UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
Cambridge International Level 3 Pre-U Certificate
Principal Subject

FURTHER MATHEMATICS **9795/02**

Paper 2 Further Applications of Mathematics **May/June 2011**

3 hours

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF20)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value for the acceleration due to gravity is needed, use $10\,\mathrm{m\,s^{-2}}$.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 120.

This document consists of **6** printed pages and **2** blank pages.

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UNIVERSITY *of* CAMBRIDGE
International Examinations

[Turn over

Section A: Probability (60 marks)

1 The independent random variables X and Y have distributions $\mathrm{N}(30, 9)$ and $\mathrm{N}(20, 4)$ respectively.

(i) Give the distribution of

$$(X_1 + X_2 + X_3) - (Y_1 + Y_2 + Y_3 + Y_4),$$

where X_i, $i = 1, 2, 3$, and Y_j, $j = 1, 2, 3, 4$, are independent observations of X and Y respectively. [2]

The time for female cadets to complete an assault course is X minutes and the time for male cadets to complete the same assault course is Y minutes.

(ii) Find the probability that the total time for three randomly chosen female cadets to complete the assault course is greater than the total time for four randomly chosen male cadets to complete the assault course. [3]

2 The discrete random variable X has a Poisson distribution with mean 12.25.

(i) Calculate $\mathrm{P}(X \leqslant 5)$. [3]

(ii) Calculate an approximate value for $\mathrm{P}(X \leqslant 5)$ using a normal approximation to the Poisson distribution. [3]

(iii) Comment, giving a reason, on the accuracy of using a normal approximation to the Poisson distribution in this case. [2]

3 The fuel economy of two similar cars produced by manufacturers A and B was compared. A random sample of 15 cars was selected from manufacturer A and a random sample of 10 cars was selected from manufacturer B. All the selected cars were driven over the same distance and the petrol consumption in miles per gallon (mpg) was calculated for each car. The results, x_A mpg and x_B mpg for cars from manufacturers A and B respectively, are summarised below, where $\bar{x}$ denotes the sample mean and n the sample size.

$$\Sigma x_A = 460.5 \qquad \Sigma(x_A - \bar{x}_A)^2 = 156.88 \qquad n_A = 15$$
$$\Sigma x_B = 334 \qquad \Sigma(x_B - \bar{x}_B)^2 = 123.97 \qquad n_B = 10$$

(i) (a) Assuming that the populations are normally distributed with a common variance, show that the pooled estimate of this common variance is 12.21, correct to 4 significant figures. [2]

(b) Construct a 95% confidence interval for $\mu_B - \mu_A$, the difference in the population means for manufacturers A and B. [6]

(ii) Comment on a claim that the fuel economy for manufacturer B's cars is better than that for manufacturer A's cars. [2]

4 **(i)** A random variable X has probability density function given by

$$f(x) = \begin{cases} \dfrac{1}{\theta}e^{-\frac{x}{\theta}} & x \geqslant 0, \\ 0 & x < 0, \end{cases}$$

where θ is a positive constant. Find $E(X^2)$. [5]

(ii) A random sample $X_1, X_2, \ldots, X_n$ is taken from a population with the distribution in part **(i)**. The estimator T is defined by $T = k\sum_{i=1}^{n} X_i^2$, where k is a constant. Find the value of k such that T is an unbiased estimator of θ^2. [5]

5 **(i)** The discrete random variable X has distribution $\text{Geo}(p)$. Show that the moment generating function of X is given by $M_X(t) = \dfrac{pe^t}{1 - qe^t}$, where $q = 1 - p$. [3]

(ii) Use the moment generating function to find

(a) $E(X)$,

(b) $\text{Var}(X)$. [7]

(iii) An unbiased six-sided die is thrown repeatedly until a five is obtained, and Y denotes the number of throws up to and including the throw on which the five is obtained. Find $P(|Y - \mu| < \sigma)$, where μ and σ are the mean and standard deviation, respectively, of the distribution of Y. [3]

6 **(i)** The continuous random variable X has a uniform distribution over the interval $0 < x < \frac{1}{2}\pi$. Show that the probability density function of Y, where $Y = \sin X$, is given by

$$f(y) = \begin{cases} \dfrac{2}{\pi\sqrt{1 - y^2}} & 0 < y < 1, \\ 0 & \text{otherwise.} \end{cases}$$ [6]

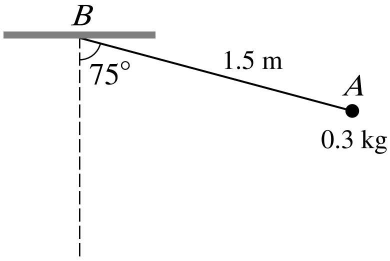
(ii) Deduce, using the probability density function, the exact values of

(a) the median value of Y, [5]

(b) $E(Y)$. [3]

Section B: Mechanics (60 marks)

7

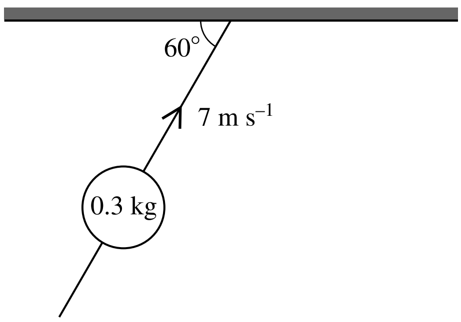


A particle of mass 0.3 kg is attached to one end A of a light inextensible string of length 1.5 m. The other end B of the string is attached to a ceiling, so that the particle may swing in a vertical plane. The particle is released from rest when the string is taut and makes an angle of 75° with the vertical (see diagram). Air resistance may be regarded as being negligible.

