = 49 \Rightarrow \boxed{w = 7}$

Now$w = 7 \Rightarrow l = 7 \times 4 = \boxed{28}$, so its perimeter

Perimeter $= 2(7 + 28) = 2(35) = \boxed{70}$

Q7. (B) The sum of the degree measures of the angles of a hexagon (six-sided polygon) is

$(6 - 2) \times 180° = 4 \times 180 = 720$

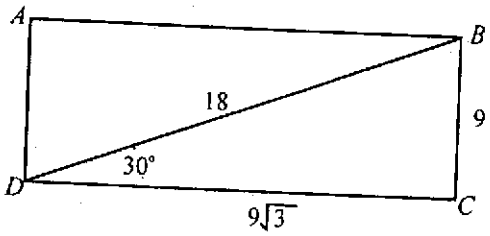
Now, sum of the ratio $= 2 + 4 + 4 + 4 + 5 + 5 = 24$

Now, degree measure of the smallest angle is

$\frac{2}{24} \times 720 = 2 \times 30 = 60$

Q8. (D) In the diagram, first we solve ΔBCD

$\frac{BC}{DB} = \sin 30° \Rightarrow \frac{BC}{18} = \frac{1}{2} \Rightarrow BC = 9$



and $\frac{DC}{BD} = \cos 30° \Rightarrow DC = BD \cos 30° \Rightarrow DC = 18 \times \frac{\sqrt{3}}{2}$

$\Rightarrow DC = 9\sqrt{3}$

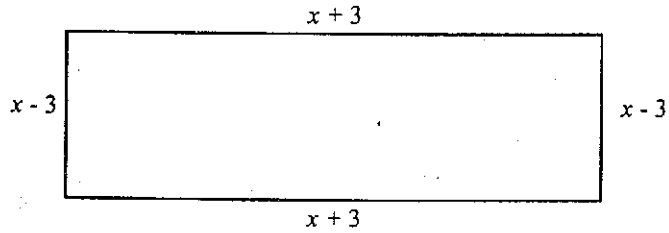
Area $= l \times w = 9 \times 9\sqrt{3} = 81\sqrt{3}$

Q9. (B) The perimeter of the rectangle $= 2(l + w)$

$= 2(9 + 9\sqrt{3})$

$= 18 + 18\sqrt{3}$

Q10.(D) Let x be the length of the square, then according to the given condition, the length and width of the rectangle is $x + 3$ and $x - 3$, respectively, as shown in the following figure



Then, area of the rectangle $A = (x + 3)(x - 3)$

$A = x^2 - 9$

But, area of the square $x^2 = 58$

Now, area of rectangle $A = 58 - 9$

$A = 49$

Q11.(A) In triangle PBQ, $PQ = \sqrt{(PB)^2 + (BQ)^2}$

$= \sqrt{(4)^2 + (3)^2} = \sqrt{25} = 5$

Hence perimeter of $PQRS = 4 \times 5 = 20$

Q12.(B) The area of the triangle $= \frac{1}{2}bh$

$= \frac{1}{2}(3)(4) = 6$

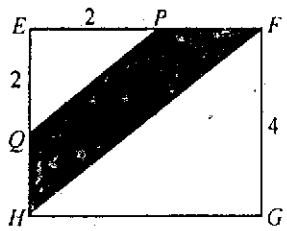
The total area of 4 triangles $= 6 \times 4 = 24$

Now, area of the triangle $ABCD = 8 \times 6 = 48$

Thus, area of the rectangle $PQRS$ = Area of rectangle $ABCD$ – Total area of the triangle

$= 48 - 24 = 24$

Q13.(C) Since P and Q are the midpoints of the sides of length 4. Therefore, EP, EQ, PF and QH are all equal to 2.



Also $PQ = \sqrt{(2)^2 + (2)^2} = \sqrt{4 + 4} = 2\sqrt{2}$

Area of $\Delta PEQ = \frac{1}{2}bh = \frac{1}{2}.2.2 = 2$

In ΔFEH, $FH = \sqrt{(EF)^2 + (EH)^2}$

$= \sqrt{16 + 16}$

$= \sqrt{32} = 4\sqrt{2}$

Perimeter of shaded region $= PQ + PF + FH + HQ$

$= 2\sqrt{2} + 2 + 4\sqrt{2} + 2$

$= 4 + 6\sqrt{2}$

Q14. Area of $\Delta EFH = \frac{1}{2}.4.4 = 8$

Area of $\Delta PEQ = \frac{1}{2} . 2 . 2 = 2$

Now area of shaded region = Area of ΔFEH – Area of ΔPEQ

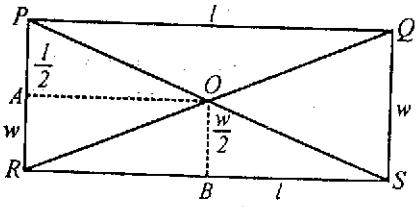
$= 8 - 2 = 6$

Q15. (B) The area of ΔPOR

$= \frac{1}{2}$ Base × Altitude

$= \frac{1}{2}(PR)(OA)$

$= \frac{1}{2}(w)(\frac{l}{2}) = \frac{wl}{4}$



Also, the area of $\Delta ORS = \frac{1}{2}$ Base × Altitude

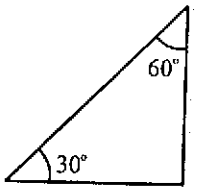
$= \frac{1}{2}(RS)(OB) = \frac{1}{2}(l)(\frac{w}{2})$

$= \frac{lw}{4}$

Q16. (B) Since angle O is acute ($\because m\angle O \angle 90$) and angle N is obtuse ($\because 90 < m\angle O \angle 180$), thus $(LN)^2 \angle x^2 + y^2$, where $(MO)^2 > x^2 + y^2$

$\Rightarrow (MO)^2 > (LN)^2 \Rightarrow MO > LN$

Q17. (C) 30 – 60 Right Triangle theorem states, that, in any right triangle with acute angle measures of 30 and 60 and with hypotenuse of length x, the length of the leg opposite the angle with measure 30(shorter leg) is $\frac{x}{2}$ and the length of the leg opposite the angle with measure 60 (longer leg) is $\frac{x}{2}\sqrt{3}$. Now the longer leg is given which is $2S$, therefore the hypotenuse will be $\frac{4S}{\sqrt{3}}$ ($\because \frac{4S}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = 2S$) and the perpendicular will be $\frac{2S}{\sqrt{3}}$ ($\because \frac{4S}{\sqrt{3}} \times \frac{1}{2} = \frac{2S}{\sqrt{3}}$). Thus the perimeter of the triangle is



$$2S + \frac{2S}{\sqrt{3}} + \frac{4S}{\sqrt{3}} = \frac{2S\sqrt{3} + 2S + 4S}{\sqrt{3}} = \frac{6S + 2\sqrt{3}S}{\sqrt{3}}$$

$$= \frac{2 . \sqrt{3} . \sqrt{3}S}{\sqrt{3}} + \frac{2\sqrt{3}}{\sqrt{3}} . S$$

$$= 2S + 2S\sqrt{3}$$

$$= S(2 + 2\sqrt{3})$$

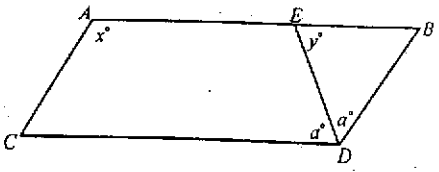
Q18. (C) The perimeter of a rectangle of area 40 is smallest when the rectangle is a square (because all sides are equal). In that case, each side is $\sqrt{40}$, which is greater than 6, and so the perimeter is greater than 24($P = 6 + 6 + 6 + 6 = 24$)

Q19. (D) Given that ED is a transversal, which is cutting AB and CD, where $AB \parallel CD$, therefore, $y = a \Rightarrow 2y = 2a$.

We know that in a parallelogram opposite angles are equal, thus

$x^o = a + a \quad \Rightarrow x = 2a$ but $a = y$

$\Rightarrow x = 2y$



Q20. (A) As, in a parallelogram, opposite angles are equal, therefore

$\therefore \quad t = 5s + 16$

and $\quad u = 3s - 12$

Now $\quad t - u = (5s + 16) - (3s - 12)$

$= 5s + 16 - 3s + 12$

$t - u = 2s + 28$ .....(i)

Because, the sum of the measure of two consecutive angles of a parallelogram is 180, therefore

$(3s - 12) + (5s + 16) = 180 \quad \Rightarrow 8s + 4 = 180$

$\Rightarrow \quad 8s = 176$

$\Rightarrow \quad \boxed{s = 22}$

Now substituting the value of s in (i), we have

$t - u = 2(22) + 28$

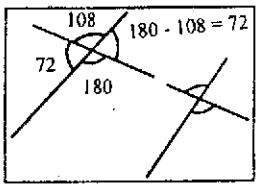
$t - u = 44 + 28 = \boxed{72}$

Q21. (A) The sum of the angles of n-gives $= (n - 2)180$

$\because \quad n = 5$, thus sum of the angles in a pentagon

$= (5 - 2)180 = 3 \times 180 = 540$

Average of each angle in a pentagon $= \frac{540}{5} = 108$



Since each interior angle has three exterior angle, so the sum of two exterior angles is $72 + 72 = 144$

Thus the sum of 5 pairs of exterior angles $= 5 \times 144$

$= 720$

Q22. E The area of the rectangle $= lw$

The area of the rectangle $PQRS = 90 = (x + 5)(x + 2)$

$\Rightarrow x^2 + 7x + 10 = 90$

and the area of the rectangle, $UVWX$

Area $= (x + 4)(x + 3) = x^2 + 7x + 12$

Which is exactly 2 more than the area of $PQRS$

hence the Area of $UVWX = 90 + 2$

$= 92$

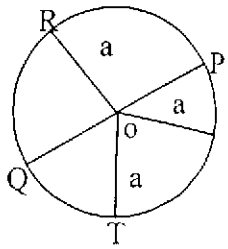
****

Chapter 4

CIRCLES

Circle:

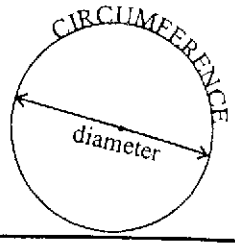
A circle is a set of all points in a plane at a given distance from a fixed point of the plane. The fixed point is the centre of the circle and the given distance is the radius. The adjacent figure is a circle of radius ***a*** unit whose centre is at the point *O*. The point *P, Q, R, S* and *T* lies on the circle, each ***a*** unit from *O*. Therefore, the following statement follows from the definition of a circle.



All radii (plural of radius) of the same circle are congruent.

Circumference:

The perimeter of a circle is called its circumference.



Note:

If "*c*" stands for the circumference of the circle and "*d*" is the diameter of the circle, then $\frac{c}{d}$ *(circumference ÷ diameter)* is the same for all circles. Its value cannot stated exactly.

The Greek letter π*(pi)* is used to stand for it.

$$\therefore \quad \frac{c}{d} = \pi$$

$$c = \pi d$$

or $$c = \pi \times 2 \times r$$

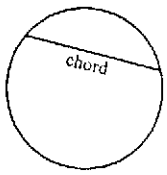
$$c = 2\pi r$$

π is a special number and is equal to 3.14159..... or $\frac{22}{7}$

Angles and Circles:

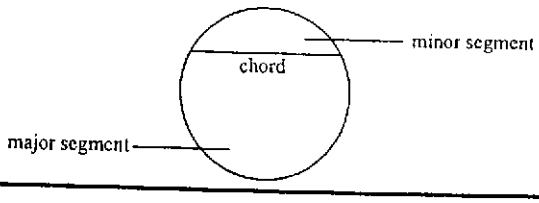
Chord:

A line joining two points on the circumference of a circle is called a chord.



Note:

A chord divides a circle into two segments.

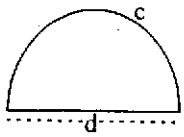


Diameter:

A chord which passes through the centre of the circle is a diameter.

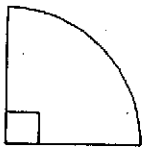
Arc Length of a Circle:

Arc length of a sector of half a circle is



$$c = \frac{\pi d}{2}$$

Arc length of a sector of quarter of a circle is



$$c = \frac{\pi d}{4}$$

Semicircles:

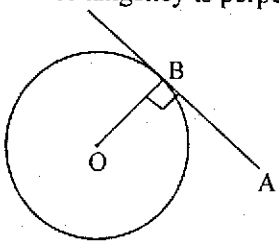
A diameter divides a circle into two congruent halves which are called semicircles.

Tangent of a Circle:

A line that intersects the circle in exactly one point is a tangent to the circle. The point of intersection is called the point of tangency.

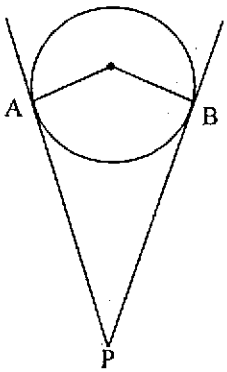
Note:

1. The radius from the centre to the point of tangency is perpendicular to the tangent.



AB is tangent to the circle with centre O. OB is perpendicular to BA.

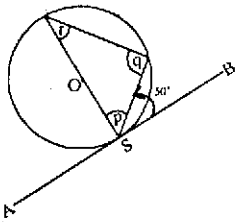
2. Tangents from the same point are equal



PA = PB

Example 1:

In the given figure, if AB is tangent to the circle. Calculate the sizes of angles, p, q, r.



Solution:

OS is the radius and AB is the tangent. By tangent – radius theorem $\angle OSB = 90^\circ$

$\angle p = 90^\circ - 50^\circ = 40^\circ$

Now $\angle q = 90^\circ$ because angle in a semicircle is right. Now angle p, q and r the angles of triangle.

$\therefore \quad \angle p + \angle q + \angle r = 180^\circ$

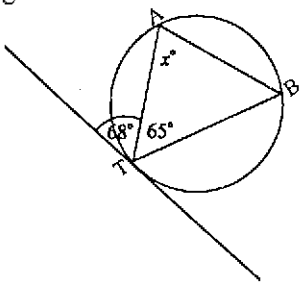
$\therefore \quad 40^\circ + 90^\circ + r = 180^\circ$

$\therefore \quad r = 180^\circ - 130^\circ$

$r = 50^\circ$

Example 2:

What is the value of x in the following diagram?

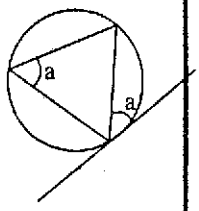


Solution:

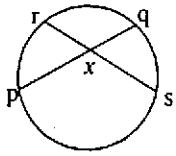
In above diagram $\angle TBA = 68^\circ$ because angles in alternate segment are equal

$\therefore \quad x^\circ + 65^\circ + 68^\circ = 180^\circ \Rightarrow x = 47^\circ$

Theorems: Tangents and Secants



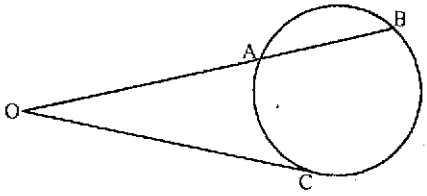
1. The angle between a tangent and a chord drawn to the point of contact is equal to the angles in the alternate segment.



2. The products of the intercepts of two intersecting chords of a circle are equal.

   That is: $px \,.\, qx = rx \,.\, sx$

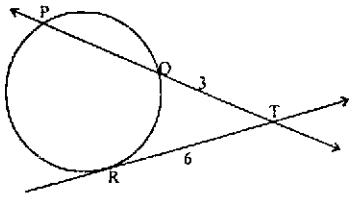
3. If a tangent and a secant intersect in the exterior of a circle, the square of the tangent segment equals the product of the secant segment and the external secant segment.



$OC^2 = OA \,.\, OB$

Example:

In the following figure. What is the value of TP?



Solution:

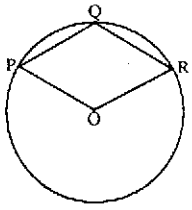
$(TR)^2 = TQ \,.\, TP$

$(6)^2 = 3 \,.\, TP$

$36 = 3 \,.\, TP$

$\Rightarrow \quad TP = 12$

Cyclic Quadrilateral:



A quadrilateral which has all its vertices laying on the circumference of a circle is called a cyclic quadrilateral.

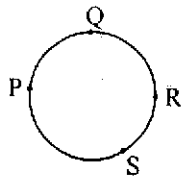
Note:

Opposite angles of a cyclic quadrilateral add up to 180°.

Common Arc Theorem:

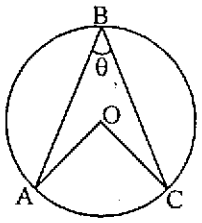
If four points on a circle are, in order and

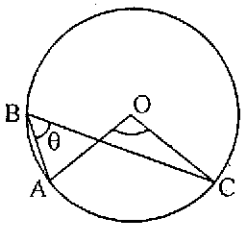
$\widehat{PQ} \cong \widehat{RS}$ Then $\widehat{PR} \cong \widehat{QS}$



Central Angle Theorems:

1. The angle subtended at the centre of a circle by an arc is twice the angle subtended at the circumference by the same arc.





In above, angle AOC = 2 × angle ABC.

2. The angle measured in degree to complete one revolution in a circle is 360°.

Example:

What size angle is subtended at the centre (O) by chord AB?

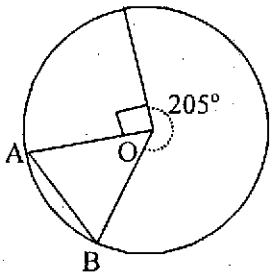
Solution:

$\angle AOB + 90° + 205° = 360°$

$\angle AOB + 295° = 360°$

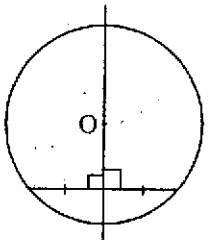
$\angle AOB = 360° - 295°$

$= 65°$



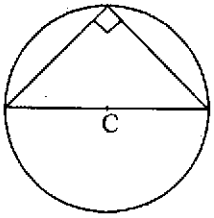
∴ Chord AB subtends an angle of 65° at the centre.

Theorem: A line from the centre of a circle through the mid-point of a chord meets the chord at right angles.



Theorem: Angle In a Semicircle:

An angle in a semicircle is a right angle.



Converse of Theorem:

If a circle passes through the ventricles of a right-angled triangle, then the hypotenuse of the triangle is a

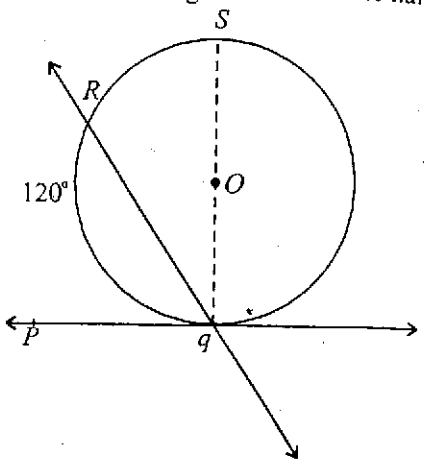
diameter of the circle.

More Angles and Arcs:

Theorem 1:

If a tangent and a secant (or chord) intersect in a point on a circle, the measure of the angle formed is one half the measure of the intercepted arc.

Explanation:



In circle O as shown in the figure, secant QR and tangent PQ intersect at point Q on the circle, forming angle PQR. The above theorem focuses upon the relationship between the measure of this angle and the degree measure of the intercepted arc, $\widehat{QR}$.

According to this theorem, $\angle PQR = \frac{1}{2}(120)$

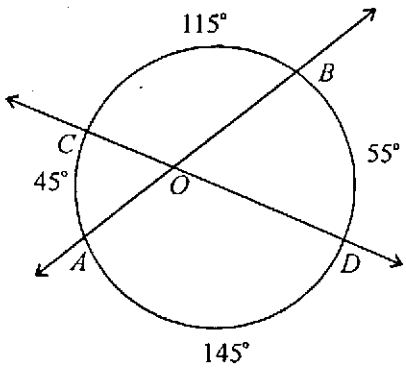
$= \frac{1}{2}(\text{Arc } QR)$

$= 60°$

Theorem 2:

If two secants (or chords) intersect in the interior of a circle, the measure of an angle formed is one half the sum of the measures of he arcs intercepted by the angle and its vertical angle.

