

UNIT A5

Recommended Prior Knowledge Units A1 to A4

Context Algebraic manipulation skills are extended to transforming formulae and factorising quadratic expressions. Graphically, students move on from straight line graphs to the graphs of quadratic functions.

Outline Earlier work on algebraic manipulation and solving equations is extended to transforming simple formulae and to using algebra for direct and inverse variation. Having expanded quadratic expressions in A4, in this unit quadratic expressions are factorised and the graphs of quadratic functions drawn.

	Learning Outcomes	Suggested Teaching Activities	Resources
22	Transform simple formulae	Start with solving an equation such as $6x + 1 = 7$, then ask the students how they would find an expression for x when $6x + 1 = y$. Move on to asking the students for simple formulae they know, such as $C = \pi d$, or $P = 2(l + w)$ and ask them to transform them to make another variable the subject. Give practice with a range of suitable simple formulae.	http://www.ex.ac.uk/cimt/mepres/allgcse/bka2.pdf for transforming formulae
12	Express direct and inverse variation in algebraic terms and use this form of expression to find unknown quantities.	Begin work on variation by using an example such as the price of a circular cloth being proportional to its area, and then moving on to the price being proportional to the radius squared. Show the students how to form the formula connecting the variables and to use the information given to find the constant of proportionality, then to use the equation they have found. Similarly for inverse variation. Throughout this topic, use examples in context as well as purely algebraic examples.	section 15.5 onwards of http://www.ex.ac.uk/cimt/mepres/allgcse/bkc15.pdf has work on variation

23	Factorise expressions of the form $x^2 + bx + c$.	Start by practising a few examples of expanding brackets and simplifying (done in A4) such as $(x + 3)(x + 5)$, $(x - 2)(x + 1)$ etc then progress to examples such as $(x + 2)(x - 1) = x^2 + 5x + 6$, asking them to fill in the gap. Finally, give them $(x - 1)(x - 8) = x^2 + 2x - 8$ etc and ask them to fill in the gaps. Some students can usually do this without further prompting and can explain how they have worked, which forms the basis for a good discussion on the principles needed to factorise quadratics. Practice is then needed in this skill.	section 10.10 of http://www.ex.ac.uk/cimt/mepres/allgcse/bkb10.pdf is about factorising quadratic expressions http://www.algebrahelp.com/worksheets/index.htm has online worksheets including one about factorising differences of squares
20	Construct tables of values and draw graphs for functions of the form $y = ax^n$ where $n = 0, 1, 2$ and simple sums of these; interpret graphs of quadratic functions; solve equations approximately by graphical methods.	Show students how to construct a table of values for a graph such as $y = x^2 - 3$, and to draw the graph, paying attention to obtaining a smooth curve which is of the correct shape at the vertex. Show the students how to use the graph to solve equations such as $x^2 - 3 = 0$ or $x^2 - 3 = 14$. Then show them how to construct tables of values for a more complex quadratic function such as $y = 2x^2 + 3x - 2$, similarly going on to draw and use the graph. [This work can be used further in unit A6 when quadratic equations are solved by factorising.]	section 13.4 of http://www.ex.ac.uk/cimt/mepres/allgcse/bkc13.pdf has questions about plotting quadratic graphs as well as other curves [see unit A8] Computer programs such as Autograph or Omnigraph are useful for using graphs to solve equations, as well as for drawing graphs.

UNIT N5

Recommended Prior Knowledge Units N1, N2, N3

Context The basis of work on percentages and positive indices in earlier units is extended in this unit.

Outline Finding a percentage increase or decrease is revisited and extended to the harder case of doing reverse percentage calculations. Whilst the calculator is being used, the opportunity is taken to emphasise efficient use of a calculator. Work on indices is extended to include negative and zero indices, their use and interpretation.

	Learning Outcomes	Suggested Teaching Activities	Resources
13	Carry out calculations involving reverse percentages, e.g. finding the cost price given the selling price and the percentage profit.	<p>Ask the students to find the price of a radio after a 5% increase, and to tell you how they did this. For those who found 5% and added it on, discuss the result Price after increase = 105% of price before = $1.05 \times$ price before, as a calculator efficient method. This leads to Price before increase = Price after $\div 1.05$, but first give them an example of a price after such an increase and see if they can work it out for themselves.</p> <p>Similarly, do examples with price decreases, finding prices after and before increases. Apply the reverse percentage method to other contexts such as prices including or excluding tax, or selling price and cost price.</p> <p>Remind students that methods such as inverse operations can be used to check their results – they should work with the price before increase that they have found and see if it gives them the correct price after increase.</p>	<p>Sale prices and percentages in local shops or newspaper advertisements; local tax rates.</p> <p>Work on reverse percentages is at section 11.9 of http://www.ex.ac.uk/cimt/mepres/allgcse/bkb11.pdf</p>