(i) Show that, at an instant when the string makes an angle of 40° with the vertical, the speed of the particle is $3.90\,\text{m}\,\text{s}^{-1}$, correct to 3 significant figures. [3]

(ii) By considering Newton's second law, along and perpendicular to the string, find the radial and transverse components of acceleration, at this same instant, and hence the magnitude of the acceleration of the particle. [3]

8



A smooth sphere of mass 0.3 kg is moving in a straight line on a horizontal surface. It collides with a vertical wall when the velocity of the sphere is $7\,\text{m}\,\text{s}^{-1}$ at 60° to the wall (see diagram). The coefficient of restitution between the sphere and the wall is 0.4.

(i) (a) Find the component of the velocity of the sphere perpendicular to the wall immediately after the collision. [2]

(b) Find the magnitude of the impulse exerted by the wall on the sphere. [2]

(ii) Determine the magnitude and direction of the velocity of the sphere immediately after the collision, giving the direction as an acute angle to the wall. [4]

9 At noon a vessel, A, leaves a port, O, and travels at $10\,\text{km}\,\text{h}^{-1}$ on a bearing of 042°. Also at noon a second vessel, B, leaves another port, P, 13 km due north of O, and travels at $15\,\text{km}\,\text{h}^{-1}$ on a bearing of 090°. Take O as the origin and $\mathbf{i}$ and $\mathbf{j}$ as unit vectors east and north respectively.

(i) Express the velocity vector of A relative to B in the form $a\mathbf{i} + b\mathbf{j}$, where a and b are constants to be determined. [2]

(ii) Express the position vector of A relative to B, at time t hours after the vessels have left port, in terms of t, $\mathbf{i}$ and $\mathbf{j}$. [2]

(iii) Explain why the scalar product of the vectors in parts **(i)** and **(ii)** is zero when the two vessels are closest together. [1]

(iv) Find the time at which the two vessels are closest together. [4]

10 A and B are two points 6 m apart on a smooth horizontal surface. A particle, P, of mass 0.5 kg is attached to A by a light elastic string of natural length 2 m and modulus of elasticity 20 N, and to B by a light elastic string of natural length 1 m and modulus of elasticity 10 N, such that P is between A and B.

(i) Find the length AP when P is in equilibrium. [4]

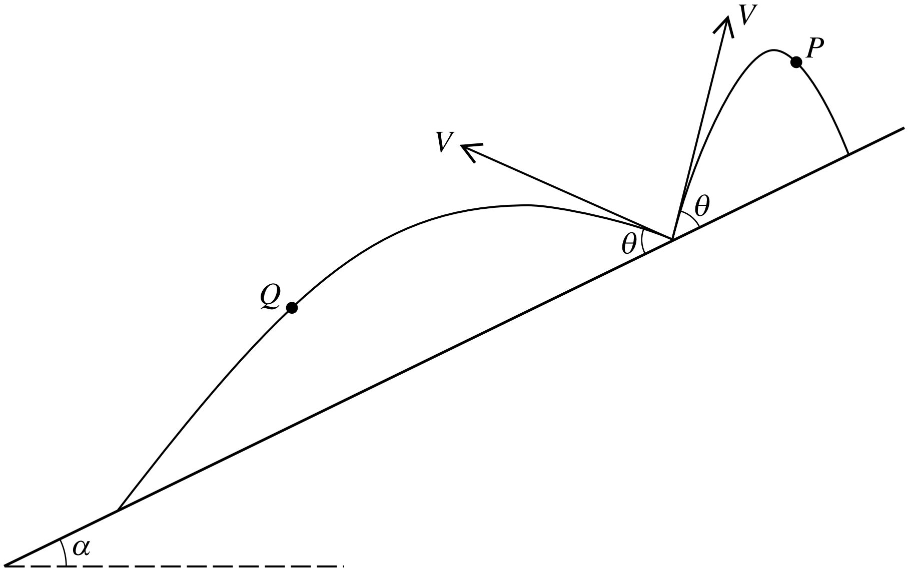
P is held at the point C, where C is between A and B and $AC = 4.5$ m. P is then released from rest. At time t seconds after being released, the displacement of P from the equilibrium position is y metres.

(ii) Show that

$$\frac{d^2y}{dt^2} = -40y.$$ [3]

(iii) Find the time taken for P to reach the mid-point of AB for the first time. [4]

11



Two particles, P and Q, are projected simultaneously from the same point on a plane inclined at α to the horizontal. P is projected up the plane and Q down the plane. Each particle is projected with speed V at an angle θ to the plane. Both particles move in a vertical plane containing a line of greatest slope of the inclined plane and you are given that $\alpha + \theta < \frac{1}{2}\pi$ (see diagram).

(i) Show that the range of P, up the plane, is given by

$$\frac{2V^2 \sin\theta}{g\cos^2\alpha}(\cos\theta\cos\alpha - \sin\theta\sin\alpha).$$ [6]

(ii) Write down a similar expression for the range of Q, down the plane. [1]

(iii) Given that the range up the plane is a quarter of the range down the plane and that $\alpha = \tan^{-1}\left(\frac{1}{2}\right)$, find θ. [5]

12 A train of mass 250 tonnes is ascending an incline of $\sin^{-1}\left(\frac{1}{500}\right)$ and working at 400 kW against resistance to motion which may be regarded as a constant force of 20 000 N.

(i) Find the constant speed, V, with which the train can ascend the incline working at this power. [5]

(ii) The train begins to ascend the incline at $6\ \mathrm{m\,s^{-1}}$ at the same power and against the same resistance. Find the distance covered in reaching a speed of $\frac{3}{4}V$. [9]

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