Explanation:



When two secants AB and CD intersect in the interior of a circle, as circle O shows to the right, two pair of vertical angles are formed. According to the given theorem

$\angle AOC = \angle DOB = \frac{1}{2}(55° + 45°)$

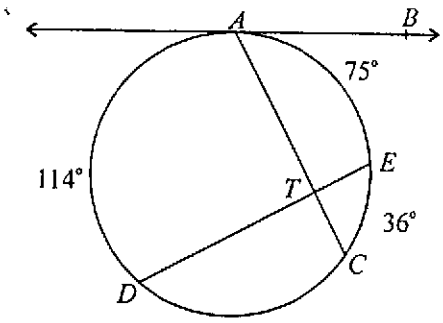
$= \frac{1}{2}(100) = 50°$

and $\angle AOD = \angle COB = \frac{1}{2}(145° + 115°)$

$= \frac{1}{2}(260)$

$= 130°$

Example 1:



Chord AC and DE intersect at T, $\overleftrightarrow{AB}$ is tangent to the circle at A.

$m\widehat{AD} = 114$, $m\widehat{EC} = 36°$ and $m\widehat{AE} = 75$

a. Find $m\angle CAB$

b. Find $m\angle ATD$

Solution: a. By theorem 1

$m\angle CAB = \frac{1}{2}m\widehat{AC}$

$= \frac{1}{2}(m\widehat{SR} + m\widehat{RT})$

$= \frac{1}{2}(75° + 36°)$

$= \frac{1}{2}(111)$

$= 55.5$

b. By theorem 2

$$m\angle ATD = \frac{1}{2}(m\widehat{EC} + m\widehat{AD})$$

$$= \frac{1}{2}(36° + 114°)$$

$$= \frac{1}{2}(150)$$

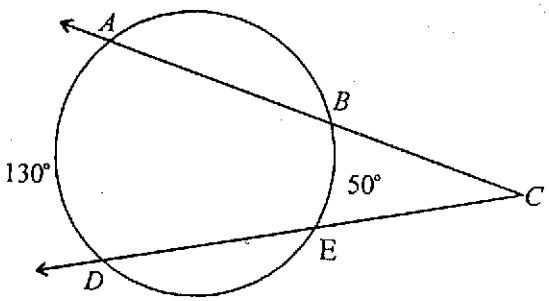
$$= 75$$

Theorem 3:

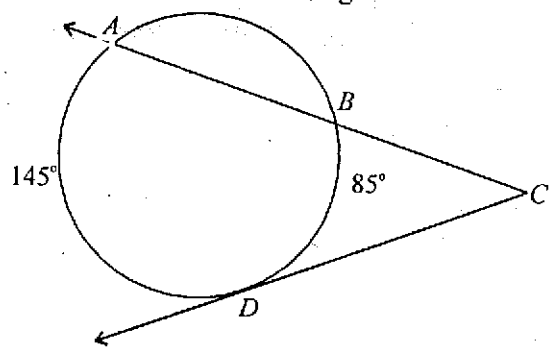
If two secants, a tangent and a secant, or two tangents intersects in the exterior of a circle, the measure of the angle formed is one-half the difference of the measures of the intercepted arcs.

Example 2: In each case of the following figure, find $m\angle C$.

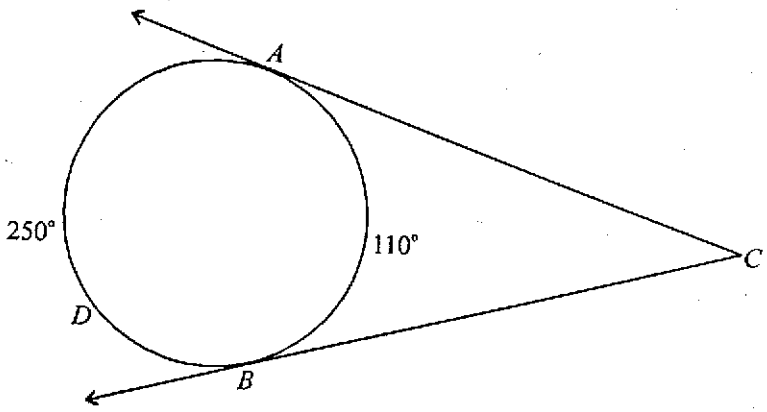
a. Two secants



b. One secant one tangent



c. Two tangents



Solution:

a. Applying theorem 3

$$m\angle C = \frac{1}{2}(m\widehat{AD} - m\widehat{BE})$$

$$= \frac{1}{2}(130 - 50)$$

$$= \frac{1}{2}(80) = 40$$

b. Applying theorem 3

$$m\angle C = \frac{1}{2}(m\widehat{AD} - m\widehat{BD})$$

$= \frac{1}{2}(250 - 110)$

$= \frac{1}{2}(140)$

$= 70$

c. Applying theorem 3

$m\angle C = \frac{1}{2}(m\widehat{ADB} - m\widehat{AB})$

$= \frac{1}{2}(250 - 110)$

$= \frac{1}{2}(150)$

$= 75$

Multiple Choice Questions (MCQs)

Q1. If the area of a circle is 81π, then its circumference is:

(A) 61π (B) 20π

(C) 18π (D) 16π

Q2. If circumference of a circle is 3π, then its area is:

(A) $\frac{7\pi}{2}$ (B) $9\pi^2$

(C) $4\pi^2$ (D) $\frac{9\pi}{4}$

Q3. If a circle is inscribed in a square of area 4, then the area of the circle is:

(A) π (B) $\frac{\pi}{2}$

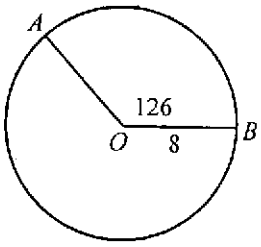
(C) $\frac{\pi}{4}$ (D) $\frac{3\pi}{4}$

Q4. If a square of area 3 is inscribed in a circle, then the area of the circle is:

(A) $\frac{9}{4}\pi$ (B) $9\pi^2$

(C) 3π (D) $\sqrt{3}\pi$

Questions 5 – 6 refer to the following figure



Q5. What is the length of arc *AB*?

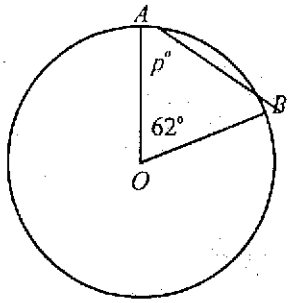
(A) 2.6π (B) 5.6π

(C) 7.6π (D) $\frac{1}{2}\pi$

Q6. What is the area of the shaded sector?

(A) 22.9π (B) 22.4π
(C) 60π (D) 62.3π

Q7. In the following figure, what is the value of p?



(A) 49 (B) 39
(C) 59 (D) 69

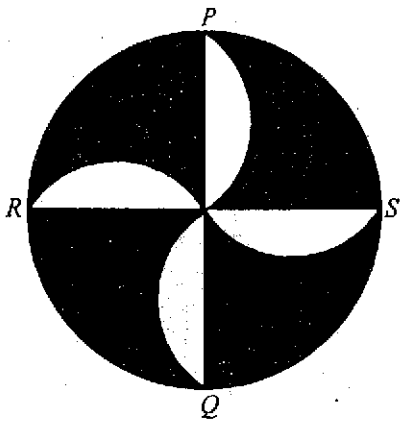
Q8. If P represents the area and W represents the circumference of the circle, then P in terms of W is:

(A) $\frac{2\pi}{W}$ (B) $\frac{4\pi^2}{W}$
(C) $\frac{2\pi^2}{W^2}$ (D) $\frac{W^2}{4\pi}$

Q9. What is the area of a circle whose radius is the diagonal of a square whose area is 9?

(A) $\sqrt{3}\pi$ (B) 12π
(C) 4π (D) 13π

Q10. In the following figure, PQ and RS are perpendicular, and each of the unshaded regions is a semicircle. What is the ratio of the white area to the shaded area?



(A) $\frac{4}{\pi}$ (B) $\frac{1}{1}$
(C) $\frac{2}{3}$ (D) $\frac{1}{2}$

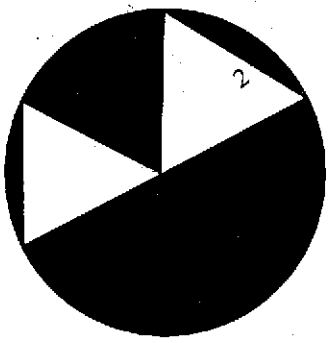
Q11. If C is the circumference of a circle of radius r, then which of the following statement is true?

(A) $\frac{C}{r} < 6$ (B) $\frac{C}{r} = 6$
(C) $\frac{C}{r} > 6$ (D) $\frac{C}{r} = \pi$

Q12. If C is the circumference of a circular disk in centimeters, and A is the area of the same circular disk in square centimeter. Then $\frac{C}{A} = \frac{A}{C}$, iff $r =$

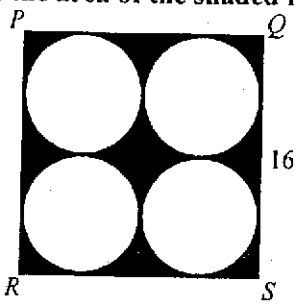
(A) 1 (B) 2
(C) 3 (D) 4

Q13. In the following figure, what is the area of the shaded region, if each of the triangle is equilateral?



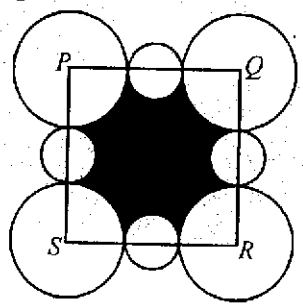
(A) 8π (B) $\frac{8}{3}\pi$
(C) 3π (D) 6π

Q14. In the following figure, *PQRS* is a square, and all the circles are tangent to one another and to the sides of the squares. What is the area of the shaded region?



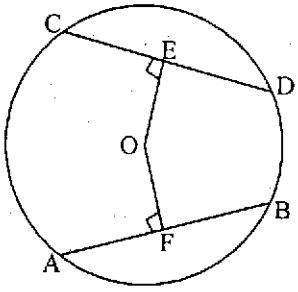
(A) 256 (B) 64π
(C) 256π (D) $64(4-\pi)$

Q15. In the following figure, the large circles have radius 4, and the small circles have diameter 4. What is the area of the square *PQRS*?



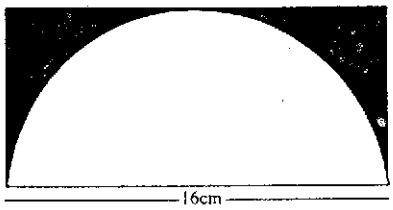
(A) 144 (B) 169
(C) 100 (D) 64

Q16. In the following figure, if AB = CD and OE = 2.5, what is the value of OF?



(A) 6.25 (B) 5
(C) 1.58 (D) 2.5

Q17. A semicircle is drawn inside a rectangle as shown.



The shaded area is closest to:

(A) 50 (B) 40
(C) 30 (D) 45

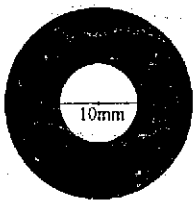
Q18. In the following figure, if the radius of the outer circle is p and the radius of each of the circles inside the larger circle is $\frac{p}{3}$, then what is the area of the shaded region?

(A) $\frac{2}{9}\pi p^2$ (B) $\frac{11}{9}\pi p^2$
(C) $\frac{22}{9}\pi p^2$ (D) $\frac{7}{9}\pi p^2$

Q19. A circle is inscribed in a square of area $\sqrt{6}$. What is the area of the circle?

(A) $\frac{3}{2}\pi$ (B) 6π
(C) π (D) 9π

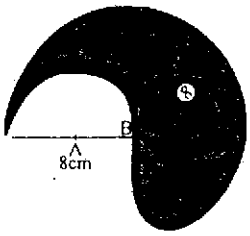
Q20. A circle of radius 5 mm is removed from the centre of a circular piece of metal of radius 7 mm to make a washer as shown below:



What is the area of the shaded region?

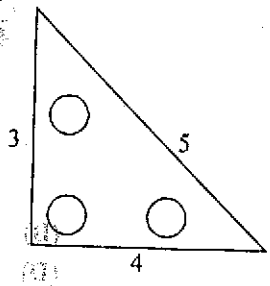
(A) 25π (B) 49π
(C) $35\pi^2$ (D) 24π

Q21. In the following metal cam, A, B and C are the centres of the semicircles shown. What is the area of the cam?



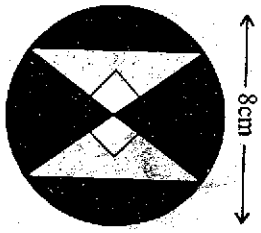
(A) 32.07 cm^2 (B) 157.08 cm^2
(C) 101.53 cm^2 (D) 201.06 cm^2

Q22. The sketch below shows a triangular copper plate with sides of 3cm, 4cm and 5cm. It has three small circular holes cut out of it. The radius of each circle is 3mm. What is the area of the copper triangle?



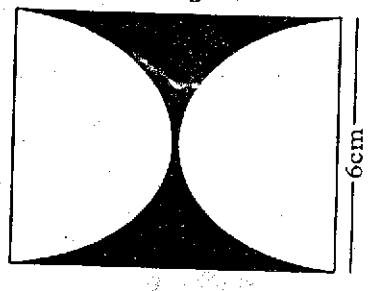
(A) 6 cm^2 (B) 5.16 cm^2
(C) 5.9973 cm^2 (D) 5.71 cm^2

Q23. What is the shaded area in the following diagram?



(A) 50.29 cm^2 (B) 16 cm^2
(C) 16.29 cm^2 (D) 34.29 cm^2

Q24. What is the shaded area in the following diagram?



(A) 36 cm^2 (B) 7.72 cm^2
(C) 28.3 cm^2 (D) 14.14 cm^2

Explanatory Answers

Q1. (C) Area of a circle: $A = \pi r^2 = 81\pi \Rightarrow r^2 = 81 \Rightarrow r = 9$

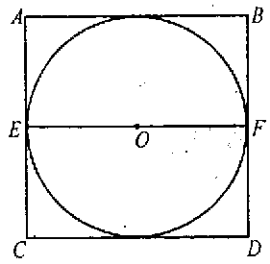
Circumference of a circle: $C = 2\pi r \Rightarrow C = 2\pi(9) = 18\pi$

Q2. (D) Circumference of a circle: $C = 2\pi r \Rightarrow 2\pi r = 3\pi$

$$\Rightarrow r = \frac{3}{2}$$

Area of a circle: $A = \pi r^2 \Rightarrow A = \pi \cdot \left(\frac{3}{2}\right)^2 \Rightarrow A = \frac{9\pi}{4}$

Q3. (A) First of all, we draw the diagram



Since the area of the square is 4 (given), therefore $AC = 2$, as in a square all sides are equal, therefore $AC = AB = BD = CD = 2$.

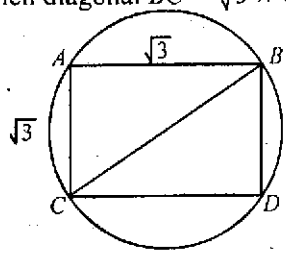
$\because$ Diameter of the circle, $EF = CD = AB \Rightarrow EF = 2$

and radius of the circle $= r = OF = OE = 1$ (half the diameter)

Hence the area of the circle with radius 1 is

$$\pi(1)^2 = \pi$$

Q4. (A) First we draw a diagram, because area of the square is 3, thus $AC = \sqrt{3}$, then diagonal $BC = \sqrt{3} \times \sqrt{3} = 3$, but BC is also the diameter of the circle, hence the diameter is 3 and radius is 1.5



Now, the area of the circle $A = \pi r^2 \Rightarrow A = \pi(1.5)^2 \Rightarrow A = 2.25\pi = \frac{9}{4}\pi$

Q5. (B) Setting a proportion

$$AB : OB :: 126 : 360$$

$$\frac{\overline{AB}}{2\pi r} = \frac{126}{360}$$

$$\overline{AB} = \left(\frac{126}{360}\right) \times 2\pi r$$

$$\overline{AB} = \left(\frac{126}{360}\right) \times 2\pi \times 8$$

$$\overline{AB} = 5.6\pi$$

Q6. (B) The area of shaded sector is $\frac{126}{360}\pi(8)^2$

$$= (0.35)\pi(64)$$

$$= 22.4\pi$$

Q7. (C) Because, the triangle is isosceles therefore, the angle B is also p, thus

$$p^\circ + p^\circ + 62^\circ = 180^\circ$$

$$\Rightarrow \quad 2p^\circ = 118 \Rightarrow p = 59$$

Q8. D Since, P is the area, so $P = \pi r^2$, and

W is the perimeter, thus $W = 2\pi r \Rightarrow r = \frac{W}{2\pi}$

$$\Rightarrow \quad P = \pi\left(\frac{W}{2\pi}\right)^2$$

$$P = \pi\frac{W^2}{4\pi^2}$$

$$\Rightarrow \quad P = \frac{W^2}{4\pi}$$

Q9. (B) Since the area of the square is 9, so its each side is 3, and the length of the diagonal will be $\sqrt{3} + \sqrt{3} = 2\sqrt{3}$

Thus area of the circle of radius $2\sqrt{3}$ is

$$A = \pi r^2 \Rightarrow A = \pi(2\sqrt{3})^2 \Rightarrow A = 12\pi$$

Q10. (B) Let the radius of the big circle be r, then its area will be πr^2, also the radius of the semicircle becomes $\frac{r}{2}$, so area of the small circle will be $\pi\frac{r^2}{2}$ and the area of the each semicircle is

$$\left(\pi\frac{r^2}{4}\right)\times\frac{1}{2} = \frac{\pi r^2}{8}$$

Then the area of the four small semicircles is

$$= 4\left(\frac{\pi r^2}{8}\right) = \frac{\pi r^2}{2}$$

So, shaded area = Total area – White area

$$= \pi r^2 - \frac{\pi r^2}{2} = \frac{\pi r^2}{2}$$

Therefore the ratio of shaded area is

$$\frac{\frac{\pi r^2}{2}}{\frac{\pi r^2}{2}} = \frac{1}{1}$$

Q11. (C) Since $C = 2\pi r \Rightarrow C = \pi(2r)$, but $2r = d$

Hence $C = \pi d \Rightarrow \pi = \frac{C}{d} \Rightarrow \frac{C}{r} = 2\pi$

$$\Rightarrow \frac{C}{2} = 2\left(\frac{22}{7}\right) > 6$$

Q12. (B) As $C = 2\pi r$ and $A = \pi r^2$, so

$$\frac{C}{A} = \frac{2\pi r}{\pi r^2} = \frac{2}{r}$$

and $\frac{A}{C} = \frac{\pi r^2}{2\pi r} = \frac{r}{2}$

Thus $\frac{C}{A} = \frac{A}{C}$, only possible, when $r = 2$

Q13. (B) Because the triangles are equilateral, then the white central angles each measure 60°, so their sum = 60 + 60 = 120. Then, the unshaded area is $\frac{120}{360} = \frac{1}{3}$ of the circle, so the shaded area of $\frac{2}{3}$ of the circle.

As the area of the circle = $\pi r^2 = \pi(2)^2 = 4\pi$

and the area of the shaded region $= \frac{2}{3}\times 4\pi = \frac{8}{3}\pi$

Q14. (D) Since $QS = 16$, thus the diameter of each circle is 8, and radius of each circle is 4.

$\therefore$ The area of each circle $= \pi r^2 = \pi(4)^2 = 16\pi$

Thus, the area of four circles $= 4(16\pi) = 64\pi$

Now, the area of the square $= 16 \times 16 = 256$

Area of the shaded region = Area of the square – Area of the circle

$= 256 - 64\pi$

$= 64(4 - \pi)$

Q15. (A) Since the radius of the large circle is 4, and diameter of the small circle is 4, so each side of the square is 4 + 4 + 4 = 12, so area of the rectangle = 12 × 12 = 144

Q16. **(D)** $\boxed{OF = 2.5}$

Because equal chords are equidistant from centre.

