Centre Number						Candidate Number				
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For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
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TOTAL	

General Certificate of Education
Advanced Level Examination
June 2013

Use of Mathematics (Pilot) USE3

Mathematical Comprehension

Friday 24 May 2013 9.00 am to 10.30 am

For this paper you must have:
- a clean copy of the Data Sheet (enclosed)
- a graphics calculator
- a ruler.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.
- You may **not** refer to the copy of the Data Sheet that was available prior to this examination. A clean copy is enclosed for your use.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 45.

Advice
- You are advised to spend 1 hour on Section A and 30 minutes on Section B.
- You do not necessarily need to use all the space provided.

Section A

Answer **all** questions.

Answer each question in the space provided for that question.

Use **Measuring inequality** *on the Data Sheet.*

1 On page 4 of the article printed on the Data Sheet, it is stated that the area under the Lorenz curve for the UK data can be found using a numerical method of integration.

Using the trapezium rule with five intervals of 0.2 gives a value of 0.328 for the area under the Lorenz curve.

This leads to an approximate value of

$$G = (0.5 - 0.328) \times 2 = 0.344$$

for the Gini coefficient.

Explain what the term 0.5 represents in this calculation for G. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 1

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QUESTION PART REFERENCE	Answer space for question 1

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2 (a) For the data for India given in **Table 2**, printed on page 6 of the Data Sheet, calculate an approximate value for the Gini coefficient using the trapezium rule with five equal intervals. *(5 marks)*

(b) Interpret this value when compared with the value of 0.344 that was obtained for the UK using this numerical method. *(1 mark)*

QUESTION PART REFERENCE

Answer space for question 2

QUESTION PART REFERENCE

Answer space for question 2

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3 Use integration to confirm that the function $f(x) = 1.04x^3 - 0.38x^2 + 0.34x$, used to model the UK inequality data, results in a value of 0.393 for the Gini coefficient.

You must show all your working. *(6 marks)*

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

Answer space for question 3

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4 On a Lorenz curve diagram, the curve that models the data always meets the line of equality at the point (1, 1). Explain why this is the case. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 4

QUESTION PART REFERENCE

Answer space for question 4

Turn over ▶

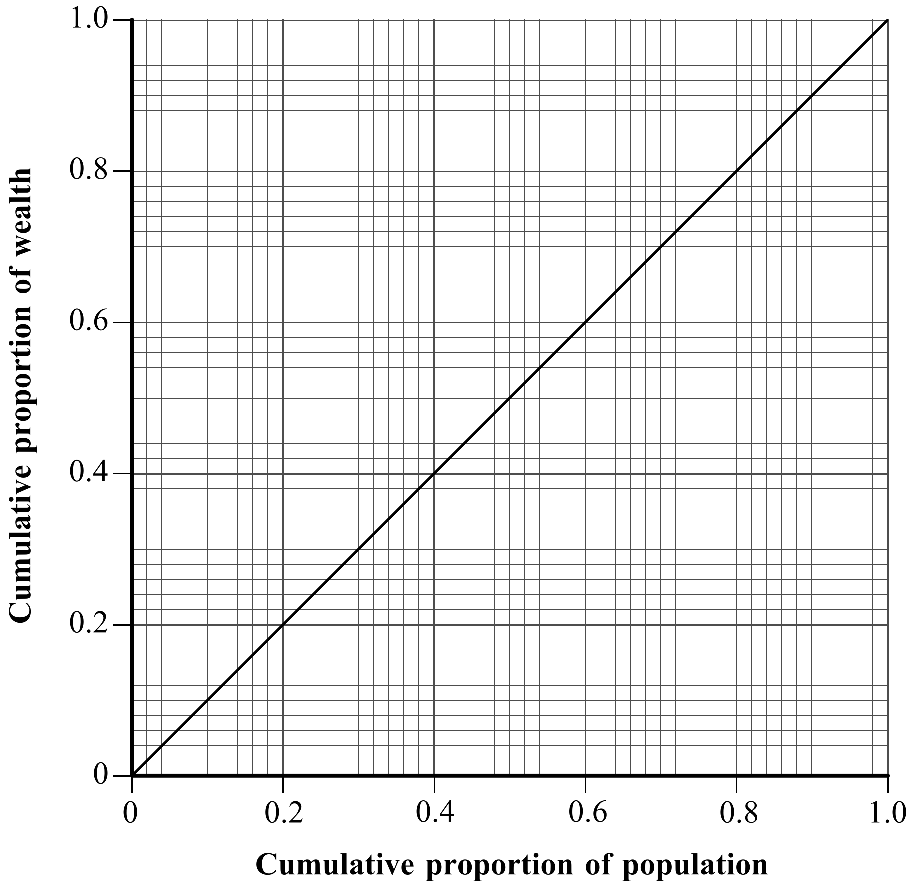
5 The Gini coefficient has a value of 1 for the case where wealth is distributed most unequally in a society.

