General Certificate of Education
June 2008
Advanced Level Examination

MATHEMATICS **MS03**
Unit Statistics 3

Friday 23 May 2008 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MS03.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of tables or calculators should normally be given to three significant figures.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The best performances of a random sample of 20 junior athletes in the long jump, x metres, and in the high jump, y metres, were recorded. The following statistics were calculated from the results.

$$S_{xx} = 7.0036 \quad S_{yy} = 0.8464 \quad S_{xy} = 1.3781$$

(a) Calculate the value of the product moment correlation coefficient between x and y. *(2 marks)*

(b) Assuming that these data come from a bivariate normal distribution, investigate, at the 1% level of significance, the claim that for junior athletes there is a positive correlation between x and y. *(4 marks)*

(c) Interpret your conclusion in the context of this question. *(1 mark)*

2 A survey of a random sample of 200 passengers on UK internal flights revealed that 132 of them were on business trips.

(a) Construct an approximate 98% confidence interval for the proportion of passengers on UK internal flights that are on business trips. *(6 marks)*

(b) Hence comment on the claim that more than 60 per cent of passengers on UK internal flights are on business trips. *(2 marks)*

3 Pitted black olives in brine are sold in jars labelled "340 grams net weight". Two machines, A and B, independently fill these jars with olives before the brine is added.

The weight, X grams, of olives delivered by machine A may be modelled by a normal distribution with mean μ_X and standard deviation 4.5 .

The weight, Y grams, of olives delivered by machine B may be modelled by a normal distribution with mean μ_Y and standard deviation 5.7 .

The mean weight of olives from a random sample of 10 jars filled by machine A is found to be 157 grams, whereas that from a random sample of 15 jars filled by machine B is found to be 162 grams.

Test, at the 1% level of significance, the hypothesis that $\mu_X = \mu_Y$. *(6 marks)*

4 A manufacturer produces three models of washing machine: basic, standard and deluxe. An analysis of warranty records shows that 25% of faults are on basic machines, 60% are on standard machines and 15% are on deluxe machines.

For basic machines, 30% of faults reported during the warranty period are electrical, 50% are mechanical and 20% are water-related.

For standard machines, 40% of faults reported during the warranty period are electrical, 45% are mechanical and 15% are water-related.

For deluxe machines, 55% of faults reported during the warranty period are electrical, 35% are mechanical and 10% are water-related.

(a) Draw a tree diagram to represent the above information. *(3 marks)*

(b) Hence, or otherwise, determine the probability that a fault reported during the warranty period:

(i) is electrical; *(2 marks)*

(ii) is on a deluxe machine, given that it is electrical. *(2 marks)*

(c) A random sample of 10 electrical faults reported during the warranty period is selected. Calculate the probability that exactly 4 of them are on deluxe machines. *(3 marks)*

5 The daily number of emergency calls received from district A may be modelled by a Poisson distribution with a mean of λ_{A}.

The daily number of emergency calls received from district B may be modelled by a Poisson distribution with a mean of λ_{B}.

During a period of 184 days, the number of emergency calls received from district A was 3312, whilst the number received from district B was 2760.

(a) Construct an approximate 95% confidence interval for $\lambda_{\mathrm{A}} - \lambda_{\mathrm{B}}$. *(6 marks)*

(b) State one assumption that is necessary in order to construct the confidence interval in part (a). *(1 mark)*

6 An aircraft, based at airport A, flies regularly to and from airport B.

The aircraft's flying time, X minutes, from A to B has a mean of 128 and a variance of 50.

The aircraft's flying time, Y minutes, on the return flight from B to A is such that

$$\mathrm{E}(Y) = 112, \quad \mathrm{Var}(Y) = 50 \quad \text{and} \quad \rho_{XY} = -0.4$$

(a) Given that $F = X + Y$:

(i) find the mean of F;

(ii) show that the variance of F is 60. *(4 marks)*

(b) At airport B, the stopover time, S minutes, is independent of F and has a mean of 75 and a variance of 36.

Find values for the mean and the variance of:

(i) $T = F + S$; *(2 marks)*

(ii) $M = F - 3S$. *(3 marks)*

(c) Hence, assuming that T and M are normally distributed, determine the probability that, on a particular round trip of the aircraft from A to B and back to A:

(i) the time from leaving A to returning to A exceeds 300 minutes; *(3 marks)*

(ii) the stopover time is greater than one third of the total flying time. *(6 marks)*

7 (a) The random variable X has a Poisson distribution with $\mathrm{E}(X) = \lambda$.

(i) Prove, from first principles, that $\mathrm{E}(X(X-1)) = \lambda^2$. *(4 marks)*

(ii) Hence deduce that $\mathrm{Var}(X) = \lambda$. *(2 marks)*

(b) The independent Poisson random variables X_1 and X_2 are such that $\mathrm{E}(X_1) = 5$ and $\mathrm{E}(X_2) = 2$.

The random variables D and F are defined by

$$D = X_1 - X_2 \quad \text{and} \quad F = 2X_1 + 10$$

(i) Determine the mean and the variance of D. *(2 marks)*

(ii) Determine the mean and the variance of F. *(3 marks)*

(iii) For **each** of the variables D and F, give a reason why the distribution is **not** Poisson. *(2 marks)*

(c) The daily number of black printer cartridges sold by a shop may be modelled by a Poisson distribution with a mean of 5.

Independently, the daily number of colour printer cartridges sold by the same shop may be modelled by a Poisson distribution with a mean of 2.

Use a distributional approximation to estimate the probability that the total number of black and colour printer cartridges sold by the shop during a 4-week period (24 days) exceeds 175. *(6 marks)*

END OF QUESTIONS

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