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General Certificate of Education (A-level)
January 2012

Mathematics MS2B

(Specification 6360)

Statistics 2B

Final

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
–*x* EE	deduct *x* marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MS2B

Question	Solution	Marks	Total	Comments
1(a)	21.05 and 21.15	B1	1	both (allow $21.04\dot{9}$ and $21.14\dot{9}$)
(b)	$\mathrm{E}(X)=0$ (symmetry)	B1		For $\mathrm{R}[-a,a]$: $\mathrm{E}(X)=0$ iff $a=0.05,\ 0.1,\ 0.5$ then:
	$\mathrm{Var}(X)=\frac{1}{12}(0.05--0.05)^2=\frac{1}{12}\times\frac{1}{100}$	M1		$\mathrm{Var}(X)=\frac{1}{12}(a--a)^2$ **or** their a = 0.049 to 0.05 used for **M1**
	$\Rightarrow\ sd(X)=\sqrt{\frac{1}{12}\times\frac{1}{100}}=\frac{1}{20\sqrt{3}}$	A1	3	or $\frac{\sqrt{3}}{60}$ or $\sqrt{\frac{1}{1200}}$ 0.0289 (3sf) A0
(c)	$\mathrm{P}(-0.01\le X\le 0.03)=0.04\times 10$ $=0.4$	B1	1	cao from correct value used $\int_{-0.01}^{0.03}10dx=[10x]_{-0.01}^{0.03}=0.4$ oe
	Total		**5**	

MS2B (cont)

Question	Solution	Marks	Total	Comments
2(a)(i)	$H_0: \mu = 61.4$ $H_1: \mu \neq 61.4$	B1		(both)
	$z_{calc} = \dfrac{65.0 - 61.4}{7.5/\sqrt{16}}$	M1		**Alternative:** $P(\bar{X} > 65.0) = P(Z > 1.92)$
	$= 1.92$	A1		$= 1 - 0.97257$ $= 0.02743$ $\geq 0.025 \quad \therefore$ Accept H_0
	$z_{crit} = \pm 1.96$ **or** (shown in / implied by diagram)	B1		Use of $t \Rightarrow$ max(B1M1A1)
	Accept H_0	Adep1		dep(B1M1) but not A1B1
	Insufficient / No evidence (at 5% level) to suggest /show **mean** (age has) **changed** (from 61.4 years.)			If incorrect **or** no hypothesis then **B0** $\Rightarrow$ max(M1A1B1) i.e. final Adep1Edep1 **not** available
	Mean (age) has **not** *changed at 1% level* (of significance)	Edep1	6	dep(Adep1)
(ii)	$61.4 - 3 \times 7.5 = 38.9 > 25$ $\Rightarrow$ none under the age of 25 years. Very unlikely any members < 25 yrs.	B1	1	$z = \dfrac{25 - 61.4}{7.5} = -4.85$ $\Rightarrow P(Z < -4.85) \approx 0$ $\Rightarrow$ none aged under 25 included
(b)(i)	$\bar{y} = \dfrac{\sum y}{n} = \dfrac{702}{12} = 58.5$	B1		($s = 2.83$)
	$s^2 = \dfrac{\sum (y - \bar{y})^2}{n-1} = \dfrac{88.25}{11} = 8.02$	B1		$\left(\sigma^2 = 7.35 \text{ or } \sigma = 2.71 \text{ iff } \sigma/\sqrt{11} \text{ used below}\right)$
	$t_{crit} = \pm 1.796$	B1		Ignore signs for t_{crit} If z used then max(B1B1B0M0A0)
	90% CI for μ: $58.5 \pm 1.796 \times \dfrac{s}{\sqrt{12}}$ 58.5 ± 1.4685 $= 57.03, 59.97$	M1		$(\text{their } \bar{y}) \pm t_{11} \times \dfrac{(\text{their } s)}{\sqrt{12}}$ **OR** $(\text{their } \bar{y}) \pm t_{11} \times \dfrac{(\text{their } \sigma)}{\sqrt{11}}$
	$= (57.0,\ 60.0)$	A1	5	
(ii)	upper limit < 61.4 $\Rightarrow$ recruitment drive lowered the average age of the club membership	B1ft	1	Must refer to 61.4 (on their CI)
	Total		**13**	