(a) On the grid below, draw a diagram that illustrates this case and shade the area **enclosed** by the Lorenz curve and the line of equality. *(2 marks)*

(b) Interpret this diagram in terms of how wealth is distributed in this case. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 5



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QUESTION PART REFERENCE

Answer space for question 5

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6 The Lorenz curve for the UK is shown in **Figure 3** printed on page 3 of the Data Sheet.

(a) Using this curve, find the proportion of wealth owned by those people in the cumulative proportion of the population from 0.5 to 0.75 . *(2 marks)*

(b) Comment on your answer in terms of the approximate gradient of the curve in this section. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 6

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QUESTION PART REFERENCE

Answer space for question 6

Turn over ▶

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7 The cumulative proportion of wealth distribution for Brazil can be modelled by the function $p(x) = 1.86x^3 - 1.34x^2 + 0.48x$.

The cumulative proportion of wealth distribution for Sweden can be modelled by the function $q(x) = 0.46x^3 + 0.54x$.

(a) Find $p'(x)$ and $q'(x)$. *(3 marks)*

(b) Hence find the values of $p'(0)$ and $q'(0)$. *(1 mark)*

(c) Interpret your results. *(2 marks)*

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

Answer space for question 7

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Section B

Read the article below carefully.

Answer **all** questions.

Answer each question in the space provided for that question.

Faster and faster?

Table 3 gives the world record for the men's 100 metre sprint since 1912. Over the years the world record has been broken many times: it appears that men can run faster and faster as the years pass. Surely there must be a limit!

The graph in **Figure 7** shows that at the end of the last century, it appeared that the record was reaching a limit. However, in the last decade the world record has been decreasing at a faster rate.

Figure 8 shows the data modelled by the linear function $T = -0.008n + 10.6$, where T seconds is the world record and n is the number of years after 1900.

Figure 9 shows the data modelled by the exponential function $T = 10.6e^{-0.0008n}$. Notice that this exponential function looks almost linear in the relatively short time for which we have data, but it is curved, and we would see this if we were to extend it over a greater length of time.

