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ASSESSMENT and QUALIFICATIONS ALLIANCE

General Certificate of Education

Mathematics 6360

MPC3 Pure Core 3

Mark Scheme

2008 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MPC3

Q	Solution	Marks	Total	Comments
1(a)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 5(3x+1)^4 \times 3$	M1		$k(3x+1)^4$
	$= 15(3x+1)^4$	A1	2	with no further errors (w.n.f.e)
(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{3x+1}$	M1		$\frac{k}{3x+1}$
		A1	2	w.n.f.e
(c)	$\frac{\mathrm{d}y}{\mathrm{d}x} =$			
	$(3x+1)^5 \times \frac{3}{3x+1} + \ln(3x+1) \times 15(3x+1)^4$	M1 A1 A1	3	product rule $uv' + u'v$ (from (a) and (b)) either term correct CSO with no further errors
	$\left(\begin{array}{l} = (3x+1)^4 \left[3 + 15\ln(3x+1)\right] \\ = 3(3x+1)^4 \left[1 + 5\ln(3x+1)\right] \end{array} \right)$			
	Total		**7**	
2(a)	$x = \cos^{-1}\frac{1}{3}$	M1		PI
	$= 1.23,\ 5.05 \quad (0.39\pi,\ 1.61\pi)$	A1,A1	3	AWRT (-1 for each error in range) SC 70.53, 289.47 B1
(b)	$\sec^2 x - 1 = 2\sec x + 2$	M1		use of $\sec^2 x = 1 + \tan^2 x$
	$\sec^2 x - 2\sec x - 3 = 0$	A1	2	AG; CSO
(c)	$\sec^2 x - 2\sec x - 3 = 0$			
	$(\sec x - 3)(\sec x + 1) = 0$	M1		attempt to solve
	$\cos x = \frac{1}{3}$ or -1 o.e	A1		
	$x = 1.23,\ 5.05,$	B1f		(2 answers in range from (a)) AWRT
	$3.14 \quad (\pi)$	B1	4	all correct and no extras in range SC 70.53, 289.47, 180 B1
	Total		**9**	

(Extra +c penalised once throughout paper)

MPC3 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\frac{dy}{dx} = -x2\sin 2x + \cos 2x$	M1 A1	2	product rule $kx\sin 2x \pm \cos 2x$ no further incorrect working
(b)(i)	$-2\alpha\sin 2\alpha + \cos 2\alpha = 0$	M1		replacing $x = \alpha$ and writing equation equal to zero (at any line)
	$\left.\begin{array}{l} 2\alpha\sin 2\alpha = \cos 2\alpha \\ 2\alpha\tan 2\alpha = 1 \end{array}\right\}$ either $2\alpha\tan 2\alpha - 1 = 0$	A1	2	AG; CSO
(ii)	$\left.\begin{array}{l} f(0.4) = 0.2 \\ f(0.5) = -0.6 \end{array}\right\}$ awrt o.e. Change of sign $\therefore 0.4 < \alpha < 0.5$	M1 A1	2	(0.9's unsubstantiated scores M0)
(iii)	$2x\tan 2x = 1$ $\left.\begin{array}{l} \tan 2x = \frac{1}{2x} \\ 2x = \tan^{-1}\left(\frac{1}{2x}\right) \end{array}\right\}$ either $x = \frac{1}{2}\tan^{-1}\left(\frac{1}{2x}\right)$	B1	1	AG; CSO
(iv)	$x_1 = 0.4$ $x_2 = 0.4480...$ $x_3 = 0.4200...$ $= 0.42$	M1 A1	2	$x_2 = 25.7$
(c)	$y = x\cos 2x$ $\left.\begin{array}{ll} u = x & du = 1 \\ dv = \cos 2x & v = \frac{\sin 2x}{2} \end{array}\right\}$	M1		$\left.\begin{array}{l} \text{differentiate one term} \\ \text{integrate one term} \end{array}\right\}$ must be $k\sin 2x$
		m1		correct substitution of their values into parts formula using $u = x$
	$\int = \frac{x\sin 2x}{2} - \int \frac{\sin 2x}{2}\,(dx)$ $= \left[\frac{x\sin 2x}{2} + \frac{\cos 2x}{4}\right]_{(0)}^{(0.5)}$	A1		
	$= \left(\frac{\sin 1}{4} + \frac{\cos 1}{4}\right) - \left(\frac{\cos 0}{4}\right)$	m1		correctly substituting values from previous 2 method marks
	$= 0.0954$	A1	5	AWRT
	Total		**14**	