Q17. **(C)** Area of the rectangle $= 16 \times 8 = 128$

Area of the circle of radius 8cm $= \pi(8)^2$

$= 64\pi$

Area of the semicircle $= \frac{64}{2}\pi = 32\pi$

$= 32(3.14) = 100.48$

$= 100.48$

Area of the shaded region = Area of rectangle – Area of the semicircle

$= 128 - 100.48$

$= 27.52$

which is closest to 30

Q18. **(D)** Area of the outer circle $= \pi(p)^2 = \pi p^2$

Area of the inner circle $= \pi\left(\frac{p}{3}\right)^2 = \frac{\pi p^2}{9}$

Total area of the inner circles

$= \frac{\pi p^2}{9} + \frac{\pi p^2}{9}$

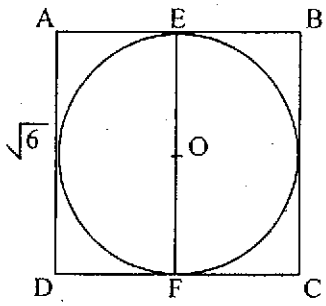
$= \frac{2}{9}\pi p^2$

Area of the shaded region $= \pi p^2 - \frac{2\pi p^2}{9}$

$= \frac{9\pi p^2 - 2\pi p^2}{9}$

$\boxed{= \frac{7}{9}\pi p^2}$

Q19. **(A)** The inscribed circle in a square of area 6 is



The side AD $= \sqrt{6}$ and also the diameter $= \sqrt{6}$. The radius of the circle O is OF $= \frac{\sqrt{6}}{2} \Rightarrow$ OF $= \sqrt{\frac{3}{2}}$

The area of the circle of radius $\sqrt{\frac{3}{2}}$ is

$$\text{Area} = \pi\left(\sqrt{\frac{3}{2}}\right)^2 \Rightarrow \boxed{\text{Area} = \frac{3}{2}\pi}$$

Q20. (D)

Area of the washer removed $= \pi r^2 = \pi(5)^2 = 25\pi$

Area of the metal $= \pi r^2 = \pi(7)^2 = 49\pi$

Area of the shaded region $= 49\pi - 25\pi = 24\pi$

Q21. (C) When we shift the semicircle of diameter 8cm in the space the shape becomes semicircle of diameter 16cm

$\therefore$ Area $= \pi r^2$

$= \pi \times (8)^2 = 64\pi$

Area of semicircle $= \frac{64\pi}{2} = 32\pi = \boxed{101.53\text{cm}^2}$

Q22. (B) Area of the triangle $= \frac{1}{2} \times \text{base} \times \text{Altitude}$

$= \frac{1}{2} \times 4 \times 3 = 6\text{cm}^2$

Now 3mm $= 0.3\text{cm}$

Area of the circle $= \pi r^2 = 0.28\text{cm}^2$

Area of 3 circles $= 3 \times 0.28 = 0.84\text{cm}^2$

Area of copper in plate $= 6 - 0.84 = \boxed{5.16\text{cm}^2}$

Q23. (D) Diameter of the circle $= 8\text{cm}$

Radius $= 4\text{cm}$

Area of the circle $= \pi r^2 = \frac{22}{7} \times 16 = 50.29\text{cm}^2$

Area of triangle $= \frac{1}{2}\text{bases} \times \text{altitude}$

$= \frac{1}{2} \times 4 \times 4$

$= 8\text{cm}^2$

Area of 2 triangles $= 2 \times 8 = 16\text{cm}^2$

Area of the shaded region $= 50.29 - 16 = \boxed{34.29\text{cm}^2}$

Q24. (B) Area of the semicircle $= \frac{1}{2}\pi d$

$= \frac{1}{2} . \pi . r^2 = \frac{1}{2}\pi(3)^2$

$= 14.14\text{ cm}^2$

Area of the 2 same semicircles $= 2 \times 14.14 = 28.28\text{cm}^2$

Area of the square $= 6 \times 6$

$= 36\text{cm}^2$

Area of the shaded region $= 36 - 28.28 = \boxed{7.72\text{cm}^2}$

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Chapter 5

AREA

Some Important Formulae

(1) Area of a rectangle or square
$= \text{length} \times \text{breadth}$

(2) Perimeter of a rectangle or square
$= 2\,(\text{length} + \text{breadth})$

(3) Area of a triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

(4) Area of a triangle of sides a, b, and c
$= \sqrt{s(s-a)(s-b)(s-c)}$

where $s = \frac{(a+b+c)}{2}$

(5) Area of a circle $= \pi r^2$

(6) Circumference of a circle
$= 2\pi r$

(7) Area of four walls of a room
$= 2(\text{length} + \text{breadth}) \times \text{height}$

(8) Area of a parallelogram
$= \text{base} \times \text{height}$

(9) Area of a trapezium
$= \frac{1}{2} \times \text{sum of two parallel sides} \times \text{width}$

(10) Area of a regular hexagon of side a
$= 6a^2\sqrt{\frac{3}{4}}$.

Model Examples

Q.1. The difference between the circumference of a circle and its diameter is 135 ft. Find the area of the circle.

$$\left(Take\,\pi = \frac{22}{7}\right)$$

Sol. Let r be the radius of the circle, then circumference of circle

$= 2\pi r$

Diameter of circle $= 2r$

By the given condition

$= 2\pi r - 2r = 135$

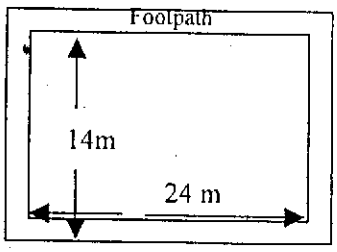
$r = \frac{63}{2}$ ft.

Area of circle $= \pi r^2$

$= \frac{22}{7} \times \frac{63}{2} \times \frac{63}{2}$ sq. ft.

$= 3118.5$ sq. ft. **Ans.**

Q.2. How many tiles 20 cm. square will be required to have a foot path 1 metre wide carried round the outside of grass plot 24 meter long by 14 metres broad?



Sol. Area of the grass plot $= 24 \times 14$
$= 336$ sq. m.

Length and breadth of the plot including the path is = 26 m and 16 m

Area of plot including path $= 26 \times 16$
$= 416$ sq. m.

$\therefore$ Area of path $= 416 - 336 = 80$ sq. m.

Area of one til $= \frac{20}{100} \times \frac{20}{100} = \frac{1}{25}$ sq. m.

$\therefore$ No. of tiles required $= \frac{80}{\frac{1}{25}} = 80 \times 25$

$= 2000$ **Ans.**

Q.3. If the length of a rectangular piece of land were 5 metres less and the breadth 2 metres more, the area would be 10 sq. m. less; but if the length were 10 metres more and breadth 5 metres more, the area would have been 275 sq. m. more. Find its length and breadth.

Sol. Let the length be 'l' breadth 'b'

$\therefore$ area $= l \times b$

Then $(l - 5)(b + 2) = lb - 10$.....(i)

And $(l + 10)(b + 5) = lb + 275$.....(ii)

From (*i*)

$lb + 2l - 5b - 10 = lb - 10$

$2l = 5b \quad \Rightarrow \quad l = \frac{5}{2}b$

From (*ii*)

$lb + 5l + 10b + 50 = lb + 275 \quad \Rightarrow \quad 5l + 10b = 225$

Putting $l = \frac{5}{2}b$

$\frac{25}{2}b + 10b = 225 \quad \Rightarrow \quad \frac{45}{2}b = 225$

$b = \frac{2 \times 225}{45} = 10$ and $l = 25$ m

Q.4. The perimeter of one square exceeds the perimeter of another square by 120 metres and the area of the larger square exceeds twice the area of the smaller square by 900 square metres. Find the length of the sides of the squares.

Sol. Let the length of one side of larger square $= x$ sq. m

Let the length of one side of smaller square $= y$ mts.

Now by the given condition the primeter ($4x$) of larger exceeds the perimeter of smaller ($4y$) by 120 sq. m

i.e., $4x - 4y = 120$

or $x = 30 + y$ ..... (*i*)

Again by the second condition

$$x^2 - 2y^2 = 900 \qquad \ldots\ldots (ii)$$

Putting the value of x from (i) in (ii), we get

$$(30 + y)^2 - 2y^2 = 900 \Rightarrow -y^2 + 60y = 0$$

or $\qquad y = 60$

Putting the value in (i) we get $\quad x = 90$

$\therefore$ Larger square side length is = 90 sq. m

Smaller square side length is = 60 sq. m **Ans.**

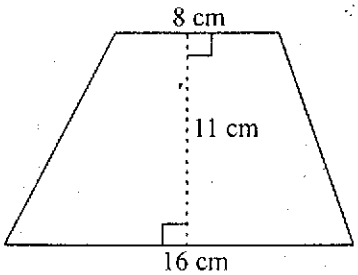
Multiple Choice Questions (MCQs)

Q1. The surface area of sphere of radius $3\frac{1}{2}$ cm is:

(A) 130 sq.cm (B) 69 sq.cm

(C) 154 sq.cm (D) 98 sq.cm

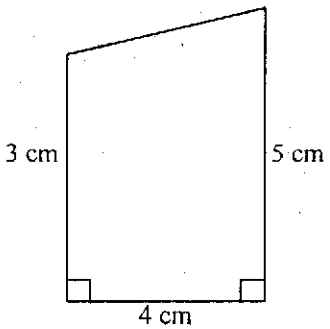
Q2. The area of the following trapezium is:



(A) 125 sq.cm (B) 132 sq.cm

(C) 139 sq.cm (D) 97 sq.cm

Q3. The area of the following figure is:



(A) 16 sq.cm (B) 15 sq.cm

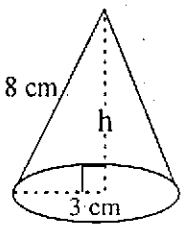
(C) 60 sq.cm (D) None of these

Q4. The height of a triangle of base 3 cm and area 9 cm is:

(A) 6 cm (B) 9 cm

(C) 18 cm (D) 22 cm

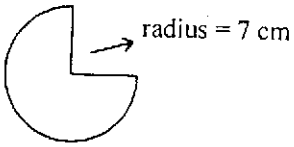
Q5. What is the surface of the following figure?



(A) 33π (B) 24π

(C) 25π (D) 11π

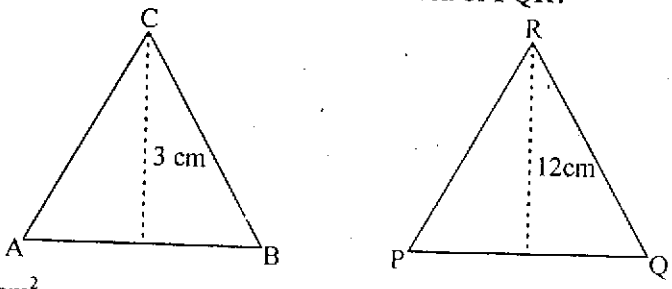
Q6. What is the area of the following figure?



(A) 125 sq.cm (B) 150.72 sq.cm

(C) 64 sq.cm (D) 56 sq.cm

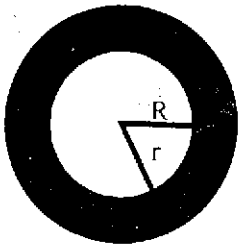
Q7. The following two triangles are similar. Find the area of PQR?



(A) $54\ cm^2$ (B) $24\ cm^2$

(C) $96\ cm^2$ (D) $59\ cm^2$

Q8. The area of the shaded region of the following figure is?



(A) $\pi^2(r^2 - R^2)$ (B) $\pi r^2 + \pi R^2$

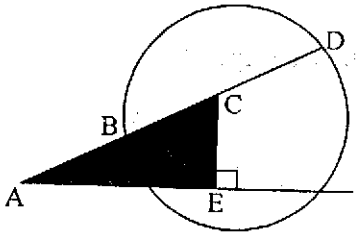
(C) $\pi^2(R + r)(R - r)$ (D) $\pi(R + r)(R - r)$

Q9. The area of the sector which contains an angle of 60° of circle of radius 7 cm, is:

(A) $25\frac{2}{3}\ cm^2$ (B) $27\frac{2}{3}\ cm^2$

(C) $41\ cm^2$ (D) $\sqrt{3}\,\frac{5}{28}\ cm^2$

Q10. In the following figure, what is the area of the right triangle? If $\overline{AE}$ = 16 cm and $\overline{BD}$ = 12 cm

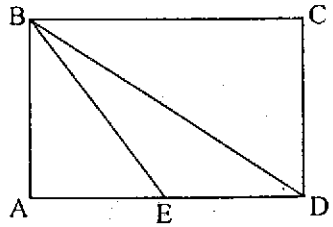


(A) 6 (B) 49

(C) 8 (D) Not possible

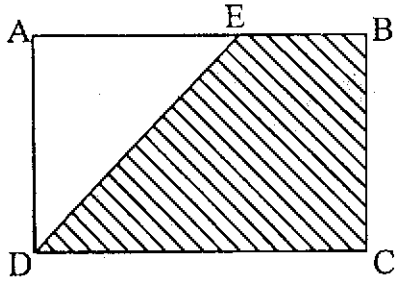
Q11. **In the figure below, ABCD is a rectangle and E is the mid point of one side, what is the area of triangle BCD?**

If $\overline{BE}$ = 5 cm and $\overline{CD}$ = 3cm



(A) 12 cm^2 (B) 25 cm^2
(C) 9 cm^2 (D) 17.23 cm^2

Q12. **In the figure below, given that AD = 6, CD = 8, AE = x. What is the area of the shaded region?**



(A) $28 - 2x$ (B) $4(14 - 4x)$
(C) $48 - 3x$ (D) $7(6 - 3x)$

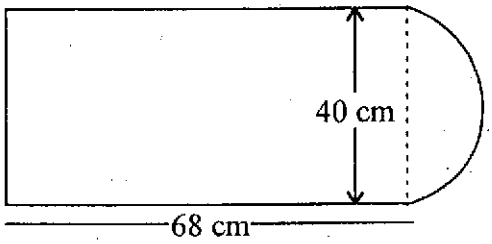
Q13. **If the radius of the circle is decreased by 20%, what happens the area?**

(A) 10% in crease (B) 20% decrease
(C) 80% increase (D) 36% decrease

Q14. **A square, with perimeter 16, is inscribed in a circle, what is the area of the circle?**

(A) 3π (B) $2\sqrt{2}\pi$
(C) 32π (D) 8π

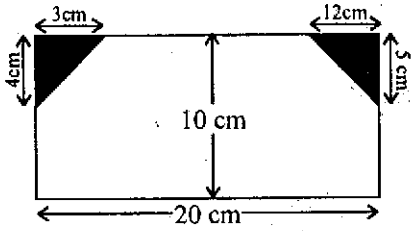
Q15. **Calculate the area of the following figure which is consists of:**



The rectangle is of 40 cm and 68 cm and half a circle of diameter 40 cm

(A) 2535 cm^2 (B) 2720 cm^2
(C) 3348 cm^2 (D) 628 cm^2

Q16. **What is the area of the following shake if the shaded areas are cut away?**



(A) 100 cm^2 (B) 125 cm^2

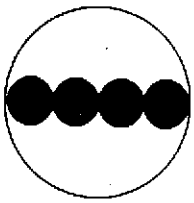
(C) 120 cm^2 (D) 164 cm^2

Q17. The length of rectangle is decreased by 15% and its width is increased by 40%. The area is:

(A) decreases by 25% (B) no effect

(C) increases by 36% (D) increases by 19%

Q18. In following figure equal circles lie along the diameter of the large circle. If the circumference of the circle is 64π. What is the area of the shaded region?



(A) 64π (B) 256π

(C) 16π (D) None of these

Explanatory Answers

Q1. (C) $A = 4\pi r^2$

$= \frac{4}{1} \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$ sq.cm

= 154 sq.cm

Q2. (B) A = Average width × Height

$= \frac{(8+16)}{2} \times 11$ sq.cm

= 132 sq.cm

Q3. (A) $A = \frac{(3+5)}{2} \times 4$

= 16 sq.cm

Q4. (A) $9 = \frac{1}{2} \times 3 \times h$

$\therefore h = \frac{2 \times 9}{3}$

= 6 cm

Q5. (A) $S = \pi rs + \pi r^2$

$= (\pi \times 3 \times 8) + \pi(3 \times 3)$

$= 24\pi + 9\pi = 33\pi$

Q6. (B) The figure is $\frac{3}{4}$ of the circle, Now

Area of the whole circle $= \pi r^2$

$\frac{3}{4}$ of the circle $= \frac{3}{4}(3.14)(18)^2$

$= \frac{3}{4} \times 200.96$

$= 150.72\ cm^2$

Q7. (A) $\frac{AB}{DE} = \frac{\text{height of the triangle ABC}}{\text{height of the triangle DCP}}$

$$\text{height} = \frac{36\text{ cm}}{4} = 9\text{cm}$$

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 12 \times 9$$

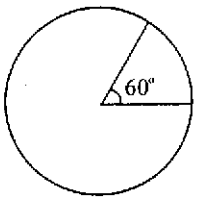
$$= 54\text{ cm}^2$$

Q8. **(D)** Area= Whole – hole

$$= \pi R^2 - \pi r^2$$

$$= \pi(R^2 - r^2) = \pi(R - r)(R + r)$$

Q9. **(A)** Sector $= \frac{\theta^o}{360^o} \times \pi r^2$



$$= \frac{60}{360} \times \frac{22}{7} \times \frac{7}{1} \times \frac{7}{1}\text{ cm}^2$$

$$= 25\frac{2}{3}\text{ cm}^2$$

Q10. **(D)** The area of the height triangle equals half the product of two legs. The point E is not on the circle, The length CE is given. We cannot find the exact value of CE. So lacking the length of CE, we cannot calculate the area.

Q11. **(A)** In the given rectangle, $\overline{AB} = \overline{CD} = 3\text{cm}$. The length of the side AE of triangle BAE can be found with the Pythagorean theorem as:

$(BF)^2 = (AB)^2 + (AE)^2 \Rightarrow 25 = 9 + (AE)^2$

$\Rightarrow (AE)^2 = 16 \Rightarrow AE = 4$

As point E is the mid point of $\overline{AD}$, the length of the rectangle is twice $\overline{AE}$ or 8 cm. The area of the right triangle BCD is half the product of the sides adjacent to the right angle

$$A = \frac{1}{2}bh = \frac{1}{2}(8)(3) = 12\text{ cm}^2$$

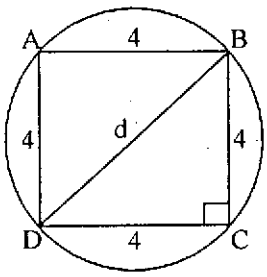
Q12. **(C)** The difference between the entire rectangle and the white triangle is the shaded area. Thus

$$(6 \times 8 = 48) - \frac{6 \times x}{2} = 48 - 3x$$

Q13. **(D)** If the radius of a circle is 1, after reducing 20% it becomes 0.8. Since Area = 0.8 × 0.8 = 0.64 = 1 – 0.64 = 0.36 = %

Q14. **(D)** Since the area of the square is 16.

