1) What is the third-quadrant value, in radians, that satisfies the equation $\sin^2 x - \sin x = \cos^2 x$?

a) $\frac{7\pi}{6}$

b) $\frac{11\pi}{6}$

c) $\frac{2\pi}{3}$

d) $\frac{4\pi}{3}$

e) $\frac{3\pi}{4}$

2) Which is a focus for the equation $\frac{x^2}{25} + \frac{y^2}{9} = 1$?

a) (5,0)
b) (4,0)
c) (3,0)
d) (0,4)
e) (0,5)

3) Given $f(x)=\frac{x+2}{x-3}$, which is the value of $f^{-1}(x)$?

a) $\frac{x-3}{x+2}$

b) $-\frac{x-2}{x+3}$

c) $\frac{3x+2}{x-1}$

d) $\frac{3x-2}{x+1}$

e) $\frac{2x-1}{3x+1}$

4) Which is the inverse function, $f^{-1}(x)$, if $f(x)=\log_5 2x$?

a) $2x^5$

b) $32x^5$

c) $25x$

d) $\frac{5^x}{2}$

e) $\left(\frac{5}{2}\right)^x$

5) In $\ln[\ln(x)] = 0$, which is the value for x?

a) 0
b) 1
c) e
d) e^x
e) x^e

6) Which is the exact value of $\cos\left(-\frac{17\pi}{6}\right)$ $\cos(-\frac{17\pi}{6})$?

a) $-\frac{1}{2}$
b) $-\frac{\sqrt{2}}{2}$
c) $-\frac{\sqrt{3}}{2}$
d) $\frac{\sqrt{3}}{2}$
e) $\frac{\sqrt{2}}{2}$

7) A regular pentagon is inscribed in a circle with radius 5.35 inches. Which is the length of one side of the pentagon?

a) 5.35 in
b) 4.33 in
c) 3.75 in
d) 6.29 in
e) 8.03 in

8) Given $\cos\theta = -\frac{\sqrt{3}}{2}$, which are all the values of θ, where $-\pi \le \theta \le \pi$?

a) $\frac{2\pi}{3}, -\frac{\pi}{3}$

b) $\frac{\pi}{6}, -\frac{\pi}{6}$

c) $\frac{2\pi}{3}, -\frac{2\pi}{3}$

d), $\frac{5\pi}{6}, -\frac{\pi}{6}$

e) $\frac{5\pi}{6}, -\frac{5\pi}{6}$

9) Which is the exact value of $\tan 75°$?

a) $1-\sqrt{3}$

b) $1+2\sqrt{3}$

c) $2-\sqrt{3}$

d) $2+\sqrt{3}$

e) $\sqrt{3}-1$

10) Which are the coordinates of the foci for $16x^2 + 9y^2 = 144$?

a) $(\sqrt{7}, 0)$, $(-\sqrt{7}, 0)$

b) $(0, \sqrt{7})$, $(0, -\sqrt{7})$

c) $(4,3)$, $(-4,-3)$

d) $(0,4)$, $(0,-4)$

e) $(3,0)$, $(-3,0)$

Answers

1) The answer is A. In $\sin^2 x - \sin x = \cos^2 x$, substituting from $\sin^2 x + \cos^2 x = 1$, or $\cos^2 x = 1 - \sin^2 x$, yields $\sin^2 x - \sin x = 1 - \sin^2 x$, or $2\sin^2 x - \sin x - 1 = 0$. Factor the quadratic to obtain $(2\sin x + 1)(\sin x - 1) = 0$, which means $\sin x = -\frac{1}{2}$ or $\sin x = 1$ For $\sin x = -\frac{1}{2}$, $x = \frac{7\pi}{6}, \frac{11\pi}{6}$, and for $\sin x = 1$, $x = \frac{\pi}{2}$. Of the three results, the only one that is a third-quadrant angle is $\frac{7\pi}{6}$.

2) The answer is B. For an ellipse, a = the length of the major axis, b = length of the minor axis, c = the focal length, and $c^2 = a^2 - b^2$. In the equation, $\frac{x^2}{25}$ has the larger denominator, indicating that the x-axis is the major axis, with major axis length equal to the square root of the denominator, or 5. The minor axis length is the square root of the smaller denominator, or 3. Since the numerators are in the form $(x-0)^2$ and $(y-0)^2$, the center of the ellipse is $(0,0)$. For $c^2 = a^2 - b^2$, c^2 = 25 – 9, or 16, with $c = \pm 4$. Thus the foci are 4 units to the right and left of the center. Therefore the two foci are $(-4,0)$ and $(4,0)$

3) The answer is C. Let $y = f(x)$, or $y = \frac{x+2}{x-3}$. Swap y and x: $x = \frac{y+2}{y-3}$, and then solve for y: $x(y-3) = y+2$, or $xy - 3x = y + 2$. Gathering y terms: $xy - y = 3x + 2$, then factor: $y(x-1) = 3x+2$, or y = $\frac{3x+2}{x-1}$. Then replace y with $f^{-1}(x)$ to obtain the inverse function $f^{-1}(x) = \frac{3x+2}{x-1}$.

4) The answer is D. First replace *y* for $f(x)$ in $f(x) = \log_5 2x$ to get $y = \log_5 2x$, then use the fact that $p = \log_b n$ is equivalent to $b^p = n$ to obtain $5^y = 2x$. Now swap *y* and *x* to obtain $5^x = 2y$, then solve for *y* to arrive at $\frac{5^x}{2} = y$. Replace this *y*-value with $f^{-1}(x)$ to obtain the inverse function $\frac{5^x}{2} = f^{-1}(x)$.

5) The answer is C. The exponential equivalent of $\ln[\ln(x)] = 0$, using the fact that $\ln a = b$ is the same as $e^b = a$, becomes $e^0 = \ln(x)$, or $1 = \ln(x)$. Then use the equivalence again to obtain $e^1 = x$, or $x = e$.

6) The answer is C. Since the angle is negative, first add multiples of 2π, or $\frac{12\pi}{6}$, until the angle is between 0 and 2π. Doing that twice obtains $cos\left(\frac{7\pi}{6}\right)$. Evaluate which quadrant $\frac{7\pi}{6}$ is in. Since $\frac{7\pi}{6}$ is between π and $\frac{3\pi}{2}$, it lies in the third quadrant. To obtain the reference angle for a third-quadrant angle, subtract π to obtain $\frac{\pi}{6}$ and then evaluate the sign of third-quadrant angles for the cosine function. Cosine is negative in the third quadrant. Putting these facts together reveals that $cos\left(\frac{7\pi}{6}\right) = -cos\left(\frac{\pi}{6}\right)$, or $-\frac{\sqrt{3}}{2}$. Therefore $cos\left(-\frac{17\pi}{6}\right) = -\frac{\sqrt{3}}{2}$.

7) The answer is D. A pentagon may be divided up into 5 congruent isosceles triangles. The central angle of each triangle is (360°/5) = 72°. This angle may be bisected, creating a pair of right triangles, with angles measuring 90°, 58°, and 32°. Since the hypotenuse of these triangles is equal to the radius of the circle, the length of the pentagon's sides may be found as L = 2(5.35×sin(36)) = 6.29 in.