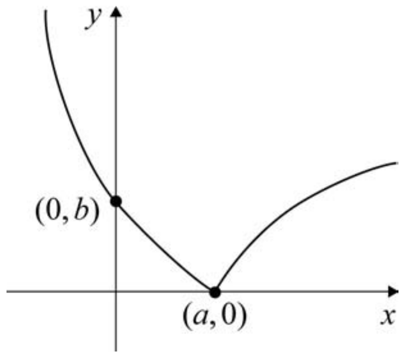
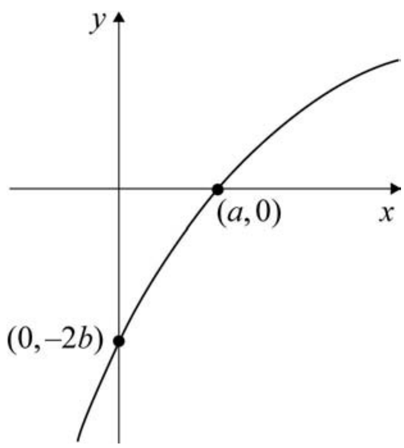
MPC3 (cont)

Q	Solution	Marks	Total	Comments
4(a)	$f(x) \geqslant 0$	B1	1	allow $f \geqslant 0,\ y \geqslant 0,\ \geqslant 0$
(b)(i)	$y = \dfrac{1}{2x-3}$			
	$x = \dfrac{1}{2y-3}$	M1		swap x and y
	$x(2y-3)=1$			
	$2xy - 3x = 1$			
	$2xy = 1 + 3x$	M1		attempt to isolate
	$y = \dfrac{1+3x}{2x} = g^{-1}(x)$ o.e.	A1	3	w.n.f.e
(ii)	$\left(g^{-1}(x)\right) \neq \dfrac{3}{2}$	B1	1	
(c)	$\left(\dfrac{1}{2x-3}\right)^2 = 9$	B1		
	$2x-3 = \pm\dfrac{1}{3}$	M1		square root and invert (condone missing $\pm$) **alternative:** attempt to solve a quadratic that comes from $4x^2 - 12 + 9 = \dfrac{1}{9}$ o.e.
	$x = \dfrac{5}{3}, \dfrac{4}{3}$ o.e.	A1	3	
	Total		**8**	

Alternative

Q	Solution	Marks	Total	Comments
4(b)(i)	$x \rightarrow \boxed{\times 2} \rightarrow \boxed{-3} \rightarrow \boxed{\text{divide into} 1} \rightarrow y$ $\dfrac{1}{2y} + \dfrac{3}{2} \leftarrow \boxed{\div 2} \leftarrow \boxed{+3} \leftarrow \boxed{\text{divide into} 1} \leftarrow y$ $\dfrac{1}{y} + 3$ M1			

MPC3 (cont)