24	Use and interpret negative and zero indices.	Start with $2^4 = 16$, $2^3 = 8$, $2^2 = 4$ and continue the pattern, to obtain $2^0 = 1$, $2^{-1} = \frac{1}{2}$ etc, generalising to $2^{-n} = 1/2^n$. Similarly for other numbers. Show the students how to use the lower buttons on their calculator with negative indices and that their calculator results are consistent with the patterns found. Then practice evaluating expressions with negative and zero indices, both with and without a calculator.	<p>http://www.mathstutor.com/MenuFrame.html has a short summary and hints about indices</p> <p>section 1.6 of http://www.ex.ac.uk/cimt/mepres/allgcse/bka1.pdf has work on indices [including fractional indices, which are in unit A7.</p> <p>http://www.algebrahelp.com/worksheets/index.htm has an online worksheet about negative indices</p>
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UNIT S5

Recommended Prior Knowledge Units S1 to S4

Context Some three dimensional work is introduced, dealing with prisms. Objects changing position are dealt with in two ways, by considering transformations and by work on loci.

Outline Mensuration is extended from two-dimensional objects to finding the volume and surface area of prisms. Informal ideas of symmetry are extended to prisms and then to formal work on transformations from a geometrical point of view [the introduction of matrices comes later in the course in units A9 and S10]. Finally, the concepts of loci are established and formal geometrical constructions used to make accurate drawings of loci.

	Learning Outcomes	Suggested Teaching Activities	Resources
27	Use and interpret vocabulary of simple solid figures: cube, cuboid, prism, cylinder.	Draw the nets of some prisms and construct the prisms. If card is available, these could be decorated (easiest to do at the net stage) and made into gift or storage boxes, perhaps with separate lids. This activity leads naturally into calculations of surface area and volume. [It could be set as a task to design a storage box taking these elements into consideration.] Discuss also the symmetry properties – having the prisms constructed also enables students to see the symmetry properties more easily.	An alternative to using card / paper is to build up a collection of prisms that may be used to demonstrate when teaching this topic. Packaging, cans, bricks etc are possible sources, or a set or different prisms carved from wood. Area and volume problems are at section 7.8 of http://www.ex.ac.uk/cimt/mepres/allgcse/bkb7.pdf symmetry properties of prisms are at section 3.6 of http://www.ex.ac.uk/cimt/mepres/allgcse/bka3.pdf
33	Solve problems involving the surface area and volume of a cuboid, cylinder, prism.		
30	Recognise symmetry properties of the prism (including cylinder)		
38	Identify and give precise descriptions of transformations connecting different figures; describe transformations.	Students have met reflection and rotation symmetry, so you can use these as an introduction to reflecting and rotating non-symmetrical objects. Use graph grids to enable descriptions and plain paper for them to create designs using reflections and rotations. If ICT facilities are available for you to use with your students, you could use the drawing facilities in specialised programs or in Word, for instance.	Loci and transformations are at http://www.ex.ac.uk/cimt/mepres/allgcse/bkc14.pdf Revision pages on transformations are at http://www.bbc.co.uk/schools/gcsebitesize/maths/shapeih/transformationshrev1.shtml . There are also test pages.
39	Describe a translation by using a vector represented by $\begin{pmatrix} x \\ y \end{pmatrix}$		

		<p>Move on to introducing enlargement and translation, – both drawing the images after transformations and describing a transformation given an object and its image. For translations, describe them using a column vector.</p> <p>Use some simple combinations of the above transformations to give students practice in both drawing and describing as a single transformation the result of a combined transformation.</p> <p>Introduce shears and stretches similarly, but pay them less attention in this unit – all the transformations will be studied further using matrices and vectors in S10.</p>	
32	<p>Use the following loci and the method of intersecting loci:</p> <p>(a) sets of points in two or three dimensions</p> <p>(i) which are at a given distance from a given point,</p> <p>(ii) which are at a given distance from a given straight line,</p> <p>(iii) which are equidistant from two given points;</p> <p>(b) sets of points in two dimensions which are equidistant from two given intersecting straight lines.</p>	<p>Introduce the idea of locus by using examples in the classroom. ‘I want to walk so that I stay 1.5m from this wall. Where can I go?’ ‘I want to stay 1 m from this chair. Where can I go?’ ‘Where can the spider be if it is 30 cm from this light bulb?’</p> <p>Progress to using pencil and paper to draw accurate scale diagrams to represent loci in two dimensions, revising geometrical constructions from unit S3 as necessary. Include examples of intersecting loci, for example, given a diagram showing the positions of villages A and B: ‘Shona lives less than 4 km from village A. She lives nearer to village B than to village A. Shade the region where Shona lives.’</p>	<p>Loci and transformations are at http://www.ex.ac.uk/cimt/mepres/allgcse/bkc14.pdf</p> <p>Revision pages on transformations are at http://www.bbc.co.uk/schools/gcsebitesize/maths/shapeih/locirev1.shtml. There are also test pages.</p>