MS2B (cont)

Question	Solution	Marks	Total	Comments
3(a)(i)	E_i: $\frac{mp}{N}$; $\frac{mq}{N}$; $\frac{np}{N}$; $\frac{nq}{N}$	B2,1	2	B1 any one correct B2 all correct (simplified)
(ii)	$\sum_i E_i = \frac{mp+mq+np+nq}{N}$ $= \frac{m(p+q)}{N} + \frac{n(p+q)}{N}$ (oe)	M1		$\sum_i E_i = \frac{mp+mq+np+nq}{N}$ $= \frac{m(p+q)+n(p+q)}{N}$ (or use of unsimplified forms)
	$= \frac{mN}{N} + \frac{nN}{N}$ $= m+n$ $= N$ (since p + q = m + n = N)	Mdep1 Adep1	 3	$= \frac{(p+q)(m+n)}{N} = \frac{N \times N}{N} = N$ (AG)
(b)	H_0: No association between Andy's results and wind conditions	B1		
	E_i:			
	17.82 | 15.18 | 33 9.18 | 7.82 | 17 27 | 23 | 50	M1		Attempt E's
	$\Rightarrow \ \lvert 0_i - E_i \rvert - 0.5 = 2.32$	M1		Yates' correction attempted
	$X^2 = 0.3020 + 0.3546 + 0.5863 + 0.6883$ = 1.93	M1 A1		Final column attempted awrt
	$\chi^2_{10\%}(1) = 2.706$	B1		correct value of χ^2 only (allow 2.71)
	$\Rightarrow$ Accept H_0	Adep1		dep (B1 for H_0)
	No association (between Andy's results and wind conditions)	Edep1	8	Appropriate conclusion dep(B1 for H_0 ; M1final column; $\chi^2_{10\%}$ = 2.706)
	Total		**13**	
(a)(ii)	An example of unsimplified values derived from $a = \frac{mp}{N}$: $\Rightarrow b = m - \frac{mp}{N}$; $c = p - \frac{mp}{N}$; $d = n - \frac{mp}{n}$ (oe)			

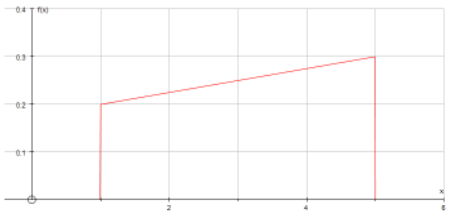
MS2B (cont)

Question	Solution	Marks	Total	Comments
4(a)(i)	Poisson	B1	1	
(ii)	$\mathrm{E}(3X-1)=3\lambda-1$	B1		
	$\mathrm{Var}(3X-1)=9\lambda$	B1	2	oe (allow $3^2\lambda$)
(iii)	$\mathrm{P}(X=x+1)=\dfrac{e^{-\lambda}\times\lambda^{x+1}}{(x+1)!}$	B1		
	$\mathrm{P}(X=x+1)=\dfrac{e^{-\lambda}\times\lambda^{x+1}}{(x+1)!}$			
	$=\dfrac{e^{-\lambda}\times\lambda^{x}\times\lambda}{(x+1)x!}$ $=\dfrac{\lambda}{x+1}\times\dfrac{e^{-\lambda}\times\lambda^{x}}{x!}$	Mdep1		dep(B1)
	$=\dfrac{\lambda}{x+1}\mathrm{P}(X=x)$	Adep1	3	AG
(b)(i)	$\lambda_{\text{car}}=500/\text{hour}$ $\lambda_{\text{coach}}=10/\text{hour}$			
	$\Rightarrow\ \lambda_{\text{vehicle}}=510/\text{hour}\ =8.5/\text{min}$	B1		for 8.5 stated / used
	$\mathrm{P}(V\geq 10)=1-0.6530$	M1		special case: $\lambda=10\Rightarrow$ B1M0A0 B1 $\Rightarrow 1-0.458$ or 0.542
	$=0.347$	A1	3	
(ii)	$\mu_{\text{car}}=836/\text{hour}$ $\mu_{\text{coach}}=22/\text{hour}$			
	$\Rightarrow\ \mu_{\text{vehicle}}=858/\text{hour}\ =14.3/\text{min}$	B1		for 14.3 stated /used
	$\mathrm{P}(V\leq 3)=\mathrm{P}(V=0,1,2,3)$			
	$=\begin{cases} e^{-14.3}\left[1+\dfrac{14.3}{1}+\dfrac{14.3^2}{2}+\dfrac{14.3^3}{6}\right] \\ e^{-14.3}\times 604.91283 \\ 0.0003726\ \text{ to }\ 0.000373 \end{cases}$	M1		All 4 terms required for **any** $\lambda>0$ M0 for use of normal approximation
	$=0.00037$ (2sf)	Adep1	3	dep M1
	Total		**12**	