∴ each side of the square is 4. In ΔBCD



$$(4)^2 + (4)^2 = (BD)^2 \Rightarrow BD = \sqrt{32} = 4\sqrt{2}$$

$$\text{The radius of the circle} = \frac{d}{2} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$$

$$\text{Area of the circle} = \pi r^2 = \pi(2\sqrt{2})^2 = \pi(4)(2) = 8\pi$$

Q15. **(C)** Area of the work surface = Area of the rectangle + area inside half a circle

Area of the rectangle $= 68 \text{ cm} \times 40 \text{ cm} = 2720 \text{ cm}^2$

Area of the circle $= \pi r^2 = 3.14 \times \left(\frac{40}{2}\right)^2 = 3.14 \times (20)^2$

$= 628 \text{ cm}^2$

Area of the shape $= 2720 \text{ cm}^2 + 628 \text{ cm}^2$

$= 3348 \text{ cm}^2$

Q16. **(D)** Area of the unshaded region = Whole Area of the rectangle – Area of first triangle – Area of second triangle

Area of rectangle $= 20 \times 10 = 200 \text{ cm}^2$

Area of first triangle $= \frac{1}{2}(3)(4) = 6 \text{ cm}^2$

Area of second triangle $= \frac{1}{2}(12)(5) = 30 \text{ cm}^2$

Area of unshaded region $= 200 - 6 - 30 = 164 \text{ cm}^2$

Q17. **(D)** The 85%L shows a 15% decrease in length 140% W represents a 40% increase in width. The news rectangle will be

Area = (new length)(new width)

$= (80\% \text{ L})(40\% \text{ W}) = \frac{85}{100}\text{L} \times \frac{40}{100}\text{W}$

$= \frac{119}{100}\text{LW}$

$= 119\% \text{ LW}$

$\therefore$ The area of new rectangle will increase $(119\% - 100\%) = 19\%$

Q18. **(B)** The circumference of big circle $= 64\pi$

We know circumference $= 2\pi r$

$\Rightarrow 64\pi = 2\pi r \Rightarrow r = 32$

From given radius of the small circle $= 8$

Area of the small circle $= \pi r^2 = \pi(8)^2$

$= 64\pi$

Area of 4 small circles = 4(area of 1 circle)

$= 4(64\pi)$

$= 256\pi$

****

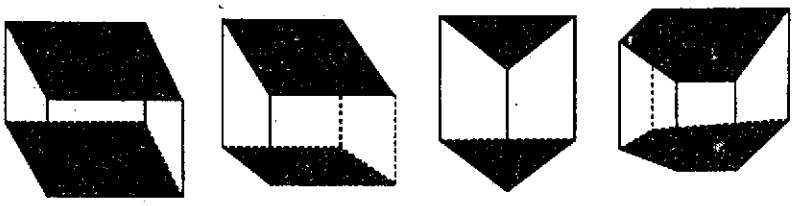
Chapter 6

SOLID GEOMETRY

Prism:

A prism is a solid figure bounded by plane faces, two of which (called the ends) are called congruent figures in parallel planes, and the others (called the sides) are parallelogram.

The solids shown below are prisms. The parts of intersecting planes that determine each prism are its faces. Two faces, F and F', are bases of each prism. The other faces are lateral (side) faces. The intersections of the faces are called edges.



1 2 3 4

Prism	Name	Description
1	Rectangular solid	All six faces are rectangles.
2	Cube	A rectangular solid in which all edges are congruent.
3	Triangular prism	The bases are triangles.
4	Pentagonal prism	The bases are pentagons.

Right Prism:

The above four prisms in the figure are right prisms because a lateral edge is perpendicular to the plane of a base.

Oblique Prism:

A prism that does not have the right prisms property is oblique prism.

Altitude of a Prism:

An altitude of a prism is a segment that is perpendicular to the planes of the bases and that has an end point in each plane. In a right prism, any lateral edge is an altitude.

Surface Area of Prism:

The total surface area of prism is the sum of its lateral surface areas and its two bases.

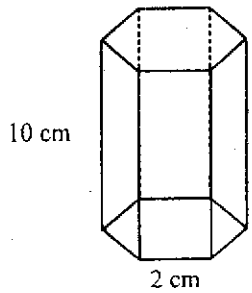
Literal Surface Area:

The literal surface area of a prism is the sum of the areas of its lateral faces.

The lateral surface area L of a right prism is the product of the perimeter P of its base and its lateral edge, e. That is

$$L = Pe$$

Example: Find the total area of the following regular hexagonal prism



Solution:

Step 1: First, find the lateral area

$L = Pe$

Here, $P = 6 \times 4 = 24$ and $e = 20$

Hence $L = 24 \times 20 = 480$

Step 2: Area of one base

Since, the area A of a regular polygon equals one half the product of its perimeter P and the apothem a

$$A = \frac{1}{2}ap$$

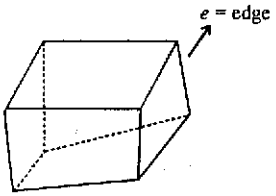
$$= \frac{1}{2} \cdot 2\sqrt{3} \cdot 24$$

$$= 24\sqrt{3}$$

Step 3: Total area

Total area $(480 + 24\sqrt{3})$ cm^2

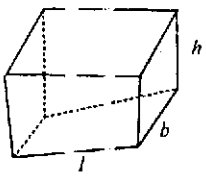
Surface Area of Rectangular Solid:



The surface area of rectangular solid

= 2(length × breadth + breadth × height + length × height)

$= 2(lb + bh + lh)$



In the case of cube, $l = b = h = e$

$\therefore$ Surface Area of a cube $= 2(e \times e + e \times e + e \times e)$

$= 2(e^2 + e^2 + e^2)$

$= 2(3e^2)$

$= 6e^2$

Volume of Rectangular Solid:

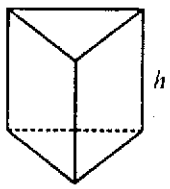
The volume of a rectangular prism is the product of its altitude h, the length of the base l, and the width of the base l and the width of the base w. That is

$$V = lwh$$

Volume of Prism:

The volume of a prism is the product of its altitude h and the area of the base, B.

$$V = Bh$$



Volume of Cube:

The volume V of a cube with edge e is the cube of e. That is

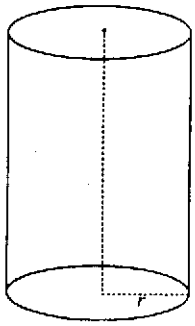
$$V = e^3$$

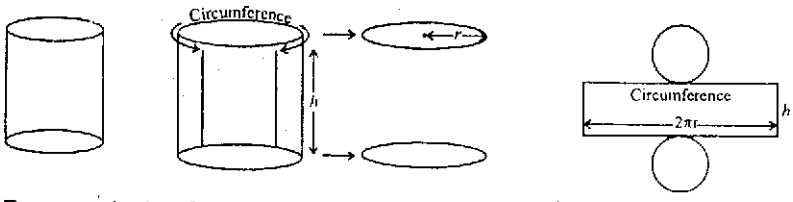
Cylinder:

A right circular cylinder is formed by the revolution of a rectangle around one of its sides. The ends of a right circular cylinder are congruent circles and the line joining the centre of the circular is perpendicular to the plane of the ends.

Surface of a Cylinder:

The surface of a cylinder consists of two congruent circular ends and a curved surface between them. If the circles are removed from the ends, and the curved surface is cut, a rectangle is produced.





From analysis of cylinder, we conclude, that

The height of the rectangle is the same as the height of the cylinder.

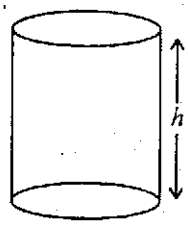
The length of the rectangle is the same as the circumference of the circle at the end.

The area of the two circles is $2\pi r^2$.

The area of the curved surface is $2\pi rh$ ($l \times b$ of the rectangle).

Surface Area of a Cylinder:

The surface area of a cylinder is double the area of one end plus the area of the curved surface.



Surface Area $= 2\pi r^2 + 2\pi rh$

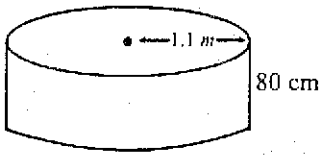
here $2\pi r^2$ is the area of the two circles

and $2\pi rh$ is the area of the curved surface.

Thus Surface area $= 2\pi r(r + h)$

Example:

Calculate the total surface area of this cylinder.



Solution:

Surface area $= 2\pi r^2 + 2\pi rh$

$= 2 \times \pi \times (1.1)^2 + 2 \times \pi \times 1.1 \times 0.8$

$= 13.1318$

$= 13.13 \text{ m}^2$

Volume of a Cylinder:

The formula for the volume of a solid cylinder is the same as the formula for the volume of a solid prism, that is

$$V = Bh$$

where B is the area of the base and h is the height. Since, for a circular cylinder, the area B of the base is πr^2, this formula reduces in this case to

$$V = \pi r^2 h$$

Multiple Choice Questions (MCQs)

Q1. The surface area of a cube is 180, its volume is:

(A) 125 (B) 30

(C) 164 (D) $30\sqrt{30}$

Q2. The volume of a cube is 216, its surface area is:

(A) 64 (B) 216

(C) 25 (D) 96

Q3. A solid metal cube of side 5 inches is placed in a rectangular tank whose length, breadth and height are 5, 6 and 7 inches, respectively. What is the volume in cubic units, of water that the tank can now held?

(A) 210 cubic units (B) 85 cubic units

(C) 125 cubic units (D) 216 cubic units

Q4. A cylinder and the cube have the same volume, if the height, h, of that cylinder is equal to the edge e of the cube, the radius of the cylinder is:

(A) $\frac{\sqrt{\pi}}{h}$ (B) $\frac{\pi}{h}$

(C) $\frac{h}{\sqrt{\pi}}$ (D) $\frac{2\pi}{h}$

Q5. If the height of a cylinder is double to its circumference, what is the volume of the cylinder in terms of its circumference, C?

(A) $\frac{C}{\pi}$ (B) $\frac{C^3}{\pi}$

(C) $\frac{C^3}{4\pi^2}$ (D) $\frac{C^3}{2\pi}$

Q6. Fatima and Maryium each roll a sheet of 9 × 18 paper to form a cylinder. Fatima tapes the two 9-inch edge together and Maryium tapes the two 18-inch edge together. Refer to this question and tell which of the following statement is true?

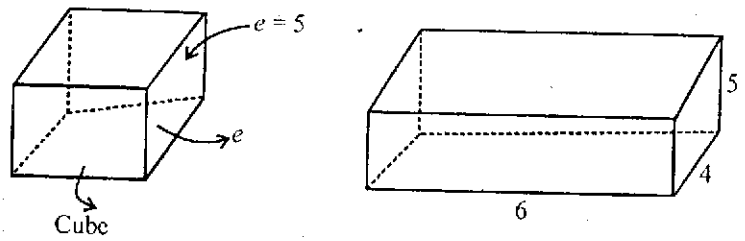
(A) The volume of Fatima's cylinder is greater than Maryium's cylinder

(B) The volume of Fatima's cylinder is less than Maryium's cylinder

(C) The volume of Fatima's cylinder is equal to the Maryium's cylinder

(D) Both cylinders have the same circumference

Q7.



Refer to the above figure, which of the following statement is true?

(A) Volume of the cube is less than the volume of the box

(B) Volume of the cube is greater than the volume of box

(C) Volume of the cube is equal to the volume of box

(D) The surface area of both cube and box are equal

Explanatory Answers

Q1. (D) ∵ Surface Area : $A = 180$

and Surface Area of the cube is $6e^2 \Rightarrow 6e^2 = 180$

$\Rightarrow e^2 = 180 \div 6 \Rightarrow e^2 = 30 \Rightarrow e = \sqrt{30}$

Therefore, edge = $e = \sqrt{30}$, hence its volume = $(\sqrt{30})^3$

$e^3 = 30\sqrt{30}$

Q2. (B) The volume of the cube: $e^3 = 216 \Rightarrow (e^3)^{1/3} = (216)^{1/3}$

$\Rightarrow e = (6^3)^{1/3} \Rightarrow e = 6$

Now Surface Area of a cube: $A = 6e^2 \Rightarrow A = 6(6)^2$

$\Rightarrow A = 216$

Q3. (B) Volume of the tank = $h \times b \times l = 5 \times 6 \times 7 = 210$ cubic units, but the volume of the solid cube, $e^3 = 5^3$.

$\Rightarrow e^3 = 125$ cubic units, therefore the tank can held 210 – 125 = 85 cubic unit of water.

Q4. (C) Volume of the cube $= e^3$ and the

Volume of the cylinder = $\pi r^2 h$

Since both are equal, therefore

$\pi r^2 h = e^3$ also $h = e$

$\therefore \quad \pi r^2 h = h^3 \Rightarrow \pi r^2 = h^2$

$$\Rightarrow \quad r^2 = \frac{h^2}{\pi} \Rightarrow r = \boxed{\frac{h}{\sqrt{\pi}}}$$

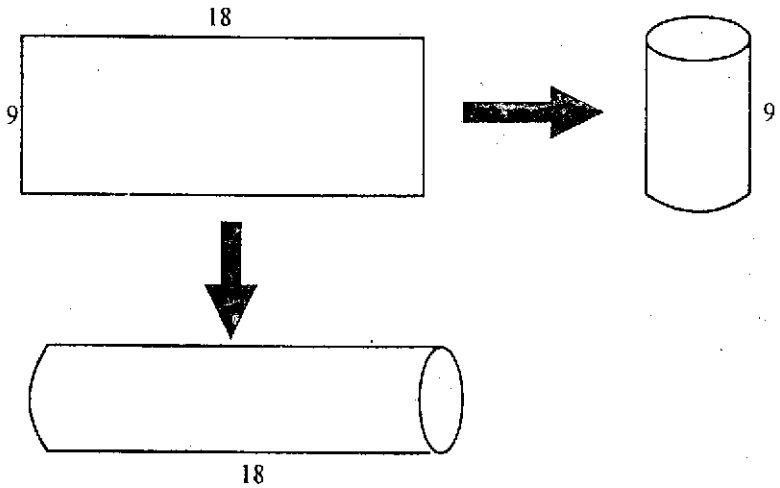
Q5. (D) Since, volume of the cylinder = $\pi r^2 h$, according to the given condition $h = 2C$.

Now $C = 2\pi r \Rightarrow r = \frac{C}{2\pi}$, therefore

$$V = \pi\left(\frac{C}{2\pi}\right)^2 \times 2C \Rightarrow V = \pi\left(\frac{C^2}{4\pi^2}\right) \times 2C$$

$$\Rightarrow V = \frac{C^3}{2\pi}$$

Q6. For sophistication, drawing the figure,



Now, the volume of the cylinder = $\pi r^2 h$

Here, we know only height only, to calculate the radius of the cylinder, we proceed as

The circumference of the cylinder made by Maryium is 9.

$$\therefore 2\pi r = 9 \Rightarrow r = \frac{9}{2\pi}$$

The circumference of the cylinder made by Fatima is 18.

$$\therefore 2\pi r = 18 \Rightarrow r = \frac{18}{2\pi} \Rightarrow r = \frac{9}{\pi}$$

Thus the volume of the cylinder made by Maryium is

$$V = \pi r^2 h \Rightarrow V = \pi\left(\frac{9}{2\pi}\right)^2 18 \Rightarrow V = \pi \times \frac{81}{4\pi^2} \times 18$$

$$\Rightarrow V = \frac{729}{2\pi}$$

and, volume of the cylinder made by Fatima is

$$V = \pi r^2 h \Rightarrow V = \pi\left(\frac{9}{\pi}\right)^2 \times 9 \Rightarrow V = \frac{729}{\pi}$$

Hence the volume of the cylinder made by Fatima is greater than the cylinder made by Maryium

Q7. (B) Volume of the cube : $V_c = 5^3 = 125$

Volume of the cube : $V_b = 4 \times 5 \times 6 = 120$

Thus the volume of the cube is greater than the volume of the box.

Chapter 7

COORDINATE GEOMETRY

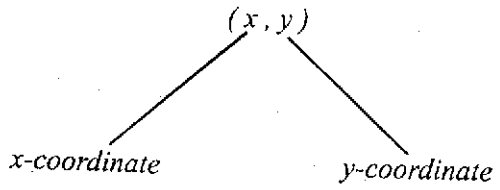
A French mathematician named Descartes invented a method of representing a pair of numbers as a point in a plane. The numbers are measured on a pair of axes which are at right angles to each other and which intersect at the origin O(0, 0).

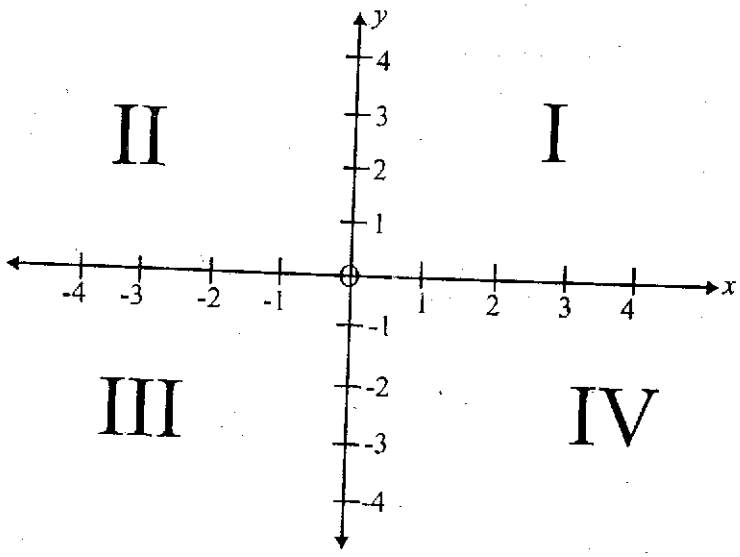
The plane is called the co-ordinate plane or the Cartesian plane.

Coordinates:

Coordinates are an ordered pairs of numbers. These ordered pairs give the position of a point using axes and an origin.

The coordinates of any point *P* are (x, y), where



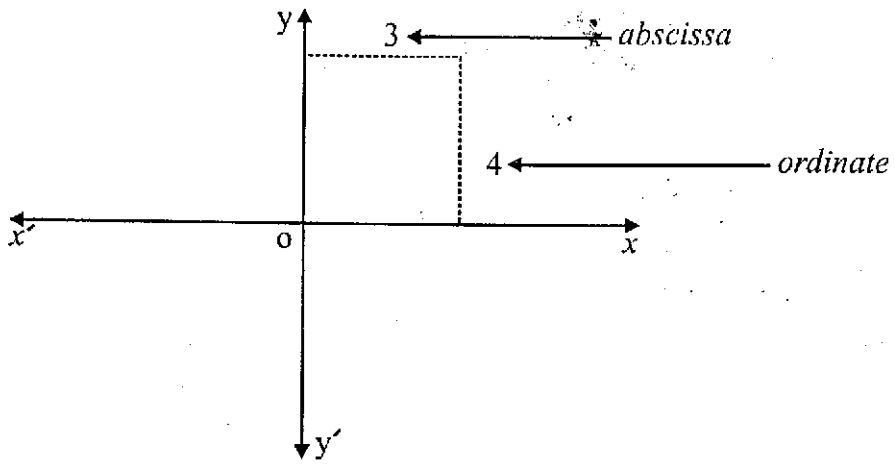


The figure above shows the (rectangular) coordinate plane. The horizontal line is called the x-axis and the perpendicular vertical line is called the y-axis. The point at which these two axis intersect, designated 0, is called the origin. The x-axis and y-axis divide the plane into four parts known as quadrants, I, II, III and IV, as shown.

Since, the coordinates of any point *P* are (*x*, *y*). The first number is always 'across'. The across axis is the *x*-axis. So the first number is called the *x*-coordinate.

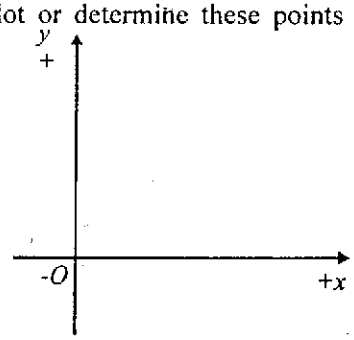
The second number is always 'up or down'. The 'up and down' axis is the *y*-axis. So the second number is called the *y*-coordinate.

Thus, in a Cartesian plane, the two perpendicular distances of the point from these axes are called the coordinates of the point. The distance of the point from the *y*-axis is known as the *x*-coordinate or the abscissa and that from the *x*-axis is known as the *y*-coordinate or ordinate of the point. Thus an order pair (3, 4) represents a point where abscissa is 3 and ordinate is 4.



Negative Coordinates:

Coordinates can be positive or negative numbers. To plot or determine these points you must have both positive and negative numbers on the axes.



On the *x*-axis:

(i) Values to the right of O are positive (+),

(ii) Values to the left of O are negative (–).

On the *y*-axis:

(iii) Values above O are positive (+),

(iv) Values below O are negative (–).