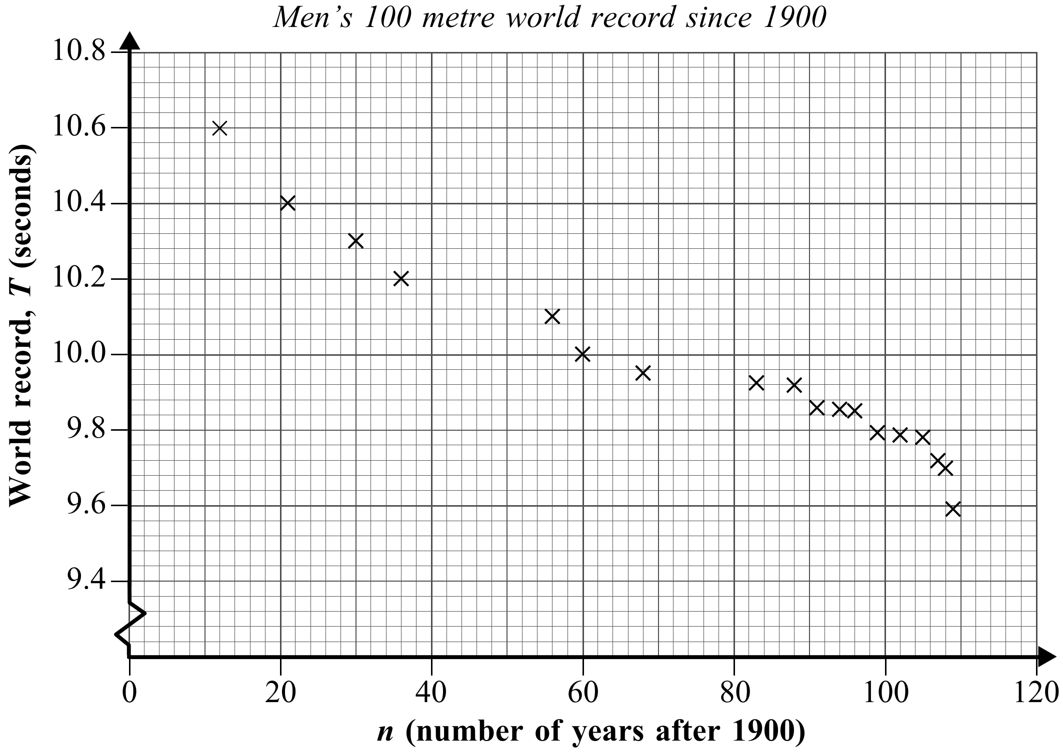
Table 3

Men's 100 metre world record since 1912

Year	n (number of years after 1900)	T (seconds)
1912	12	10.6
1921	21	10.4
1930	30	10.3
1936	36	10.2
1956	56	10.1
1960	60	10.0
1968	68	9.95
1983	83	9.93
1988	88	9.92
1991	91	9.86
1994	94	9.85
1996	96	9.84
1999	99	9.79
2002	102	9.78
2005	105	9.77
2007	107	9.72
2008	108	9.69
2009	109	9.58

Figure 7

Men's 100 metre world record since 1900



Using such functions allows you to use mathematics to find interesting features, such as the rate of change in the world record over time, and to predict likely values in the future.

For example, the exponential function gives $\dfrac{\mathrm{d}T}{\mathrm{d}n} = -0.00848\mathrm{e}^{-0.0008n}$, suggesting that, in the year 2000, the rate of decrease in the world record was 0.00783 seconds per year. This contrasts with a value of 0.008 seconds per year for the linear model.

Figure 8

Men's 100 metre world record data, modelled by the linear function $T = -0.008n + 10.6$

