Centre Number						Candidate Number				
Surname										
Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
TOTAL	

General Certificate of Education
Advanced Level Examination
June 2011

Use of Mathematics (Pilot) USE3

Mathematical Comprehension

Thursday 26 May 2011 9.00 am to 10.30 am

For this paper you must have:
- a clean copy of the Data Sheet (enclosed)
- a graphics calculator
- a ruler.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- You may **not** refer to the copy of the Data Sheet that was available prior to this examination. A clean copy is enclosed for your use.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 45.

Advice
- You are advised to spend 1 hour on Section A and 30 minutes on Section B.

Section A

Answer **all** questions in the spaces provided.

Use **Skydiving – no time for differential equations!** *on the Data Sheet.*

1 (a) The article considers the case of a skydiver with zero initial velocity and with $\frac{\mathrm{d}v}{\mathrm{d}t} = g$.

Show clearly that $v = gt$ in this case. *(2 marks)*

(b) For this skydiver, find how long it would take to reach a downward vertical speed of $40\,\mathrm{m\,s^{-1}}$. *(1 mark)*

QUESTION PART REFERENCE

2 Use the data given in **Figure 3**, on the data sheet, to confirm that the function $v = gt$ does not provide a suitable model to describe how v varies with t. *(2 marks)*

QUESTION PART REFERENCE

Do not write outside the box

3 Show, for the case of a skydiver with initial velocity v_0, how

$$t = -\frac{1}{k}\ln(g - kv) + c \quad \text{leads to} \quad v = \frac{g}{k} - \left(\frac{g}{k} - v_0\right)\mathrm{e}^{-kt}$$

(5 marks)

QUESTION PART REFERENCE

Do not write outside the box

4 Explain how the expression $v = \frac{g}{k} - \left(\frac{g}{k} - v_0\right) \mathrm{e}^{-kt}$ suggests that the terminal velocity of a skydiver is $v_\infty = \frac{g}{k}$. *(2 marks)*

QUESTION PART REFERENCE

5 The velocity of the skydiver in the article is given by $v = 53.6(1 - e^{-0.183t})$. Find how long it takes this skydiver to reach a speed of $40\,\mathrm{m\,s^{-1}}$, giving your answer to three significant figures. *(3 marks)*

QUESTION PART REFERENCE

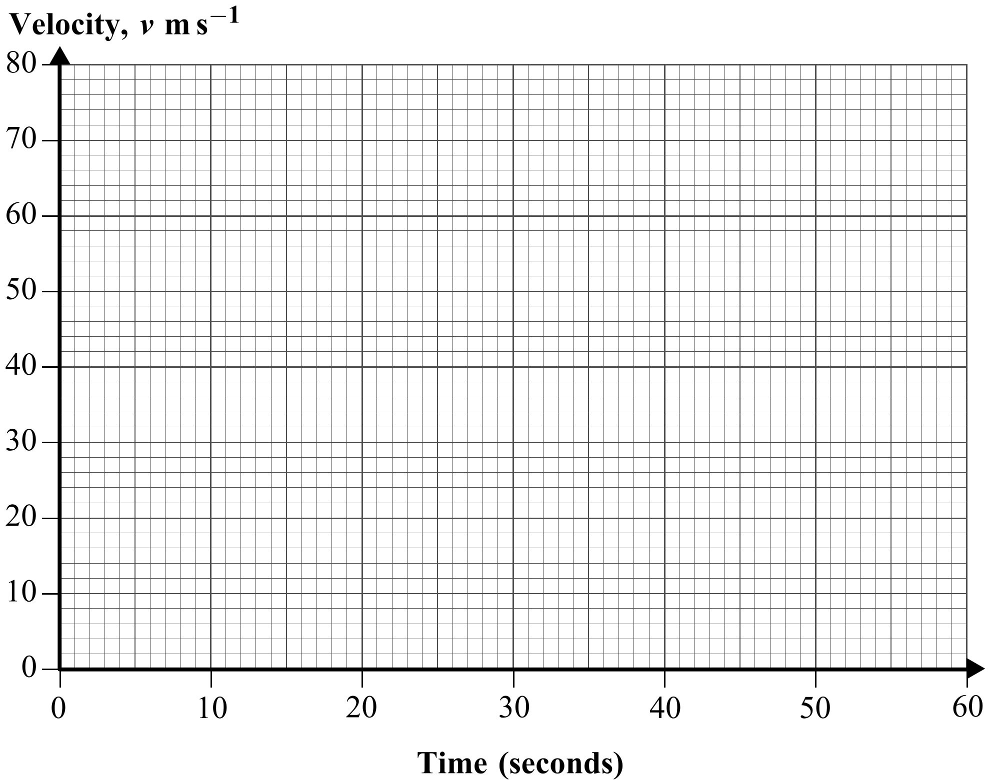
Do not write outside the box

6 A skydiver has velocity given by $v = 70(1 - e^{-kt})$.

(a) Find the value of k. *(2 marks)*

(b) On the grid at the bottom of this page, sketch a graph of the function you have found in part **(a)**, indicating all significant features clearly. *(2 marks)*

QUESTION PART REFERENCE



Do not write outside the box

7 For the case of the skydiver in the article, $v = 53.6 - 53.6\,e^{-0.183t}$.

(a) Show clearly how this leads to the expression $x = 53.6[t + 5.46(e^{-0.183t} - 1)]$, for the vertical distance, x metres, fallen by the skydiver after time t seconds. *(5 marks)*

(b) For large values of t, the expression for x in terms of t approximates to $x = 53.6(t - 5.46)$.