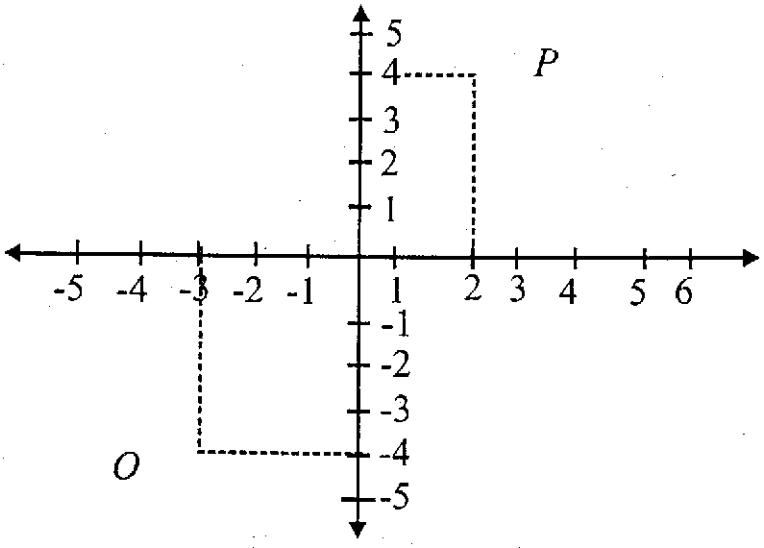
Remember

(i) The coordinates of the origin are (0, 0).

(ii) Any point on the *x*-axis has its ordinate zero.

(iii) Any point on the *y*-axis has its abscissa zero.

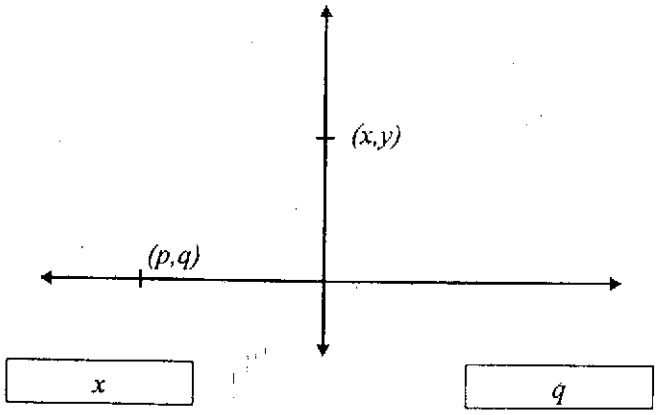
Example 1:



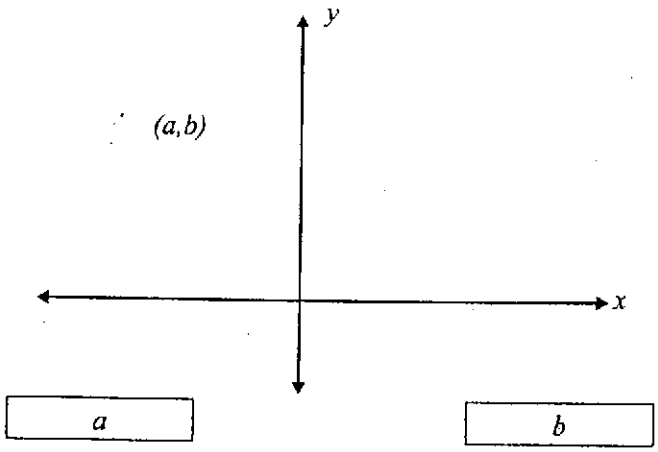
In the above graph, the (x, y) coordinates of point P are (2, 4), it is because P is 2 units to the right of the y-axis (*i.e.*, $x = 2$) and 4 units above the x-axis (*i.e.*, $y = 4$). In the same way, the (x, y) coordinates of point Q are (–3, –4), and the origin O has the coordinates (0, 0).

Column A **Column B**

Example 2:



Example 3:



Solution 2:

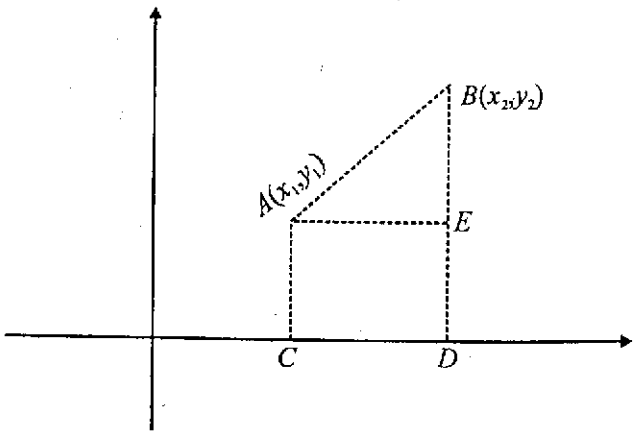
In the Cartesian plane, (p, q) lies on the x-axis, we know, on x-axis the y-coordinate of a point is zero, hence, $q = 0$. Similarly, on the y-axis, the x-coordinate of a point is zero. Now, because, point (x, y) lies on y-axis, hence $x = 0$. Thus, the correct answer is "=".

Solution 3:

In the second example, point (a, b) lies in the second quadrant and in second quadrant a is negative and b is positive, so $a < b$, is the correct answer. The answer is <.

Distance between Any Two Points in A Cartesian Plane:

Consider two points A and B with coordinates (x_1, y_1) and (x_2, y_2). Join A and B. Draw perpendicular



$\overline{AC}$ and $\overline{BD}$ on the x-axis from A and B respectively. Also from A, draw $\overline{AE}$ perpendicular to $\overline{BD}$. Then from figure

$$AE = CD$$

$= OD - OC$

$= x_2 - x_1$

$BE = BD - ED$

$= y_2 - y_1$

$\therefore$ In right-angled triangle ABE

$AB^2 = AE^2 + BE^2$

$= (x_2 - x_1)^2 + (y_2 - y_1)^2$

$\therefore \quad AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Thus

The distance d between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Note:

The distance of a point $P(x, y)$ from the origin is

$$OP = \sqrt{x^2 + y^2}$$

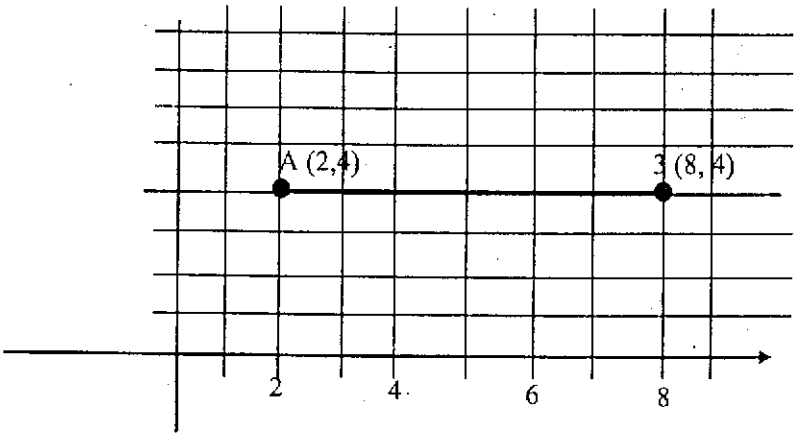
The Midpoint Formula

The coordinates (x_m, y_m) of the midpoint M of the segment whose endpoints are $P(x_1, y_1)$ and $P_2(x_2, y_2)$ are

$$x_m = \frac{x_1 + x_2}{2} \text{ and } y_m = \frac{y_1 + y_2}{2}$$

Example 4:

Find the coordinates of the midpoint $M(x_m, y_m)$ of $\overline{AB}$.



Solution:

Since $\overline{AB}$ is horizontal, the y-coordinate of any point on $\overline{AB}$ is 4. Thus y coordinate of M is 4.

Now $\quad x_m = \dfrac{x_1 + x_2}{2}$

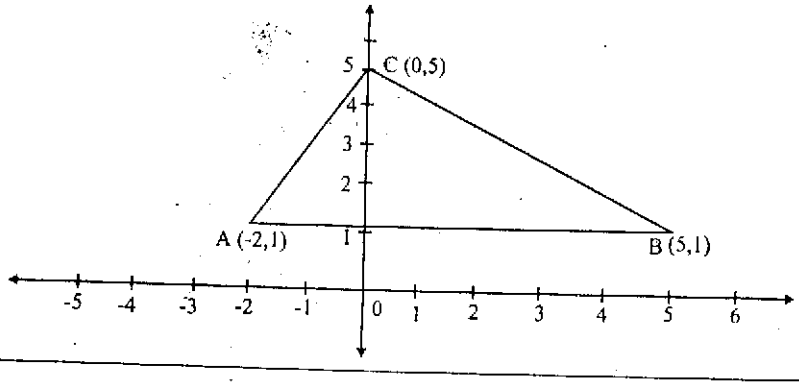
$= \dfrac{2 + 8}{2}$

$= \dfrac{10}{2}$

$= 5$

$\therefore \quad M(x_m, y_m) = (5, 4)$

Example 5-6 refer to a triangle in the following figure



Example 5:

What is the area of ΔABC?

A. 9 B. 15

C. 14 D. 12

Solution:

The base of the triangle ABC is AB, and A(–2, 1), B(5, 1) lies on a same horizontal line so length of AB is,

$$AB = \sqrt{(5+2)^2 + (1-1)^2}$$

$$AB = \sqrt{7^2} = 7$$

Now, the altitude of the triangle is the distance between point C to line AB, Hence

$$\text{Altitude} = \sqrt{(0-0)^2 + (1-5)^2} = \sqrt{-4^2} = \sqrt{16}$$

$$= 4$$

$$\text{Thus, the Area} = \frac{1}{2}(\text{Base})(\text{Altitude})$$

$$= \frac{1}{2}(7)(4)$$

$$= (7)(2)$$

$$= 14 \qquad (C)$$

Example 6:

What is the perimeter of ΔABC?

A. $14 + \sqrt{41}$ B. $\sqrt{41} + 6\sqrt{5}$

C. $\sqrt{41} + 14\sqrt{5}$ D. $7 + \sqrt{41} + 2\sqrt{5}$

Solution:

The perimeter of the triangle ABC is $AB + BC + CA$.

From example, 5 $AB = 7$, Now we find BC and AC

$$\therefore \quad BC = \sqrt{(0-5)^2 + (5-1)^2} = \sqrt{25+16} = \sqrt{41}$$

$$AC = \sqrt{(0+2)^2 + (5-1)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

Hence .Perimeter $= AB + BC + CA$

$$= 7 + \sqrt{41} + 2\sqrt{5} \quad (D)$$

Slope of a Line:

The gradient is the slope of a line. A line that slopes upwards ↗ from left to right has a positive gradient, while a line that slopes downwards ↘ from left to right has a negative gradient. In mathematics, the gradient or slope is represented by the letter m. Thus

Gradient (m) of a straight line $= m = \frac{\text{Rise}}{\text{Run}} = \frac{\text{Difference in } y \text{ values}}{\text{Difference in } x \text{ values}}$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$

Example 7:

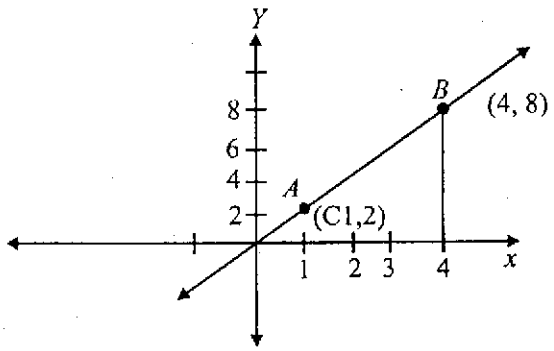
Find the gradient (slope) of line AB.

Solution:

$$m = \frac{\text{Difference between } y\text{-coordinate}}{\text{Difference between } x\text{-coordinate}}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{8-2}{4-1} = \frac{6}{3} = 2$$



Note:

1. The slope of any horizontal line is 0.
2. The slope of any vertical line is undefined.
3. Two non-vertical lines are parallel (||) if and only if they have the same slope.
4. Two non-vertical lines are perpendicular if and only if their slopes are negative reciprocals.

Example 8:

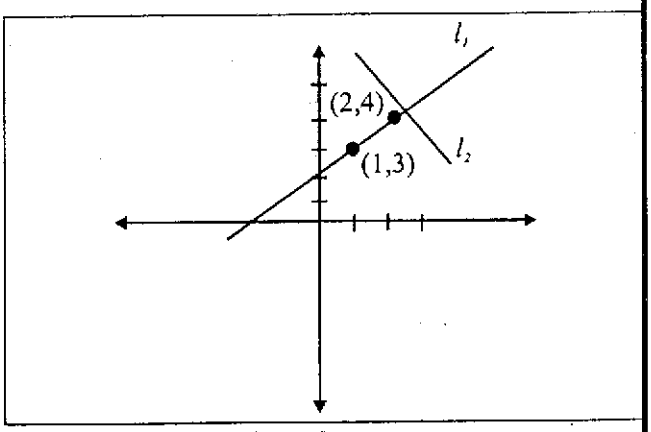
Column A	Column B
Line l_1 passes through (1, 3) and (2, 4) Line l_2 is perpendicular to l_1	
The slope of l_1	The slope of l_2

Solution:

First we sketch the line l_1, and then draw line l_2 perpendicular to l_2. The line l_1 slopes upward so it has positive slope and the line l_2 is sloped downward having negative slope, so

$$m_1 > m_2$$

where m_1 is the slope of line l_1 and m_2 is the slope of line l_2.



Multiple Choice Questions (MCQs)

Q1. If $A(-2, 3)$ and $B(5, -1)$ are the endpoints of one side of a square $ABCD$, what is the area of the square?

(A) 20 (B) 65

(C) 30 (D) 35

Q2. **If $A(3, 2)$ and $B(7, 2)$ are two vertices of a rectangle, which of the following could not be the another vertices of that rectangle?**

(A) (3, 7) (B) (7, 3)
(C) (3, −7) (D) (−3, 7)

Q3. **A circle whose center is at (3, 4) passes through the origin. Which of the following points is not on the circle?**

(A) (−1, 3) (B) (−1, 1)
(C) (0, 0) (D) (7, 7)

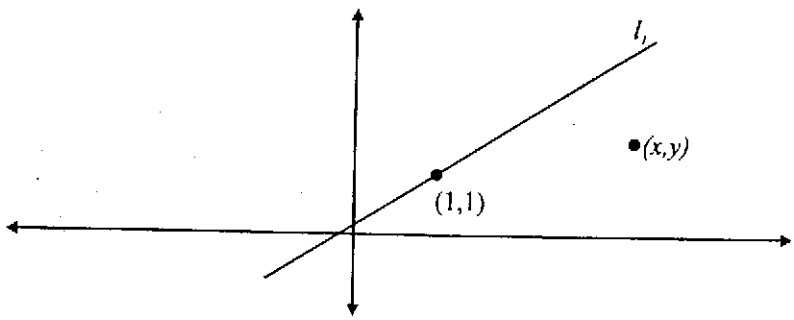
Q4. **If a line passes through the points (x, y) and $\left(\frac{1}{x}, y\right)$, then its slope is**

(A) $\frac{1}{x}$ (B) 0
(C) $\frac{1-x^2}{x}$ (D) 1

Q5. **The slope of the line passing through $(-b, b)$ and $(3b, a)$ is 1 and $b \neq 0$, which of the following is an expression for a in terms of b?**

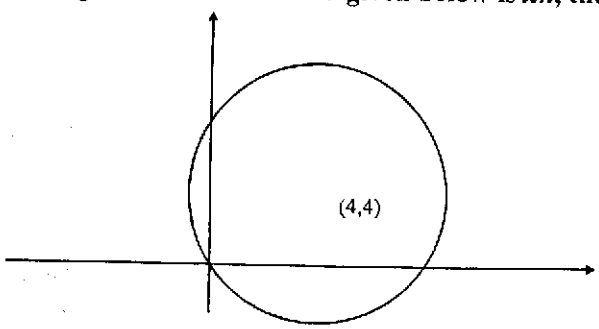
(A) $\frac{1}{4b}$ (B) $3b$
(C) $5b$ (D) $2b$
(E) $4b$

Q6. **In the figure given below, $x - y$ is**



(A) positive (B) negative
(C) less than zero
(D) less than or equal to zero
(E) cannot find from the given information

Q7. **If the area of the following circle with center O given below is $x\pi$, then x = ?**



(A) 28π (B) 32π
(C) 9π (D) 7π

Direction, **Questions 8 – 9** are referred to parallelogram $ABCD$, whose coordinates are $A(-4, 2)$, $B(-3, 6)$, $C(4, 6)$, $D(3, 2)$.

Q8. What is the area of the parallelogram $ABCD$?

(A) 24 (B) 21
(C) 29 (D) 28

Q9. What is the perimeter of parallelogram $ABCD$?

(A) $14+\sqrt{17}$ (B) $16+\sqrt{7}$
(C) $2+\sqrt{17}$ (D) $2(7+\sqrt{17})$

Explanatory Answers

Q1. (B) $S = AB = \sqrt{(5+2)^2+(-1-3)^2}$

$= \sqrt{(7)^2+(-4)^2}$

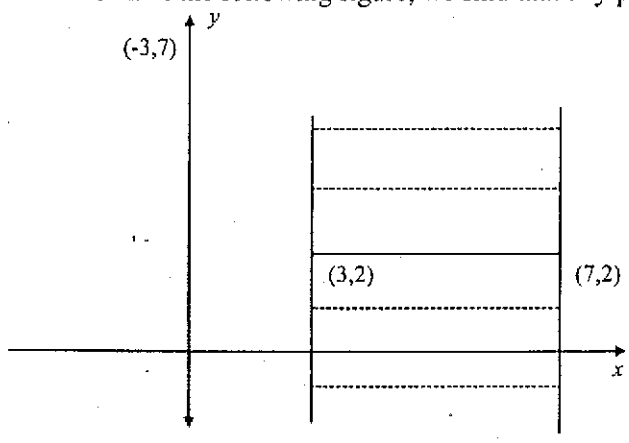
$= \sqrt{49+16}$

$S = \sqrt{65}$

$S^2 = (\sqrt{65})(\sqrt{65})$

$S^2 = 65$

Q2. (D) By drawing diagram as shown in the following figure, we find that any point whose x-coordinate is

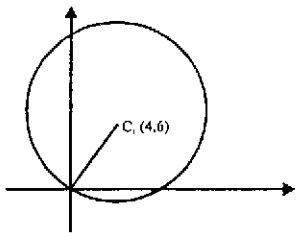


3 or 7 could be another vertex, so $(-3, 7)$ is not possible.

Q3. (A) First of all, we draw a diagram. Thus its radius is the distance from the origin to its center. By using distance formula

$r = \sqrt{(3-0)^2+(4-0)^2}$

$= \sqrt{9+16} = \sqrt{25} = 5$



Since the distance of any point of a circle from the center of that circle is same, so any point whose distance is greater than 5 would not be on the circle. Now check these points given in the options:

A: $(-1, 3) \Rightarrow r = \sqrt{(3+1)^2+(4-3)^2} = \sqrt{16+1} = \sqrt{17}$ which is not equal to 5, hence $(-1, 3)$ will not lie on the circle.

B: $(-1, 1) r = \sqrt{(3+1)^2+(4-1)^2} = \sqrt{16+9} = \sqrt{25} = 5$

Since its distance from the center of the given circle is equal to the radius of the given circle, so given point will lie on the circle. Similarly, we can prove the options C, D & E.

