General Certificate of Education
June 2008
Advanced Subsidiary Examination

MATHEMATICS   **MPC2**
Unit Pure Core 2

Thursday 15 May 2008   9.00 am to 10.30 am

For this paper you must have:
* an 8-page answer book
* the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
* Use black ink or black ball-point pen. Pencil should only be used for drawing.
* Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
* Answer **all** questions.
* Show all necessary working; otherwise marks for method may be lost.

Information
* The maximum mark for this paper is 75.
* The marks for questions are shown in brackets.

Advice
* Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 (a) Write $\sqrt{x^3}$ in the form x^k, where k is a fraction. *(1 mark)*
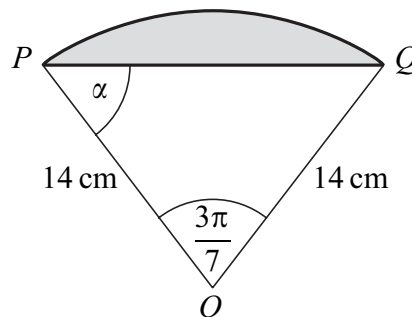
(b) A curve, defined for $x \geqslant 0$, has equation

$$y = x^2 - \sqrt{x^3}$$

(i) Find $\dfrac{dy}{dx}$. *(3 marks)*

(ii) Find the equation of the tangent to the curve at the point where $x = 4$, giving your answer in the form $y = mx + c$. *(5 marks)*

2 The diagram shows a shaded segment of a circle with centre O and radius $14\,\text{cm}$, where PQ is a chord of the circle.

In triangle OPQ, angle $POQ = \dfrac{3\pi}{7}$ radians and angle $OPQ = \alpha$ radians.
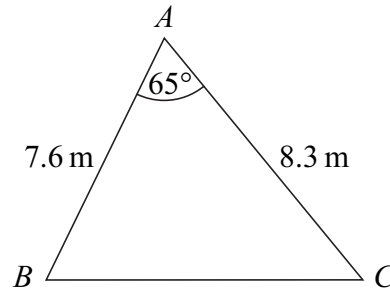
(a) Find the length of the arc PQ, giving your answer as a multiple of π. *(2 marks)*

(b) Find α in terms of π. *(2 marks)*

(c) Find the **perimeter** of the shaded segment, giving your answer to three significant figures. *(2 marks)*

3 A geometric series begins

$$20 + 16 + 12.8 + 10.24 + ...$$

   (a) Find the common ratio of the series. *(1 mark)*

   (b) Find the sum to infinity of the series. *(2 marks)*

   (c) Find the sum of the first 20 terms of the series, giving your answer to three decimal places. *(2 marks)*

   (d) Prove that the nth term of the series is 25×0.8^n. *(2 marks)*

4 The diagram shows a triangle ABC.

The size of angle BAC is 65°, and the lengths of AB and AC are 7.6 m and 8.3 m respectively.

   (a) Show that the length of BC is 8.56 m, correct to three significant figures. *(3 marks)*

   (b) Calculate the area of triangle ABC, giving your answer in m^2 to three significant figures. *(2 marks)*

   (c) The perpendicular from A to BC meets BC at the point D.

   Calculate the length of AD, giving your answer to the nearest 0.1 m. *(3 marks)*

5 (a) Write down the value of:

   (i) $\log_a 1$; *(1 mark)*

   (ii) $\log_a a$. *(1 mark)*

   (b) Given that

$$\log_a x = \log_a 5 + \log_a 6 - \log_a 1.5$$

   find the value of x. *(3 marks)*

6 The nth term of a sequence is u_n.

The sequence is defined by

$$u_{n+1} = pu_n + q$$

where p and q are constants.

The first three terms of the sequence are given by

$$u_1 = -8 \qquad u_2 = 8 \qquad u_3 = 4$$

(a)   Show that $q = 6$ and find the value of p.   *(5 marks)*

(b)   Find the value of u_4.   *(1 mark)*

(c)   The limit of u_n as n tends to infinity is L.

    (i)   Write down an equation for L.   *(1 mark)*

    (ii)   Hence find the value of L.   *(2 marks)*

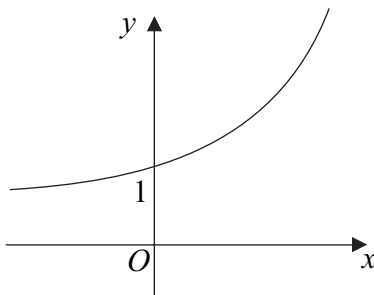**7**   (a)   The expression $\left(1 + \dfrac{4}{x^2}\right)^3$ can be written in the form

$$1 + \frac{p}{x^2} + \frac{q}{x^4} + \frac{64}{x^6}$$

By using the binomial expansion, or otherwise, find the values of the integers p and q.   *(3 marks)*

(b)   (i)   Hence find $\displaystyle\int \left(1 + \frac{4}{x^2}\right)^3 \, dx$.   *(4 marks)*

    (ii)   Hence find the value of $\displaystyle\int_1^2 \left(1 + \frac{4}{x^2}\right)^3 \, dx$.   *(2 marks)*

8 The diagram shows a sketch of the curve with equation $y = 6^x$.

(a) (i) Use the trapezium rule with five ordinates (four strips) to find an approximate value for $\int_0^2 6^x \, dx$, giving your answer to three significant figures. *(4 marks)*

(ii) Explain, with the aid of a diagram, whether your approximate value will be an overestimate or an underestimate of the true value of $\int_0^2 6^x \, dx$. *(2 marks)*

(b) (i) Describe a single geometrical transformation that maps the graph of $y = 6^x$ onto the graph of $y = 6^{3x}$. *(2 marks)*

(ii) The line $y = 84$ intersects the curve $y = 6^{3x}$ at the point A. By using logarithms, find the x-coordinate of A, giving your answer to three decimal places. *(4 marks)*

(c) The graph of $y = 6^x$ is translated by $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ to give the graph of the curve with equation $y = f(x)$. Write down an expression for $f(x)$. *(2 marks)*

9 (a) Solve the equation $\sin 2x = \sin 48°$, giving the values of x in the interval $0° \leqslant x < 360°$. *(4 marks)*

(b) Solve the equation $2 \sin \theta - 3 \cos \theta = 0$ in the interval $0° \leqslant \theta < 360°$, giving your answers to the nearest $0.1°$. *(4 marks)*

END OF QUESTIONS

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