MS2B (cont)

Question	Solution	Marks	Total	Comments
5(a)				
	n | Outcome | $P(N=n)$ 1 | H | **0.5** ($\frac{1}{2}$) 2 | TH | **0.25** ($\frac{1}{4}$) 3 | TTH | 0.125 ($\frac{1}{8}$) 4 | TTTH | **0.0625** ($\frac{1}{16}$) 5 | TTTTA | 0.0625 ($\frac{1}{16}$)	B2,1		B1 for **one** correct entry for n = 1, 2, 4 B2 for all 3 correct Can be implied by correct $\mathrm{E}(N)$
	$\mathrm{E}(N)=\left(1\times\frac{1}{2}\right)+\left(2\times\frac{1}{4}\right)+\left(3\times\frac{1}{8}\right)+\left(4\times\frac{1}{16}\right)+\left(5\times\frac{1}{16}\right)$	M1		$\sum_{n=1}^{n=5} n\times\mathrm{P}(N=n)$ (all 5 terms attempted /seen/ implied)
	$=\frac{1}{2}+\frac{2}{4}+\frac{3}{8}+\frac{4}{16}+\frac{5}{16}=\frac{31}{16}$ $=1\frac{15}{16}$ (1.9375)	A1	4	(awfw 1.93 to 1.94)
(b)	m | Outcome | $P(M=m)$ 1 | H | $\frac{1}{4}$ 2 | TH | $\frac{3}{4}\times\frac{1}{4}=\frac{3}{16}$ 3 | TTH | $\left(\frac{3}{4}\right)^2\times\frac{1}{4}=\frac{9}{64}$ 4 | TTTH | $\left(\frac{3}{4}\right)^3\times\frac{1}{4}=\frac{27}{256}$ 5 | TTTTA | $\left(\frac{3}{4}\right)^4\times 1=\frac{81}{256}$	B3,2,1	3	(given) (given) (B1 any one correct) (B2 any 2 correct) (B3 all 3 correct)
(c)(i)	$\mathrm{P}(J,R)$:			
	$\mathrm{P}(1,1)=\frac{1}{2}\times\frac{1}{4}=\frac{1}{8}$ (oe) $\mathrm{P}(2,2)=\frac{1}{4}\times\frac{3}{16}=\frac{3}{64}$ (oe) $\mathrm{P}(3,3)=\frac{1}{8}\times\frac{9}{64}=\frac{9}{512}$ (oe)	M1		e.g 0.125 attempt at any $\mathrm{P}(n,n)$
	$\mathrm{P}(4,4)=\frac{1}{16}\times\frac{27}{256}=\frac{27}{4096}$ (oe)	A1		any 1 correct **to 3sf**
	$\mathrm{P}(5,5)=\frac{1}{16}\times\frac{81}{256}=\frac{81}{4096}$ (oe)	A1		all 5 correct **to 3sf**
	$p=\sum_{n=1}^{n=5}\mathrm{P}(n,n)$	m1		$\sum_{n=1}^{n=5}\mathrm{P}(n,n)$ with all 5 values attempted
	$\Rightarrow\quad p=\frac{221}{1024}$ (0.2158)	A1	5	(awfw 0.215 to 0.217)
(ii)	$=3\times\left(\frac{221}{1024}\right)^2\times\left(\frac{803}{1024}\right)$	M1		(either term with **their *p*** used) $(0<p<1)$
	$+\left(\frac{221}{1024}\right)^3$	M1		(second term with **their *p*** used) $(0<p<1)$
	$\mathrm{P}(X\geq 2)=\mathrm{P}(X=2)+\mathrm{P}(X=3)$	Mdep1		dep (M1M1)
	$=0.120$ (3dp)	A1	4	(allow 0.119; 0.12; 0.121)
	Total		**16**	