Give a brief explanation of why you know that this expression is not suitable for small values of t. *(2 marks)*

QUESTION PART REFERENCE

Do not write outside the box

8 For the skydiver in the article, explain the relationship between the shapes of the graphs of v against t (**Figure 4** on the Data Sheet) and x against t (**Figure 5** on the Data Sheet), for:

(a) $0 \leqslant t \leqslant 20$; *(2 marks)*

(b) $30 \leqslant t \leqslant 60$. *(2 marks)*

QUESTION PART REFERENCE

Section B

Read carefully the article below and answer **all** questions in the spaces provided.

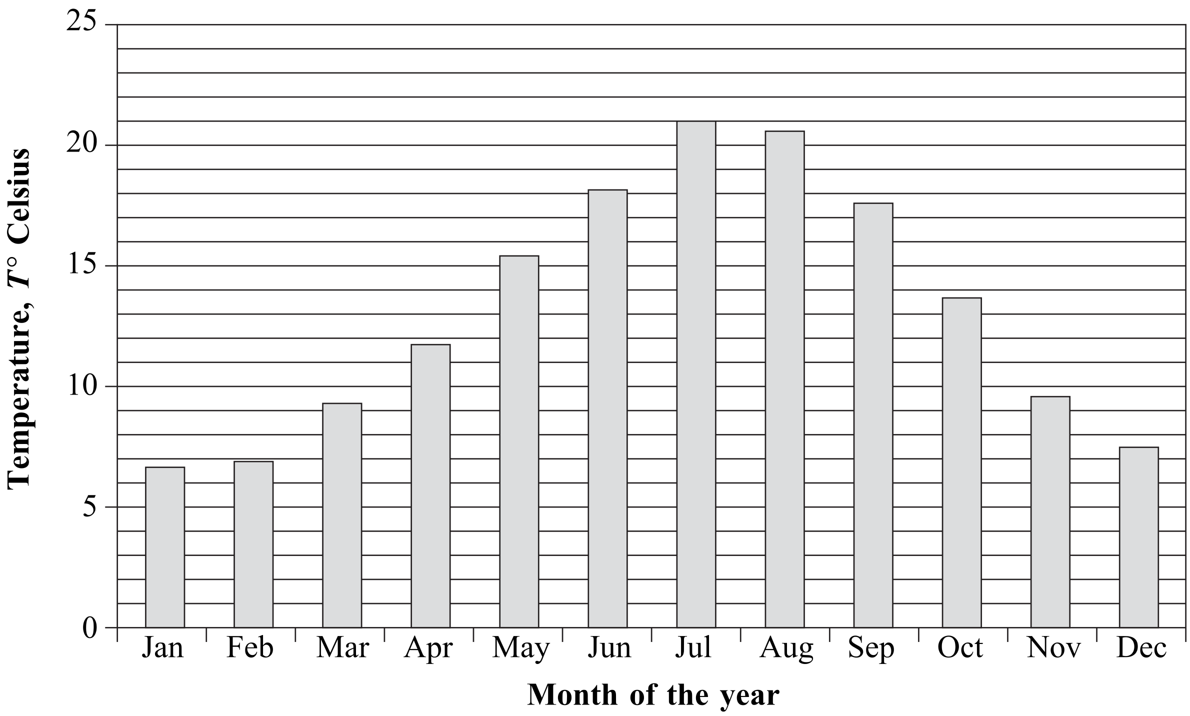
Annual temperature cycle

For London, the mean daily maximum temperature, $T°$ Celsius, for each month of the year is given in **Figure 6**. These data are plotted as a bar chart in **Figure 7**.

Figure 6 *Table giving the mean daily maximum temperature, $T°$ Celsius, in London for each month of the year*

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Temperature	6.6	6.9	9.3	11.7	15.4	18.1	21.0	20.5	17.5	13.6	9.5	7.4