Q4. (B) Slope of the given line $= \dfrac{y_2 - y_1}{x_2 - x_1}$

Here $y_2 = y$, $y_1 = y$ and $x_2 = \dfrac{1}{x}$, $x_1 = x$, thus

$$m = \frac{y - y}{\frac{1}{x} - x} = \frac{0}{\frac{1}{x} - x} = 0$$

Q5. (C) The slope of the line passing through two points (x_1, y_1) and (x_2, y_2) is given by

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Here $y_2 = a, y_1 = b$ and $x_2 = 3b, x_1 = -b, m = 1$

Hence, $1 = \frac{a - b}{3b + b}$

$4b = a - b \Rightarrow a = 4b + b$

$\Rightarrow a = 5b$

Q6. Since the line l passes through origin (0, 0) and (1, 1), if any point (p, q), other than (1, 1) on the line l, then any point (p, r), below the point (p, q) will be less than (p, q). Thus $a > b \Rightarrow a - b > 0$

Q7. (B) Since the line segment joining (4, 4) and (0, 0) is the radius r of the circle

$\therefore \quad r = \sqrt{(4-0)^2 + (4-0)^2} \Rightarrow r = \sqrt{16 + 16}$

$\Rightarrow \quad r = \sqrt{32} \Rightarrow r = 4\sqrt{2}$

Since Area, $A = \pi r^2 \quad \Rightarrow A = \pi(4\sqrt{2})^2$

$\Rightarrow A = \pi(16(2))$

$\Rightarrow A = 32\pi$

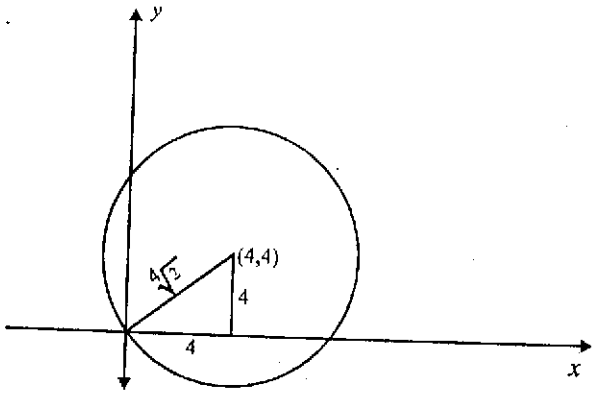
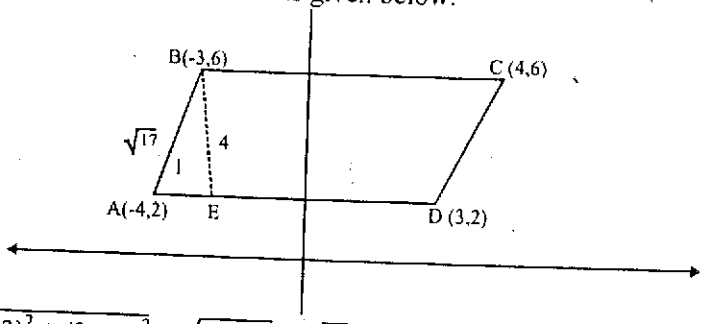


Diagram for solution question 8 and 9 is given below:



$AB = \sqrt{(-4+3)^2 + (2-6)^2} = \sqrt{1 + 16} = \sqrt{17}$

$AE^2 = AB^2 - BE^2 \Rightarrow AE^2 = (\sqrt{17})^2 - (4)^2$

$\Rightarrow AE^2 = 1 \Rightarrow AE = 1$

$AD = \sqrt{(-4-3)^2 + (2-2)^2} \Rightarrow AD = \sqrt{49 + 0} \Rightarrow AD = 7$

Q8. (D) Since the base is 7 and the height is 4, thus

Area = 7 × 4 = 28 square unit.

Q9. (D) Since AD and BC are each 7 and AB and CD both are equal to $\sqrt{17}$, thus the perimeter is

$7 + \sqrt{17} + 7 + \sqrt{17} = 14 + 2\sqrt{17} = 2(7 + \sqrt{17})$

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Chapter 8

COUNTING AND PROBABILITY

In mathematics, some questions begin with "How much, how many,....." In these type of questions you are being asked to count something: how many bananas Sonia bought, how many Rupees did Rizwan spend, how many pages did Fatima read, how many numbers are required to satisfy a certain formula, or how many ways are there to complete a given task. Usually these problems can be solved by simple arithmetic. Sometimes it helps to use basic probability rules, and Vann diagrams. In this chapter, we shall deal with such rules and counting principles that helps to solve such problems.

Using Arithmetic to Count:

This method is illustrated by the following three examples:

Example 1:

Osama bought some chocolates. If he entered the shop with Rs. 215 and left with Rs. 195, how much did the chocolates cost?

Example 2:

Usman was selling tickets for the magic show. One day he sold tickets numbered 98 through 111. How many tickets did he sell that day?

Example 3:

In a line outside the utility store, Hamza is the 29th person and Osama is the 38th person. How many people are there between Hamza and Osama?

Solution:

These questions require a simple subtraction. In example 1, Osama did spend (215 – 195 = 20) 20 on chocolates; in example 2, however, Usman sold 14 tickets; and in example 3, only 8 persons are on line between Hamza and Osama.

In example 1, we simply subtract the amount after purchasing and before purchasing. In example 2, you need to subtract and then add 1: 111 – 98 = 13 + 1 = 14. And in example 3, you need to subtract and then subtract 1 more: 38 – 29 = 9 – 1 = 8.

In these examples, the issue is that, whether or not the first and last numbers are included or excluded.

From above examples, we find the following rules.

To find how many numbers there are between two numbers, apply following rules:

1. If exactly one of the endpoints is included, then subtract these values.
2. If both endpoints values is included, then subtract these values and add 1 in the answer.
3. If endpoints values are not included, then
   (i) Subtract these values, and
   (ii) Subtract 1 more from the answer.

The Counting Principle:

The Sum Rule

If a first task can be done in m ways and a second task in n ways, and if these tasks cannot be done at the same time, then there are $m + n$ ways to do either task.

Example 1:

A student can choose a computer project from one of the three lists. The three lists contain 21, 13 and 17 possible projects, respectively. How many possible projects are there to choose from?

Solution:

The student can choose a project from the first list in 21 ways, from the second list in 13 ways, and from the third list in 17 ways. Hence there are 21 + 13 + 17 = 51 projects to choose from.

The Product Rule:

Suppose that a procedure can be broken down into two tasks. If there are m ways to do the first task and n ways to do the second task after the first task has been done, then there are $m \times n$ ways to do the procedure.

The following examples illustrate how the product rule is used:

Example 1:

There are 28 computers in a computer center. Each computer has 22 parts. How many different parts to a computer in the center are there?

Solution:

The procedure of choosing a part consists of two tasks, first picking a computer and then picking a part on this computer. Since there are 28 ways to choose a computer and 22 ways to choose the part no matter which computer has been selected, the product rule shows that there are 616 (28×22) parts.

Example 2:

Sana has 6 different baskets in the basement. She is going to bring up 2 of them and placed 1 in her den and 1 in her bedroom. In how many ways can she choose which baskets go in each room?

Solution:

The first job was to pick 1 of the 6 baskets and place it in the bedroom. That could be done in 6 ways. The second job is to pick a second basket and place in the den. That could be done by choosing any of the remaining 5 baskets. So there are $6 \times 5 = 30$ ways to place 2 of the baskets.

The Inclusion-Exclusion Principle:

When two tasks can be done at the same time, we cannot use the sum rule to count the number of ways to do one of the two tasks. Adding the number of ways to do each task leads to an overcount, since the ways to do both tasks are counted twice. To correctly count the number of ways to do one of the two tasks, we add the number of ways to do each of the two tasks and then subtract the number of ways to do both tasks. This technique is called the principle of inclusion-exclusion.

Permutations:

Each one of the total arrangements that can be made by taking some or all of a number of different objects is called permutation. This actually presents any arrangement of a set of objects in a definite order. Permutations of the set of letters a, b, c, d taken all at a time are $abcd$, $acbd$, $bacd$ etc. An ordered arrangement of r elements of a set is called an r-permutation. For example, permutations of $n = 4$ letters a, b, c, d taken $r = 2$ at time are ab, ac, ad, bc, bd, ba, etc. Thus there are twelve permutation of four letters, taken two at a time.

The number of r-permutations of a set with n elements is denoted by ${}^{n}p_r$ or $P(n, r)$.

Note:

We can find ${}^{n}p_r$ using the product rule.

Theorem:

The number of r-permutations of a set with n distinct elements is

$$ {}^{n}p_r = n(n-1)(n-2) \ldots\ldots (n-r+1) $$

Example 1:

How many different arrangements are there of the letters A, B, C, D and E?

A. 24 B. 20 C. 25 D. 120

Solution:

In this problem, the first job is to choose one of the five letters to write in the first position; there are 5 ways to complete that job. The second job is to choose one of the remaining 4 letters to write in the second position; there are 4 ways to complete that job. The third job is to choose one of the 3 remaining letters to write in the third position; there are 3 ways to complete that job. The fourth job is to choose one of the two remaining letters to write in the fourth position; there are 2 ways to complete that job. At last, the fifth job is to choose the only remaining letter and to write it.

$$ 5 \times 4 \times 3 \times 2 \times 1 = 120 $$

Using Permutation:

The 5 letters can be written in

$^5P_4 = 5(5-1)(5-2)(5-3)(5-4)$

$= 5 \times 4 \times 3 \times 2 \times 1$

$= 120$ ways

Example 2:

In how many different ways can 6 persons be seated on a bench?

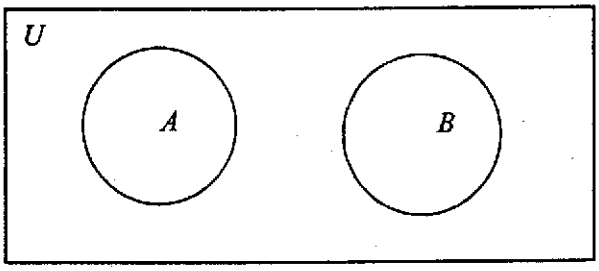
Solution:

Six persons can be seated in different position on a bench is

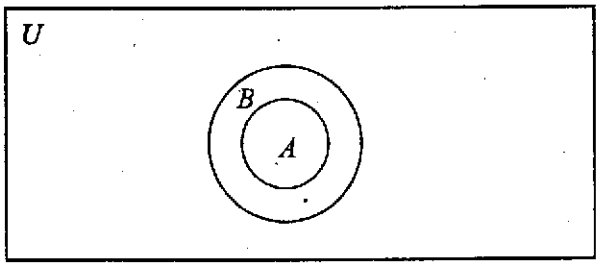
$$= 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720 \text{ ways}$$

Venn Diagrams:

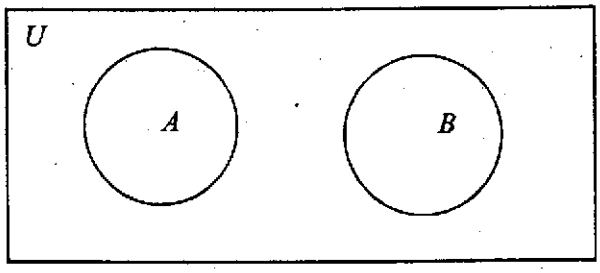
In a Venn diagram, a rectangular region represents a universal set and regions bounded by simple closed curves represent other sets, which are subsets of the universal set. In the following figure, the circular regions represents set A and set B and the remaining position of rectangle representing the universal Set U.



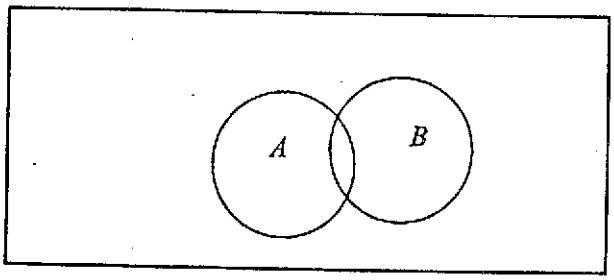
If $A \subset B$, then the circle representing A will be entirely within the circle representing B as shown in the following figure



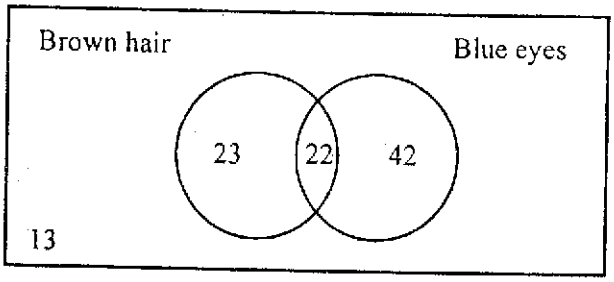
If A and B are disjoint, *i.e.*, have no elements common, then the circle representing A will be separated from the circle representing B as shown in the following figure



However, if A and B are two arbitrary sets, then it is possible that some elements are in A but not in B, some are in B but not in A, some are in both A and B, and some are in neither A nor B; hence in general we represent A and B; as in following figure



Many verbal statements can be translated into equivalent statements about sets which can be determined by Vann diagrams. To illustrate this assume that department of Pharmacy of Punjab University has 100 students. The following diagram, which divides the rectangle into four regions, shows the distribution of these students in the brown hair and blue eyes.

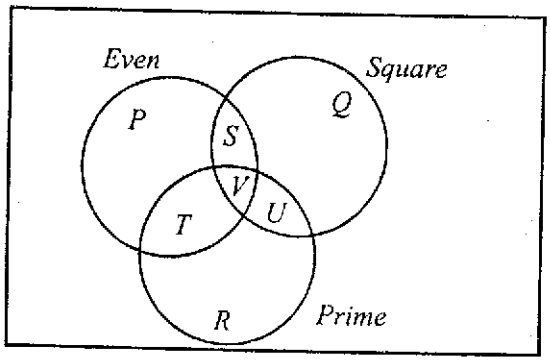


The 22 written in the part of the diagram where the two circles overlap represents the 22 students who have both brown hair and blue eyes. The 23 written in the circle on the right represents the 23 students who have brown hairs but not blue eyes, while the 42 written in the left circle represents the 42 students who have blue eyes but not brown hairs. At last, the 13 written in the rectangle outside the circles represents the 13 students who have neither brown hair nor blue eyes. The numbers in all four regions must add up to the total number of the students.

22 + 23 + 42 + 13 = 100. In this diagram, we see that, there are 64 students having blue eyes – 22 who are also having brown hair and 42 not having brown hair. Similarly, there are 23 + 22 = 45 students having brown hair. Note that, if we add both brown hair and blue hair, then we find 64 + 45 = 109 students—more than the number of students in the department. That's because 22 names are on both lists and so have been counted twice. The number of students having brown hair and blue eyes are 109 – 22 = 87. Those 87 together with the 13 students who have neither brown hair nor blue eyes make up the total of 100.

Example:

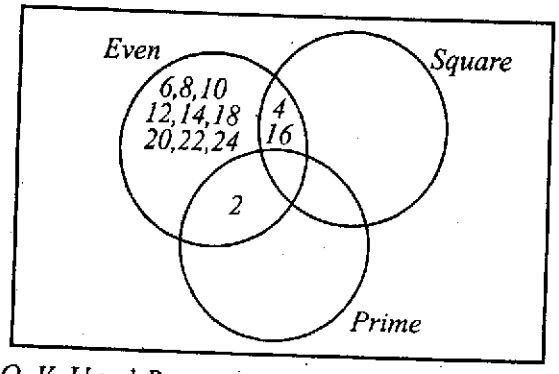
If the integers from 1 through 25 are each placed in the following Venn diagram. Then identify the empty region.



A. Q only B. V only
C. U and V only D. R only
E. Q, V, U and R only

Solution:

Put each of the number from 1 through 25 in the appropriate region



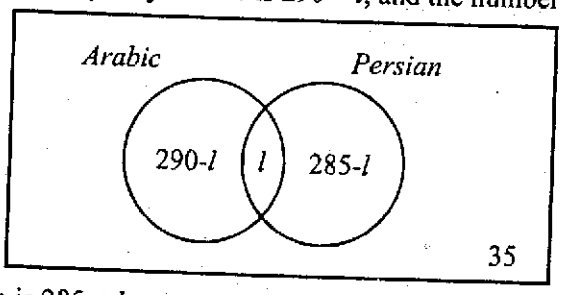
From above figure, we see that Q, V, U and R are empty regions. Hence the true choice is E.

Example:

There are 560 students of 10th class at Muslim High School, 290 of them study Arabic and 285 study Persian. If 35 students study neither language, how many study both?

Solution:

First of all, draw a Venn diagram. Let l represents the number of students who study both Persian and Arabic. Then the number of students who study only Arabic is $290 - l$, and the number of



students who study only Persian is $285 - l$.

The number of students who study at least one of the languages is $560 - 35 = 525$, thus

$$525 = (290 - l) + l + (285 - l)$$

$\Rightarrow \quad 525 = 290 - l + l + 285 - l$

$\Rightarrow \quad 525 = 575 - l \Rightarrow l = 575 - 525 \Rightarrow l = 50$

Thus 50 students study both languages.

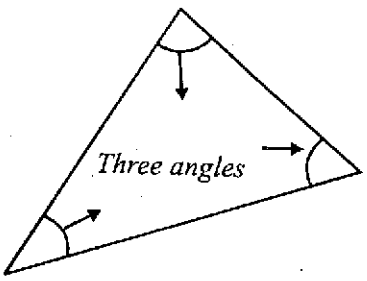
Probability:

Definition:

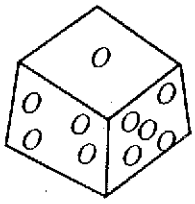
Probability is the numerical evaluation of a chance that a particular event would occur.

Probability is about how likely something is to happen. We often use the word chance for probability.

Something always happen, we say they are certain to happen. For example, if you draw a triangle it has three sides and three angles. That's certainty!



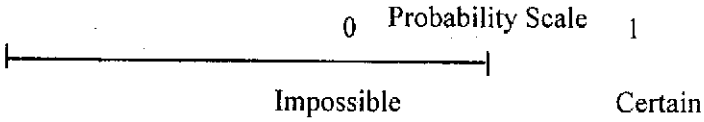
Something will never happen. We say they are impossible. For example, if you roll an ordinary dice you will never get a 7 or 8. That's impossible.



Something are not certain, but not impossible either. They may or may not happen. For example, if you toss a coin it may land heads, or it may not.

Scale of Probability:

In mathematics when we talk about the probability of 'something' happening, we call the something an event. Probability is a measure of how likely an event is to happen. We use a number, not words, to describe its size. We can give a probability a number from 0 to 1.



On this Probability Scale:

An event that is impossible is given a probability of 0. An event that is certain is given a probability of 1.

For example:

The probability that you get a 7 on an ordinary dice is 0, the probability that a triangle you draw will have three angles is 1.

All other probabilities on this scale are between 0 and 1. They must be less than 1. We write them as proper fractions or decimal fractions.

Remember:

In a proper fraction, the top number is smaller than the bottom.

To compare probabilities, we compare the sizes of the fractions. The less likely an event is to happen, the smaller the fraction. The more likely an event is to happen, the larger the friction.

Some Basic Definitions:

Equally Likely Events:

A set of events is said to be equally likely if none of them is expected to occur in preference to the other.

For example, when a fair coin is tossed, then occurrence of head or tail are equally likely events and there is no reason to expect a 'head' or a 'tail' in preference to the other.

Exhaustive Events:

A set of events is said to be exhaustive when a random experiment always results in the occurrence of at least one of them.

For example, if a die is thrown, then the events

$$E_1 = \{1, 2\}, E_2 = \{2, 3, 4\}$$

are not exhaustive as we cannot get 5 as outcome of the experiment which is not the member of any of the events E_1 and E_2. While, if $E_3 = \{1, 2, 3\}$ and $E_2 = \{2, 4, 5, 6\}$, then the set of events E_3 and E_4 is exhaustive.

Independent Events:

Two events are said to be independent, if the occurrence of one does not depend on the occurrence of the other.

For example, when a coin is tossed twice, the event of occurrence of head in the first throw and the event of occurrence of head in the second throw are independent events.

Mutually Exclusive Events:

A set of events is said to be mutually exclusive if occurrence of one of them precludes the occurrence of any of the remaining events.

In other words, events $E_1, E_2, \ldots\ldots E_n$ are mutually exclusive if and if only

$$E_i \cap E_j = \phi \qquad (i \neq j)$$

Complement of An Event:

The complement of an event E, denoted by $\overline{E}$ or E' or E^c, is the set of all sample points of the space other than the sample points in E.

For example, when a die is thrown, we get the sample space

$$S = \{1, 2, 3, 4, 5, 6\}$$

If $E = \{1, 4, 5, 6\}$, then $\overline{E} = \{2, 3\}$

It is noted that $E \cup \overline{E} = S$

Calculating Probabilities:

When all the outcomes of an activity are equally likely, you can calculate the probability of an event happening.

$$\text{Probability of an event} = \frac{\text{Number of favourable outcomes for that event}}{\text{Total number of possible outcomes}}$$

This is sometimes called the probability fraction.