MS2B (cont)

Question	Solution	Marks	Total	Comments
6(a)		B2,1	2	B2 for st. line from $(1,0.2)$ to $(5,0.3)$ B1 st. line (m > 0) from $x = 1$ to $x = 5$.
(b)	$\mathrm{E}(X)=\frac{1}{40}\int_1^5 x(x+7)\,dx$	M1		Ignore limits
	$=\frac{1}{40}\left(\frac{x^3}{3}+\frac{7x^2}{2}\right)_1^5$	A1		Ignore limits
	$=\frac{1}{40}\left(\frac{125}{3}+\frac{175}{2}-\frac{1}{3}-\frac{7}{2}\right)$			
	$=3\frac{2}{15}$	A1	3	cao (accept $3.13\dot{3}$ or $\frac{47}{15}$ oe ***exact***)
(c)	$\mathrm{F}(x)=\int_1^x \frac{1}{40}(x+7)\,dx$	M1		$\mathrm{F}(x)=\int\left(\frac{x}{40}+\frac{7}{40}\right)dx$
	$=\frac{1}{40}\left[\frac{x^2}{2}+7x\right]_1^x$	A1		$=\frac{x^2}{80}+\frac{7x}{40}+c \Rightarrow$ (M1A1)
	$=\frac{1}{80}\left(x^2+14x-1-14\right)$			$\mathrm{F}(1)=0 \Rightarrow c=-\frac{1}{80}-\frac{7}{40}=-\frac{15}{80}$ **or** [use of F(5) = 1]
	$=\frac{1}{80}\left(x^2+14x-15\right)$	Adep1		$\Rightarrow \mathrm{F}(x)=\frac{1}{80}\left(x^2+14x-15\right)$
	$=\frac{1}{80}(x+15)(x-1)$	Adep1	4	$\mathrm{F}(x)=\frac{1}{80}(x+15)(x-1)$ (AG)
(d)(i)	$\mathrm{P}(2.5\le X\le 4.5)=\mathrm{F}(4.5)-\mathrm{F}(2.5)$			Trapezium Rule
	$=\frac{1}{80}(19.5\times 3.5-17.5\times 1.5)$	M1		$\frac{1}{2}\left(\frac{23}{80}+\frac{19}{80}\right)\times 2$
	$=\frac{42}{80}=\frac{21}{40}$ (0.525)	A1	2	$=\frac{42}{80}=\frac{21}{40}$
(ii)	$\mathrm{F}(m)=\frac{1}{2}$	B1		$\int_1^m \frac{1}{40}(x+7)dx=0.5$ (B1)
	$\Rightarrow \frac{1}{80}\left(m^2+14m-15\right)=\frac{1}{2}$	M1		Correct equation formed
	$(\times 80)$ $\Rightarrow m^2+14m-15=40$ $m^2+14m-55=0$	Adep1	3	AG
(e)	$m=\frac{-14\pm\sqrt{196+220}}{2}=\frac{-14\pm 20.396}{2}$	M1		Correct attempt at solving quadratic (by formula, oe).
	$m=\frac{-14+20.396}{2}$ (since m > 1) $m=3.198$ (3dp)	A1	2	cao
	Total		**16**	
	TOTAL		**75**	