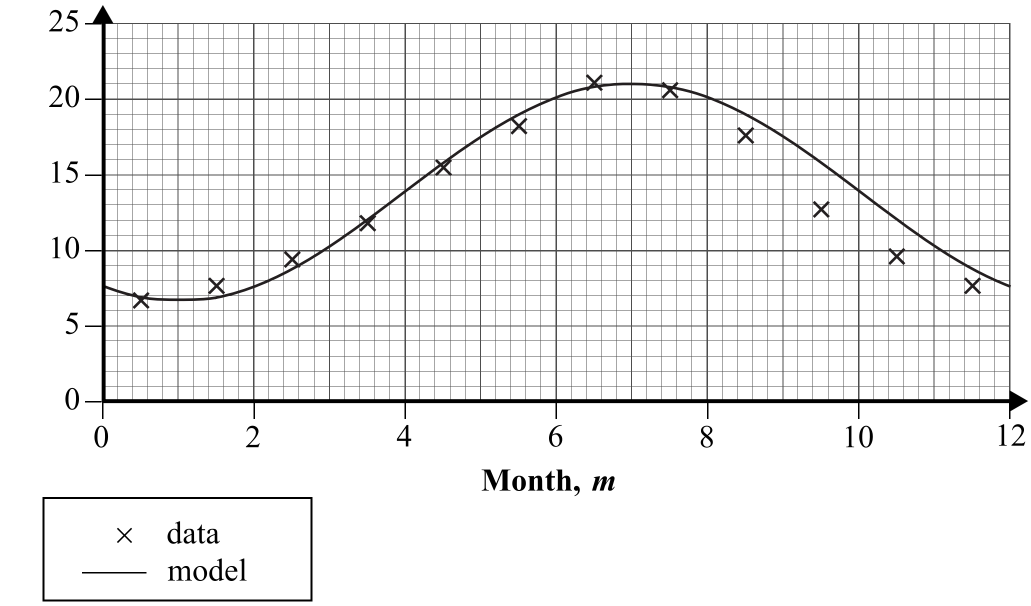
Figure 7 *Bar chart giving the mean daily maximum temperature, $T°$ Celsius, in London for each month of the year*



Like many annual cycles of this type, it appears that a trigonometric function might be suitable to model these data. **Figure 8** shows the data values plotted against the number of months, m, into the year. The temperature is plotted at the mid-point of each month, so the value for January is plotted at $\frac{1}{2}$, the value for February at $1\frac{1}{2}$ and so on. This wave is modelled by the function $T = 13.8 + 7.2\sin\left(\frac{\pi}{6}m - \frac{2\pi}{3}\right)$, the graph of which is also shown in **Figure 8**, for $0 \leqslant m \leqslant 12$. As you can see, this proves to be a relatively good model for the data.

Do not write outside the box

Figure 8 *Data for mean daily maximum temperature, T° Celsius, in London plotted for each month of the year together with the function* $T = 13.8 + 7.2\sin\left(\frac{\pi}{6}m - \frac{2\pi}{3}\right)$ *used to model this*



Although it is not necessary, the function used to model the data allows you to use calculus to determine some key features of how the mean daily maximum temperature, T° Celsius, varies over the year. For example:

(i) finding $\frac{dT}{dm}$ allows you to determine that the rate of increase of T is a maximum when $m = 4$;

(ii) the average, $\overline{m}$, of the mean daily maximum temperature over the summer months can be found using integration:

$$\overline{m} = \frac{1}{6}\int_3^9 \left[13.8 + 7.2\sin\left(\frac{\pi}{6}m - \frac{2\pi}{3}\right)\right] dm = 17.8$$

Do not write outside the box

9 Use the model

$$T = 13.8 + 7.2\sin\left(\frac{\pi}{6}m - \frac{2\pi}{3}\right)$$

to confirm that T has a maximum of 21 when $m = 7$, that is, during the month of July. *(2 marks)*

QUESTION PART REFERENCE

10 (a) Find the rate of change of the function $T = 13.8 + 7.2\sin\left(\frac{\pi}{6}m - \frac{2\pi}{3}\right)$ with respect to m and use this to show that the rate of change of daily maximum temperature is greatest for the first time when $m = 4$. *(4 marks)*

(b) (i) Use the expression that you have for $\frac{\mathrm{d}T}{\mathrm{d}m}$ to find the value of m for $0 \leqslant m \leqslant 12$ for which $\frac{\mathrm{d}T}{\mathrm{d}m}$ is a minimum. *(3 marks)*

(ii) Interpret this in terms of the situation. *(1 mark)*

QUESTION PART REFERENCE

Do not write outside the box

11 Show clearly, using calculus and setting out all steps of your calculations, that the average, $\overline{m}$, of the mean daily maximum temperature over the summer months is

$$\overline{m} = \frac{1}{6}\int_3^9 \left[13.8 + 7.2\sin\left(\frac{\pi}{6}m - \frac{2\pi}{3}\right)\right] \mathrm{d}m = 17.8$$

(3 marks)

QUESTION PART REFERENCE

Do not write outside the box

12 The table below shows the mean daily minimum temperature, $T°$ Celsius, for each month of the year.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Temperature	1.0	1.1	2.4	3.6	6.3	9.1	11.4	11.2	9.3	6.6	3.5	2.0

Find a trigonometric function of the form $T = A + B\sin\left(\frac{\pi}{6}m - \frac{2\pi}{3}\right)$, where m is the number of months into the year, to model these data. *(2 marks)*

QUESTION PART REFERENCE

END OF QUESTIONS

There are no questions printed on this page

DO NOT WRITE ON THIS PAGE
ANSWER IN THE SPACES PROVIDED

ACKNOWLEDGEMENT OF COPYRIGHT-HOLDERS AND PUBLISHERS

Section B: © Crown copyright 2000, the Met Office.