To calculate the probability of an event, you may need to list all the outcomes and the favourable outcomes first.

Note:

Outcomes which give the 'event' you are interested in are called favourable outcomes for that event, and, outcomes which have an equal chance of happening are called equally likely outcomes.

Example:

Sadaf throw an ordinary dice. Write down the probability that she gets

a) a four b) more than two c) a seven

Solution:

Sample space: Possible outcomes = {1, 2, 3, 4, 5, 6}

Total number of possible outcomes = 6

a) Event: get a four

Favourable outcome: 4

Number of favourable outcomes = 1

Probability of getting a 4 $= \frac{1}{6}$

b) Event: get more than two

Favourable outcomes: 3, 4, 5, 6

Number of favourable outcomes = 4

Probability of getting more than 2 $= \frac{4}{6} = \frac{2}{3}$

c) Event: 'get a seven'

No. of favourable outcomes = 0

Probability of getting a seven $= \frac{0}{6} = 0$

Example:

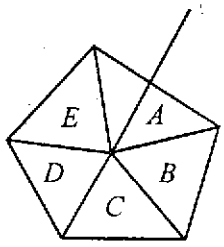
This spinner is made from a regular pentagon. When Fatima spins it once what is the probability that she get

a) the letter B

b) the vowel

c) a consonant

d) the letter F



Solution:

Sample space : Possible outcomes = {A, B, C, D, E}

Total number of possible outcomes = 5

a) Event: 'get a letter B'
Favourable outcomes = B
No. of favourable outcomes = 1
Probability of favourable outcomes $= \frac{1}{5}$

b) Event: 'get a vowel'
Favourable outcomes = A, E
No. of favourable outcomes = 2
Probability of favourable outcomes $= \frac{2}{5}$

c) Event: 'get a consonant'
Favourable outcomes : B, C, D
No. of favourable outcomes : 3
Probability of favourable outcomes $= \frac{3}{5}$

d) Event: 'get a letter F'
Favourable outcomes: 0
Probability of favourable outcomes $= \frac{0}{5} = 0$

Some Important Results on Probability:

1. $0 \le P(E) \le 1$, *i.e.*, the probability of occurrence of an event is a number laying between 0 and 1.
2. $P(\phi) = 0$, *i.e.*, probability of occurrence of an impossible event is zero.
3. $P(S) = 1$, *i.e.*, probability of occurrence of a sure event is 1.
4. $P(\overline{E}) = 1 - P(E)$, where $\overline{E}$ is the event that will not occur.
5. $P(\overline{E}) + P(E) = 1$
6. If A and B are mutually exclusive events, then

$$P(A \cup B) = P(A) + P(B)$$

Product Rule:

When more than one event occurs (*e.g.*, tossing 2 coins, planting 5 seeds, choosing 3 people, throwing 2 die), multiply the probabilities together.

$$P(AB) = P(A) \,.\, P(B)$$

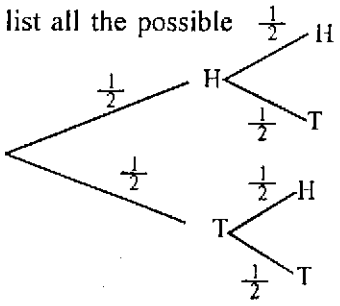
Tree Diagrams:

When using the product rule there may be more than one possible result. For example, the result when tossing two coins could be HT or TH (Head / tail or Tail / head).

When there is more than one possible result we add them together

$$P(A \text{ or } B) = P(A) + P(B)$$

This is called the additional rule of probability. A tree diagram helps to list all the possible outcomes. Tree diagrams combine the addition and product rules.



Example 1:

If 2 coins are tossed, find the probability of tossing a head and a tail.

Solution:

$P(\text{head or tail}) = P(HT) + P(TH)$

$$= \left(\frac{1}{2} \times \frac{1}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2}\right)$$

$$= \frac{1}{4} + \frac{1}{4}$$

$$= \frac{1}{2}$$

Example 2:

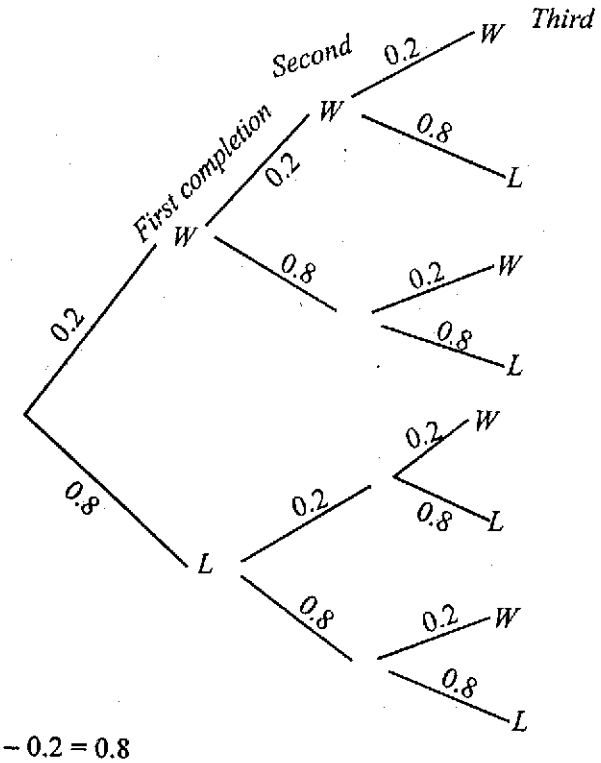
Fahad has probability of 0.2 of winning a prize in a competition. If he enters 3 competitions, find the probability of his winning:

a) 2 competition

b) at least 1 competition

Solution:

First, we construct a tree diagram of the example



a) Probability of losing = 1 – 0.2 = 0.8

$P(2W) = P(WWL) + P(WLW) + P(LWW)$

$= (0.2 \times 0.2 \times 0.8) + (0.2 \times 0.8 \times 0.2) + (0.8 \times 0.2 \times 0.2)$

$= 0.032 + 0.032 + 0.032$

$= 0.096$

b) $P(\text{at least one } W) = 1 - P(LLL)$

$= 1 - (0.8 \times 0.8 \times 0.8)$

$= 1 - 0.512$

$= 0.488$

Multiple Choice Questions (MCQs)

Q1. Ayesha completed questions 4 – 18 of a mathematics exercise in 30 minutes. At this rate, how long, in minutes, will it take her to complete questions 27 – 55?

(A) 59 (B) 29

(C) 30 (D) 58

Q2. Munir was born on August 14, 1934 and died on February 28, 1999. What was his age, in years, at the time of his death?

(A) 64 (B) 65

(C) 66 (D) 68

Q3. How many three-digit number have only even digits?

(A) 48 (B) 58

(C) 500 (D) 300

Q4. There are 28 players in a college cricket team. What is the probability that at least 3 of them

have their birthday in the same month?

(A) 0 (B) $\frac{1}{5}$

(C) 1 (D) $\frac{1}{2}$

Q5. A bag has 7 marbles, one of each of colours, green, blue, brown, yellow, red, white and pink. If 6 marbles are removed from the bag, what is the probability that the red one was removed?

(A) $\frac{1}{7}$ (B) $\frac{4}{3}$

(C) $\frac{2}{7}$ (D) $\frac{6}{7}$

Q6. A bag contains 20 marbles: 6 green, 10 brown, and 4 white. If one marble is removed randomly, what is the minimum number that must be removed to be certain that you have at least 2 marbles of each colour

(A) 16 (B) 18

(C) 10 (D) 15

Q7. In a squash tournament that has 75 entrants, a player is eliminated whenever he loses a match. How many matches will be played in the entire tournament?

(A) 74 (B) 18

(C) 34 (D) 36

Explanatory Answers

Q1. (D) Ayesha completed 18 – 4 + 1 = 15 mathematics exercises in 30 minutes. It means that she complete one exercise every 2 minutes. Thus, to complete 55 – 27 + 1 = 29 questions would take 29 × 2 = 58 minutes.

Q2. (A) Munir's last birthday was August 1998, when he turned 1998 – 1934 = 64.

Q3. (A) We use counting principle to solve this problem. The first digit can be chosen in any 3 ways (2, 4, 6) whereas the second, third can be chosen in any 4 ways (0, 2, 4, 6). Therefore the total number of 4-digit numbers all of whose digits are even is 3 × 4 × 4 = 48.

Q4. (C) Suppose, there were no month in which at least 3 players had a birthday, then each month would have the birthdays of at most 2 players. But it is not possible. Now, if there were two birthdays in January, 2 in February, ... and 2 in December, that would be 24 players only. Now, it is sure that with more than 24 players, (28 given) at least one month will have 3 or more birthdays. This is the sure event. The probability of the sure event is 1. Hence *C* is the correct choice.

Q5. (D) It is an equally likely event, any one of the 7 marbles will be the one that is not removed, so the probability that red one is left is $\frac{1}{7}$ and the probability that it is removed is $\left(1-\frac{1}{7}\right)=\frac{6}{7}$.

Q6. (B) You might have a chance to remove 10 brown ones in a row, followed by all 6 green ones. At that point you have removed 16 marbles, and you still wouldn't have even 1 white one. Now, the next two marbles must both be white. Hence the answer is 16 + 2 = 18.

Q7. (A) Since the winner never loses and the other 74 players each lose once. Since each match has exactly one loser, there must be 74 matches.

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Chapter 9

INTERPRETATION OF DATA

The Mathematical Reasoning section also include few questions about tables, charts and graphs. In these type of questions, you should know how to:

1. Read and understand information that is given.
2. Calculate, analyze and apply the information given.
3. Manipulating and predicting some future trends.

Two types of questions are asked based on the same set of data.

- The first question is quite simple and easy. It requires only that you read the facts or information in the table or graph.
- The second question is usually somewhat difficult, in which you are asked questions about Interpret, manipulate or predict data.

Let's start by looking at a pictogram.

Pictograms:

A pictograms uses simple symbols or pictures to show data. Pictograms gives you a quick impression of the information.

It is quite easy to compare data in a pictograms. In pictogram, you just compare how many pictures each item has.

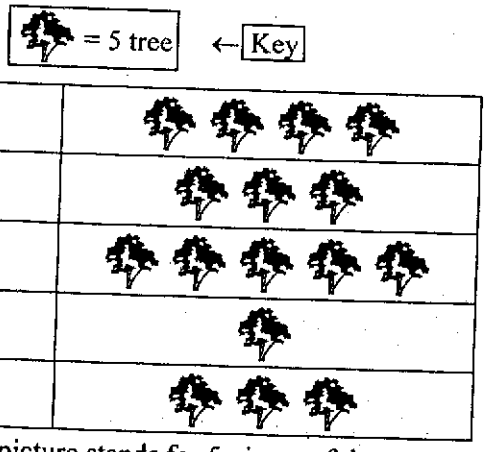
Remember

When you look at a pictogram.

(i) Read the title carefully, it tells you what the pictogram is about.

(ii) Read and understand the key, the key shows you what each little picture stands for.

In the following pictogram

Title → Trees in National Park 

Each picture stands for 5 pieces of data.

Example 1:

What is the total number of trees in National Park.

Solution:

Just read the number of trees and multiply them by 5 *i.e.*, $16 \times 5 = 80$

Example 2:

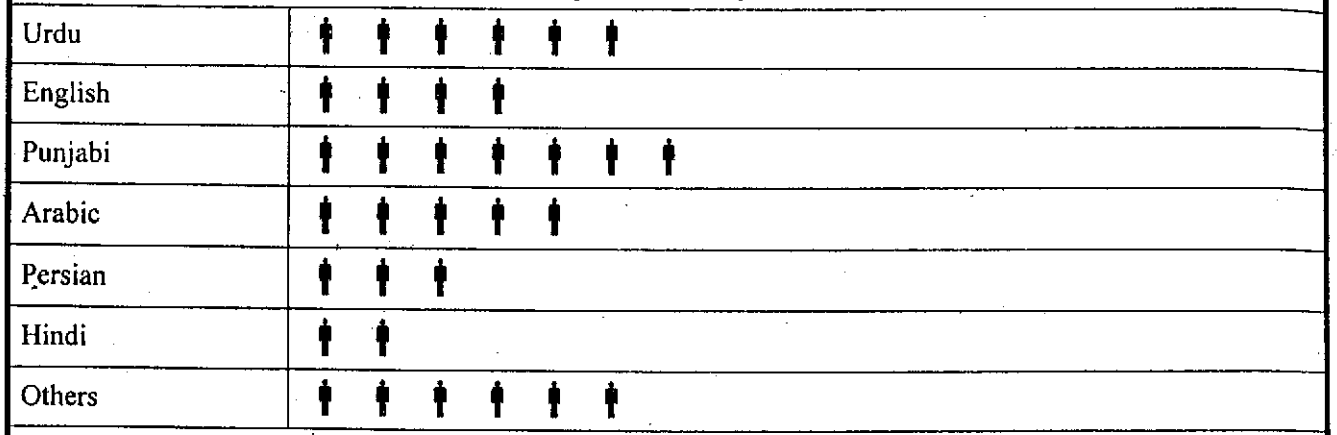
What percent of oak trees with respect to other trees in the park?

Solution:

There are, $4 \times 5 = 20$, trees of oak in the park and total number of trees is $16 \times 5 = 80$

$\therefore$ The %age of each oak trees in the park $= \frac{20}{80} \times 100 = 25\%$

Number of Students Enrolled in M.A. (Languages)
In Punjab University in 2005



Examples 3–5 refer to the above graph. Each 🧍 represents 30 students.

Example 3:

What is the total number of students enrolled language classes in Punjab University?

Solution:

Just read the pictogram and multiply the figure by 30,

$$33 \times 30 = 990$$

Example 4:

What is the average (Arithmetic Mean) number of students studying each language, if "other" category includes three languages.

Solution:

Total number of students $= 33 \times 30 = 990$

No. of languages $= 6 + 3 = 9$

Average $= 990 \div 9$

$= 110$

Example 5:

If the number of students studying English next year is the same as the number taking Arabic this year, by what percent will the number of students taking English increase?

Solution:

The number of students taking English next year = number of students taking Arabic this year = 150

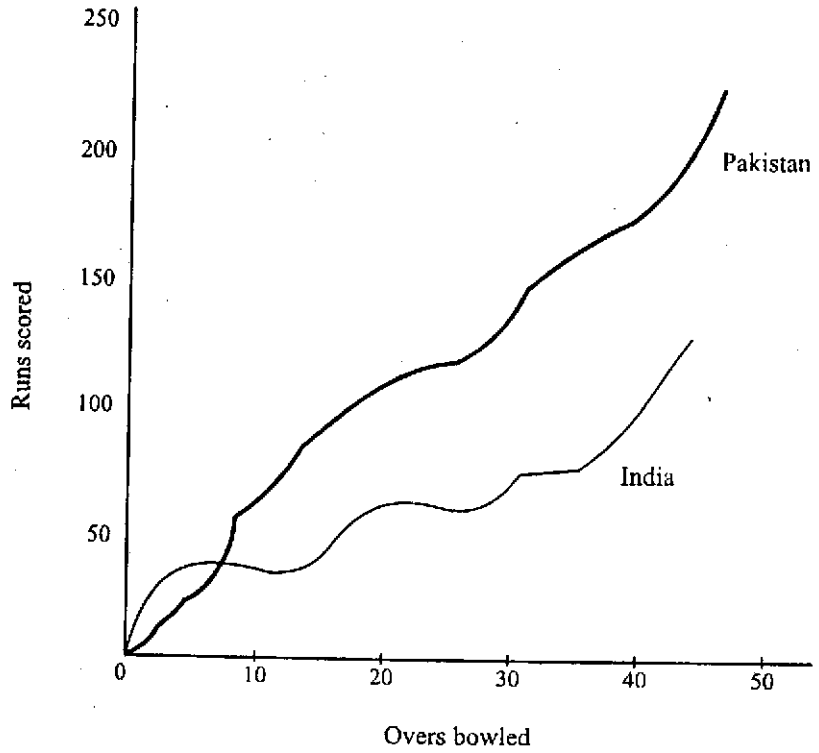
Increment $= 150 - 120 = 30$

%age increase $= \frac{30}{120} \times 100 = 25\%$

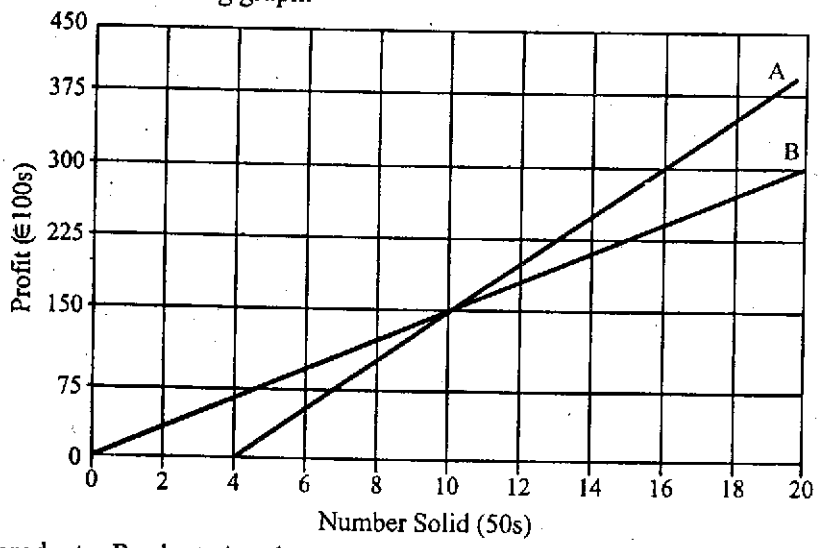
Line Graphs:

A line graph specifies how one or more quantities change over time. These types of graphs appear on the television screen during one-day cricket matches. The graph shows the accumulation of runs by each team as the match progresses. In this graph, Pakistan have already finished batting. India is still batting.

From graph, many questions could be asked. Here are some



Example 6:
How many scores did Pakistan score in his innings?
Solution:
The solid line in the graph represents Pakistan inning. You can see that at the end of 50 overs the grey line is at 250.
Hence, Answer is 250
Example 7:
How many runs have India scored so far?
Solution:
India have scored 125 in 40 overs.
Example 8:
How many runs had Pakistan scored at the same stage in their innings?
Solution:
175
Examples 9 – 11 refer to the following graph.



A firm produces 2 products. Product *A* makes a profit only when more than 200 items are sold; after that a

steady profit is made. Product B makes a steady profit of € 10 on each item. Use the above graph to solve the following examples.

Example 9:

What is the profit for selling 1000 items of A?

Solution:

Since the product A starts its profit when more than 200 items have been sold, thus its profit is

$$300 \times 100 = 30{,}000 \text{ €}$$

Example 10:

What does the profitability of A reach that of B?

Solution:

When 500 items of A are sold.

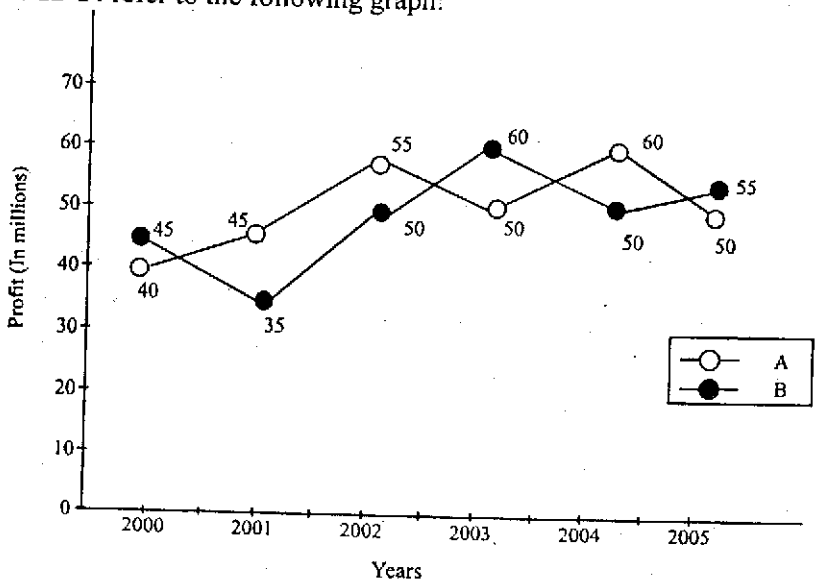
Example 11:

What is the profit from selling 800 items of B?