Q	Solution	Marks	Total	Comments
5(a)(i)		B1 B1	 2	shape coordinates
(ii)		B1 B1	 2	shape coordinates
(b)(i)	Translation	M1		**OR** I stretch M1 I + (II or III)
	$\begin{bmatrix}-1\\0\end{bmatrix}$	A1		II SF 4 III ⁄⁄ y-axis A1 (I + II + III)
	Stretch I SF 4 II ⁄⁄ y-axis III	M1		$\text{I}+(\text{II or III})$ Translation M1
		A1		I + II + III $\begin{bmatrix}-1\\-2\end{bmatrix}$ A1 B1
	Translation $\begin{bmatrix}0\\-2\end{bmatrix}$	B1		both
	All correct and no mistakes on order etc	A1	6	All correct A1
	Alternative:			
	$y=4\ln(x+1)-2=4\left[\ln(x+1)-\frac{1}{2}\right]$	(B1)		
	Translation	(M1)		
	$\begin{bmatrix}-1\\-\frac{1}{2}\end{bmatrix}$	(A1)		
	Stretch I SF 4 II ⁄⁄ y-axis III	(M1) (A1)		$\text{I}+(\text{II or III})$ I + II + III
	All correct and no mistakes on order etc	(A1)	(6)	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
5(b)(ii)	$y = 4\ln(x+1) - 2$			
	$x = 0 \quad y = -2$	B1		
	$y = 0$			
	$4\ln(x+1) = 2$			
	$\ln(x+1) = \frac{1}{2}$	M1		isolate $\ln(x+1) =$ or $(x+1)^4$
	$x+1 = e^{\frac{1}{2}}$	A1		$x+1 = e^k$
	$x = e^{\frac{1}{2}} - 1$ o.e.	A1	4	CSO isw
	Total		**14**	
6(a)	$y = \left(e^{3x}+1\right)^{\frac{1}{2}}$			
	$\frac{dy}{dx} = \frac{1}{2}\left(e^{3x}+1\right)^{-\frac{1}{2}} \times 3e^{3x}$	M1		$\frac{1}{2}\left(e^{3x}+1\right)^{-\frac{1}{2}}$
		A1		e^{3x}
		A1		$\frac{3}{2}$ (allow $\frac{1}{2} \times 3$) w.n.f.e
	$x = \ln 2:$			
	$\frac{dy}{dx} = \frac{3}{2}\left(e^{\ln 8}+1\right)^{-\frac{1}{2}} \times e^{\ln 8}$	M1		correct substitution into their $\frac{dy}{dx}$ (must use ln8 or $\ln 2^3$)
	$= \frac{3}{2} \times \frac{1}{3} \times 8$			
	$= 4$	A1	5	
(b)	x / y: 0.25 / 1.765(5); 0.75 / 3.238(5); 1.25 / 6.597(1); 1.75 / 13.84(1)	B1		correct x values
		B1		3 or 4 correct y values 4 s.f. or better
	$\int = 0.5 \times \sum y$ P.I	M1		
	$= 12.7$	A1	4	sc 12.7 with no working $\frac{2}{4}$
(c)	$v = \pi\int y^2 dx$			
	$= (\pi)\int\left(e^{3x}+1\right)(dx)$	M1		
	$= (\pi)\left[\frac{1}{3}e^{3x} + x\right]_{(0)}^{(2)}$	A1		$ke^{3x} + x$
	$= (\pi)\left[\left(\frac{1}{3}e^6 + 2\right) - \left(\frac{1}{3}e^0 + 0\right)\right]$	m1		correct substitution into f $(\int e^{3x})$
	$= \pi\left[\frac{1}{3}e^6 + \frac{5}{3}\right]$	A1	4	CSO
	$\left(= \frac{\pi}{3}(e^6 + 5)\right)$			
	Total		**13**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
7(a)	$y = \dfrac{\sin\theta}{\cos\theta}$			
	$\dfrac{\mathrm{d}y}{\mathrm{d}\theta} = \dfrac{\cos\theta\cos\theta - \sin\theta(-\sin\theta)}{\cos^2\theta}$	M1 A1		$\dfrac{\pm\cos^2\theta \pm \sin^2\theta}{\cos^2\theta}$
	$= \dfrac{1}{\cos^2\theta}$ o.e.			$(1+\tan^2\theta)$
	$= \sec^2\theta$	A1	3	AG; CSO
(b)	$x = \sin\theta$ $x^2 = \sin^2\theta$			
	$\cos^2\theta = 1 - x^2$ $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$	M1		use of $\cos^2\theta + x^2 = 1$
	$= \dfrac{x}{\sqrt{1-x^2}}$	A1	2	AG; CSO
	OR LHS = $\dfrac{\sin\theta}{\sqrt{1-\sin^2\theta}}$ $= \dfrac{\sin\theta}{\cos\theta}$ $= \tan\theta$ AG			
(c)	$\displaystyle\int \frac{1}{(1-x^2)^{\frac{3}{2}}}\,\mathrm{d}x$			
	$x = \sin\theta$ $\mathrm{d}x = \cos\theta\,\mathrm{d}\theta$ o.e.	M1		$\dfrac{\mathrm{d}x}{\mathrm{d}\theta} = \pm\cos\theta$
	$\displaystyle\int = \int \frac{\cos\theta\,(\mathrm{d}\theta)}{(1-\sin^2\theta)^{\frac{3}{2}}}$	m1		all in terms of θ
	$\displaystyle= \int \frac{\cos\theta}{(\cos^2\theta)^{\frac{3}{2}}}\,(\mathrm{d}\theta)$	A1		
	$\displaystyle= \int \sec^2\theta\,(\mathrm{d}\theta)$	A1		
	$= \tan\theta$			
	$= \dfrac{x}{\sqrt{1-x^2}}\ (+c)$	A1	5	CSO including $\mathrm{d}\theta$'s
	Total		**10**	
	TOTAL		**75**	

Alternative

7(a)	$y = \dfrac{\tan\theta}{1}$			
	$\dfrac{\mathrm{d}y}{\mathrm{d}\theta} = \dfrac{1\sec^2\theta - 0}{1^2}$	M1 A1		
	$= \sec^2\theta$	A1		