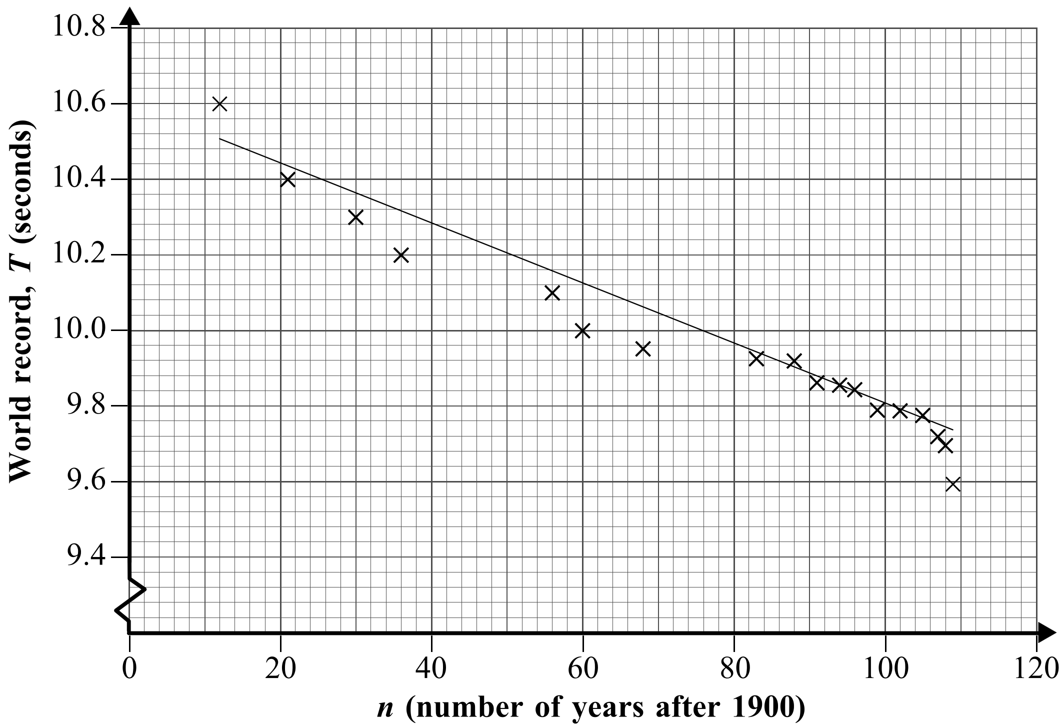
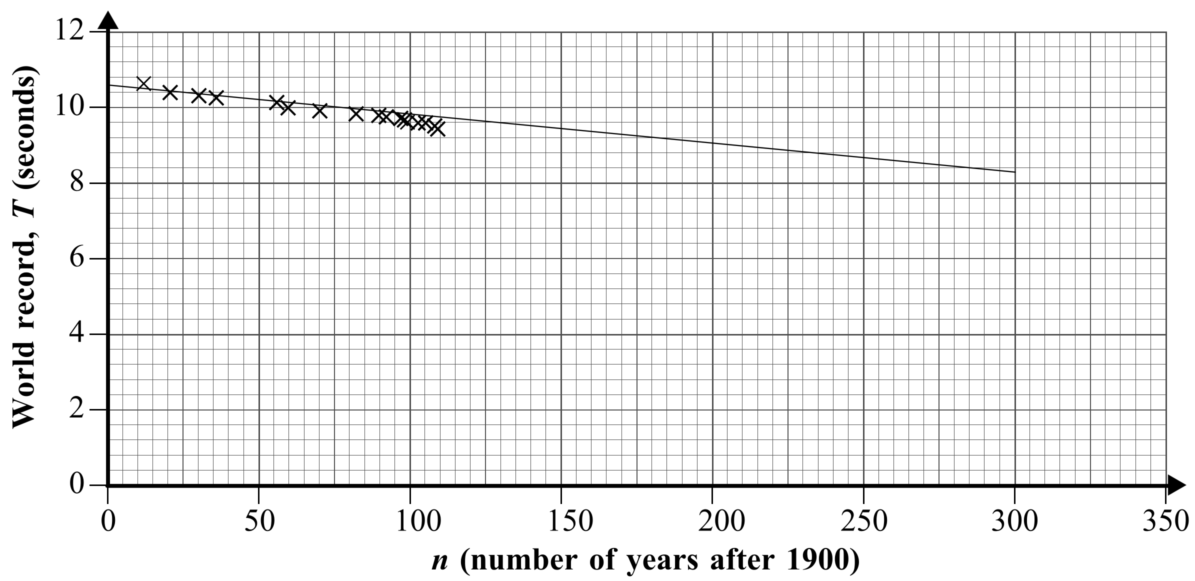


Figure 9

Men's 100 metre world record data, modelled by the exponential function $T = 10.6\mathrm{e}^{-0.0008n}$



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8 In **Figure 8**, the men's 100 metre world record data, T seconds, is modelled by the straight line

$$T = -0.008n + 10.6$$

where n is the number of years after 1900.

(a) Interpret the meaning of the values -0.008 and 10.6 in this model. *(2 marks)*

(b) Use the model to find the world record it predicts for 2020. *(2 marks)*

QUESTION PART REFERENCE

Answer space for question 8

QUESTION PART REFERENCE

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9 In **Figure 9**, the men's 100 metre world record data, T seconds, is modelled by the function

$$T = 10.6e^{-0.0008n}$$

where n is the number of years after 1900.

(a) What value does this model predict for the 100 metre world record in 1900?

(1 mark)

(b) In what year does this model predict that the world record will reach 9.58 seconds, that is the value of the actual record in 2009?

You must show all your working. *(4 marks)*

QUESTION PART REFERENCE

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10 Find and comment on the long-term predictions of the models in question **8** and question **9**. *(3 marks)*

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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11 (a) For the model $T = 10.6e^{-0.0008n}$, find $\frac{dT}{dn}$ when $n = 0$. *(2 marks)*

(b) Interpret this value. *(1 mark)*

QUESTION PART REFERENCE

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QUESTION PART REFERENCE

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