Solution:

€ 230,00

Directions: **Examples 12-14** refer to the following graph:



Example 11:

Firm A spent 3,50,000 in the year 2002. What is the income of Firm A in that year?

A. 15,42,500 **B.** 20,00,000

C. 15,25,000 **D.** 90,00,000

Solution: D

Profit of firm A in 2002 is 60,00,000

Income = Profit + Expenditure

= 3,50,000 + 55,00,000

= 900000

Example 12:

If the expenditure of both the firms A and B in the year 2004 was equivalent, then what was the ratio between the income of firm A to firm B?

A. 15 : 6 **B.** 6 : 15

C. 6 : 5 **D.** 16 : 5

Solution: C

Let X be the expenditure of the firm A, since expenditure of the firms A and B are equivalent, then set the ratio

Firm A		Firm B
$X + 60{,}00{,}000$	:	$X + 50{,}00{,}000$

$$\Rightarrow x + 60{,}00{,}000 : x + 50{,}00{,}000 \Rightarrow \text{ratio is ; } \boxed{6 : 5}$$

Example 13:

In which of the following years was the maximum percentage of growth/decline with respect to the previous

years in case of company *B*?

A. 2000 – 2001 B. 2001 – 2002

C. 2002 – 2003 D. 2003 – 2004

Solution: Profit in 2000 = 450000

Profit in 2001 = 350000

Decline = 450000 – 350000 = 100000

$= \frac{100000}{350000} \times 100 = 29\%$ (approx.)

Bar Charts:

A bar chart uses bars, side by side, to display data. The bars can go up or across the page. The length of a bar stands for the size of the data it shows. This makes the data easy to compare. Just compare the lengths of the bar.

Reading Bar Charts:

To read information from bar charts, here are some points to remember:

■ **Title**

Make sure that you know what the bar chart is about.

■ **Axes**

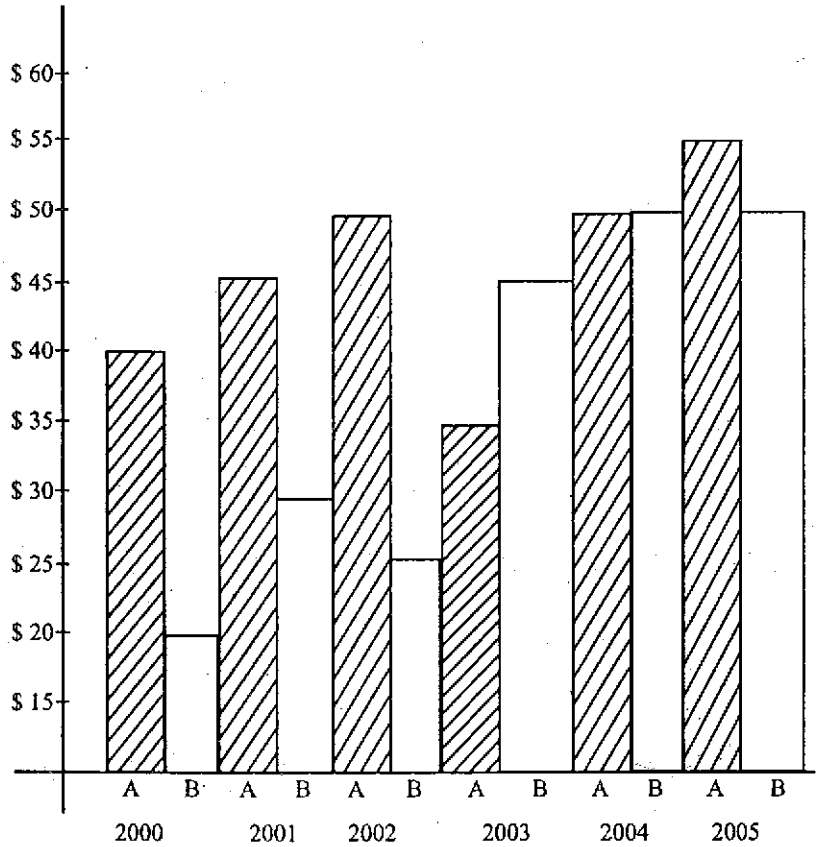
Carefully, check the labels on the axes.

■ **Scale**

Carefully, look at the scale on the number line.

Directions: **Examples 14 – 18** refer to the following graph.

Price per share of Stocks *A* and *B* on June 5 of 6 years



Example 14:

What is the difference, in dollars, of a share of stock *A*, between the highest and lowest value?

A. $10 B. $15

C. $20 D. $25

Solution:

According to the bar graph, the highest value of the share of stock *A* was \$55 in 2005 and the lowest value the share of stock A was \$35 in 2003. Thus

Difference = 55 – 35 = \$20. (C)

Example 15:

In which year, there was the greatest difference between the values of the share of the stock *A* and a share of stock *B*?

A. 2003 B. 2004

C. 2002 D. 2005

Solution:

According to the bar chart, clearly in 2002. There is greatest difference between the share of stock *A* and *B*. (C)

Example 16:

In which year, the ratio of the value of a share of stock *A* to the value of a share of stock *B* the greatest?

A. 2 : 1 B. 3 : 1

C. 5 : 1 D. 1 : 1.5

Solution: According to the bar chart, period from 2003 to 2005, the values of the shares of two stocks are very close, so we neglect these years. Now, there is a greatest difference between the shares of stock *A* and *B* in 2002, which is 50 : 25 ⇒ 2 : 1.

Answer is option A.

Example 17:

n which year was the percent increase in the value of a share of stock B the greatest?

A. 2001 B. 2002

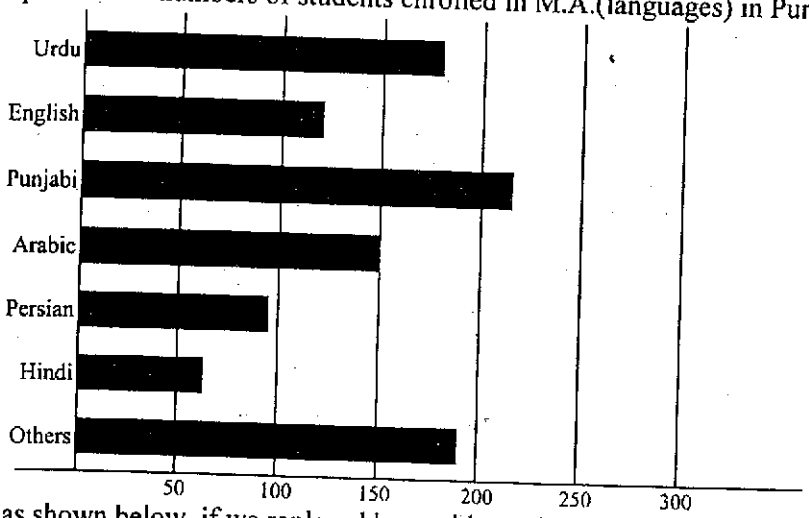
C. 2003 D. 2004

olution: According to the bar chart, the gradient of the line emerging in case of stock *B* in 2002 is the leepest.

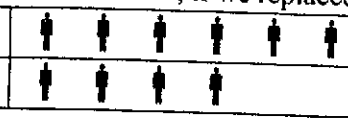
lote:

n a bar graph, the taller the bar, the greatest is the value of the quantity.

he following bar graph shows a numbers of students enrolled in M.A.(languages) in Punjab University.



is is a same graph as shown below, if we replaced bars with symbols.

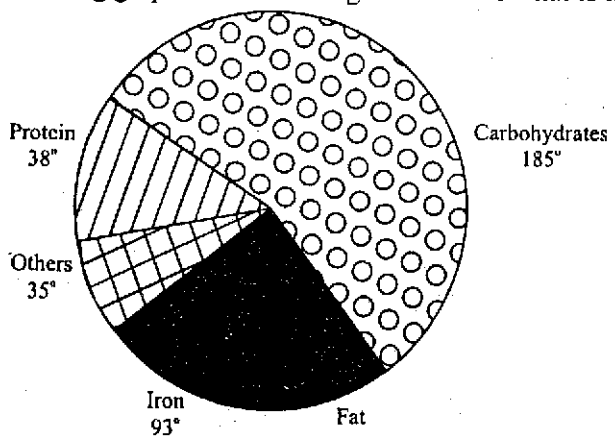


Punjabi	
Arabic	
Persian	
Hindi	
Others	

Pie Chart:

A pie chart is a circular diagram used to display data. It shows how the data are divided into group, so it looks like a pie cut into slices.

Example 17-18 refer to the following graph. The following chart shows what is in cereal.



Example 17:

Calculate the angle for fat on the diagram.

A. 23° B. 32°
C. 10° D. 20°

Solution: $360° - 185° - 37° - 35° - 93° = 10°$

Example 18:

An average 'serving' of this cereal weights 45g. How much protein is in this?

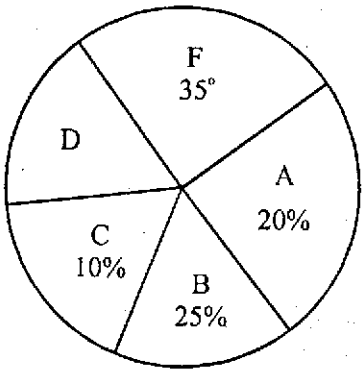
A. 1.26 g B. 2.35 g
C. 7 g D. 4.625 g

Solution: $\frac{37}{360}$ of 45 g $= 45 \times \frac{37}{360} = 4.625$ g

Multiple Choice Questions (MCQs)

Direction: Question 1-2 refer to the following graph.

Grades achieved on the Final Exam in Physics.



Q1. If 250 students took the exam, how many earned grades of D?

(A) 35 (B) 25
(C) 10 (D) 29

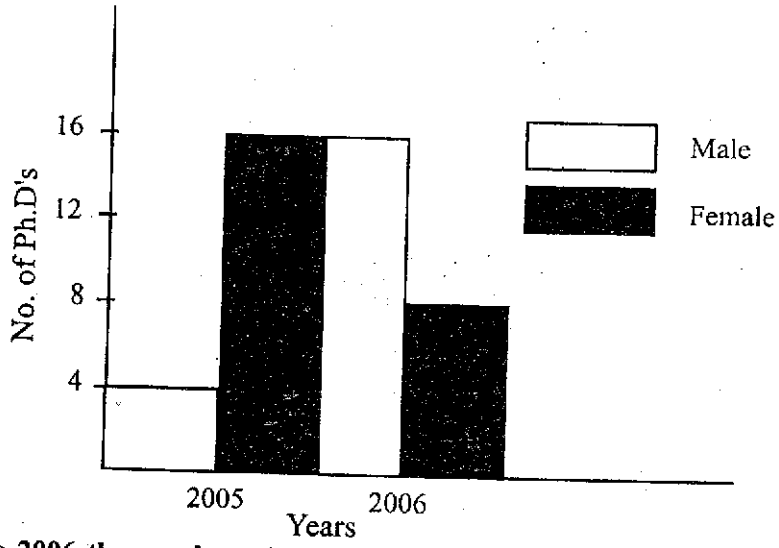
Q2. If 500 students took the exam, how many earned the grades of C?

(A) 50 (B) 20
(C) 35 (D) 10

Q3. What percent of the students who failed the exam would have had to pass it, in order for percent of students passing the exam to be at least 77% out of 500?

(A) 15% (B) 21%
(C) 23% (D) 27%

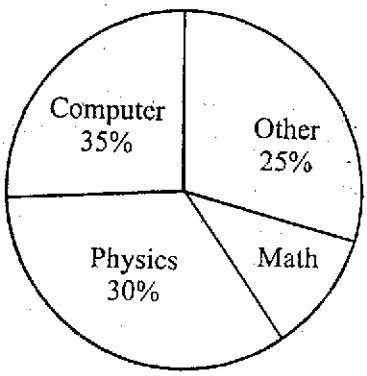
Direction: The following bar chart shows the number of male and female who earned Ph.D's in physics at Punjab University in 2005 and 2006:



Q4. From 2005 to 2006 the number of male earning Ph.D.'s increased by s%, and the number of female earning Ph.D's decreased by t%. What is the value of $s-t$?

(A) 225 (B) 200
(C) 275 (D) 250

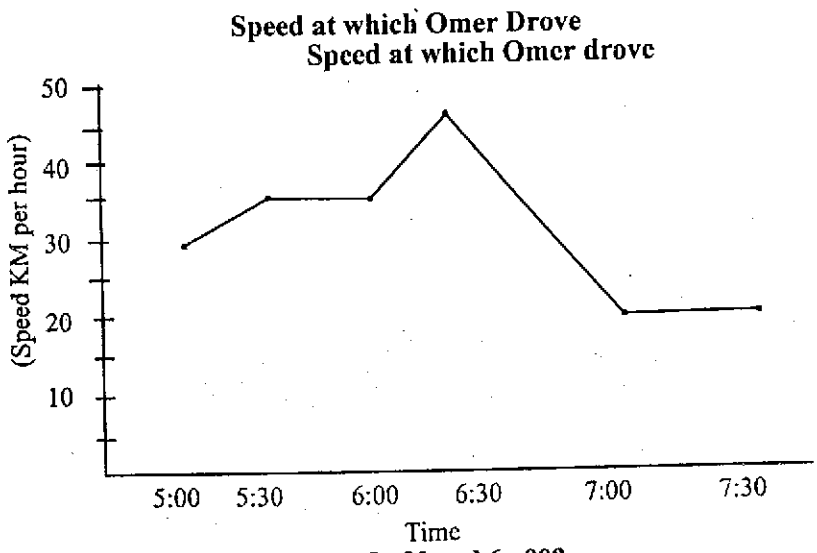
Q5. Refer to the following pie chart



every student at Crescent High School is taking exactly one science subject. This distribution has been illustration in above circular diagram. If the "other" category in order of number of students taking each subject, consists of biology, geology and astronomy. Then which of the following statement is true?

(A) Number of students taking math is equal to the number of students taking astronomy.
(B) Number of students taking math is less than the number of students taking astronomy.
(C) Number of students taking math is greater than the number of students taking astronomy.
(D) Cannot find from the given information.

Direction: **Questions 6 – 8** refer to the following graph.



Q6. **How far, in KM, did Omer drive between 5 : 30 and 6 : 00?**

(A) 20 (B) $17\frac{1}{2}$

(C) 35 (D) $12\frac{1}{2}$

Q7. **What was Omer's average speed, in KM per hour, between 5 : 30 to 6 : 30?**

(A) 25.25 (B) 36.5

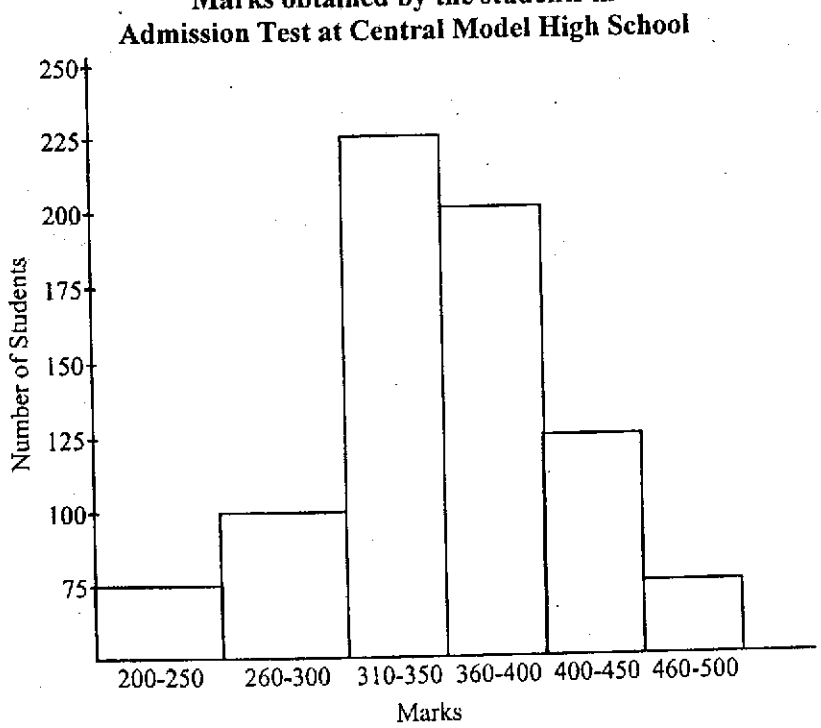
(C) 42.5 (D) 35

Q8. **For what percent of time was Omer driving at 35 km per hour or faster?**

(A) 35 (B) 25

(C) 27 (D) 50

Question 9-11 refer to the following graph.



Q9. **How many students took the admission test?**

(A) 400 (B) 375

(C) 600 (D) 800

Q10. What percent of the students had scored of less than 400?

(A) 50 (B) 75

(C) 25 (D) 65

Q11. How many candidates had scored between 250 to 350?

(A) 325 (B) 225

(C) 125

(D) cannot be exactly determined from the given information

Explanatory Answers

Q1. (B) In the given pie diagram

$$A\% + B\% + C\% + D\% + F\% = 100\%$$

$$20\% + 25\% + 10\% + D\% + 35\% = 100\%$$

$\Rightarrow$ $90\% + D\% = 100\%$

$\Rightarrow$ $D\% = 10\%$

Now, 10% of 250 is $= 250 \times \frac{10}{100} = 25.$

Q2. (A) 10% of 500 $= 500 \times \frac{10}{100} = 50.$

Q3. No. of passed students $= 500 \times \frac{77}{100} = 385.$

Thus for the passing rate to have been at least 77%, no more than 115 students, which is 23% of 500.

Q4. (D) No. of male students in 2005 = 4

No. of male students in 2006 = 16

Increase $= 16 - 4 = 12$

% Increase $= \frac{12}{4} \times 100\% = 300\%$

$\Rightarrow$ $s = 300$

During this period, the number of female students fell from 16 to 8, a decrease of 8.

$\therefore$ %age decrease $= \frac{8}{16} \times 100\% = 50\%$

$\Rightarrow$ $t = 50$

Hence $s - t = 300 - 50$

$= 250$

Q5. (B) Let m = number of students taking math

Then:

$25\% + 35\% + 30\% + m\% = 100\%$

$\Rightarrow$ $90\% + m\% = 100\%$

$\Rightarrow$ $m\% = 10\% \Rightarrow m = 10$

which is less than the number of students taking astronomy.

Q6. (B) Since Omer is driving at constant rate 35 km/hour during. 5 : 30 to 6 : 00. Thus in half hour he drove

$$\frac{1}{2} \times 35 = 17\frac{1}{2} \text{ km}$$

Q7. (D) From the graph, we see that, clearly from 5 : 30 to 6 : 00 Omer's average speed was clearly 35 km/hour, and from 6:00 to 6:30 Omer's speed steadily increased from 35 to 40 km/hour, so during 6:00 to 6:30 his average speed was $\frac{35+40}{2} = \frac{75}{2} = 37.5$ km/hour.

Thus, in the given hour, his average speed was

$$\frac{37.5+35}{2} = 36.25 \text{ km/h}$$

Q8. (D) From the graph, we see that from 5 : 30 to 6 : 45 $\left(1\frac{1}{2} \text{ hour}\right)$ the car is driven 35 or more than 35 km/hour which is half time of the total time (5:00 to 7:30) $2\frac{1}{2}$ hours.

Hence, the answer is 50%.

Q9. (D) Just add the number of candidates by reading the graph carefully

$$75 + 100 + 225 + 200 + 125 + 75 = 800.$$

Q10.(B) Number of students who scored less than 400

$$= 75 + 100 + 225 + 200 = 600.$$

Thus, 600 students had scored below or equal to 400 out of 800 candidates. Hence

$$\frac{600}{800} \times 100 = 75\%$$

Q11.(D) We cannot find exact number of students who scored between 250 to 350, because in every interval (260 – 300, 200, 250, etc.) every lower term is included in the previous term.

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