

### **General Certificate of Education**

## Mathematics 6360

MPC1 Pure Core 1

# **Mark Scheme**

2009 examination - January series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

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#### Key to mark scheme and abbreviations used in marking

M	mark is for method			
m or dM	mark is dependent on one or more M marks and is for method			
A	mark is dependent on M or m marks and is for accuracy			
В	mark is independent of M or m marks and is for method and accuracy			
E	mark is for explanation			
√or ft or F	follow through from previous	MC		
	incorrect result	MC	mis-copy	
CAO	correct answer only	MR	mis-read	
CSO	correct solution only	RA	required accuracy	
AWFW	anything which falls within	FW	further work	
AWRT	anything which rounds to	ISW	ignore subsequent work	
ACF	any correct form	FIW	from incorrect work	
AG	answer given	BOD	given benefit of doubt	
SC	special case	WR	work replaced by candidate	
OE	or equivalent	FB	formulae book	
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme	
–x EE	deduct x marks for each error	G	graph	
NMS	no method shown	c	candidate	
PI	possibly implied	sf	significant figure(s)	
SCA	substantially correct approach	dp	decimal place(s)	

#### No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

### MPC1

MPC1 Q	Solution	Marks	Total	Comments
1(a)	M(3,2)	B1 B1	2	B1 for each coordinate
(b)	Gradient $AB = \frac{-2-6}{5-1} = \left(\frac{-8}{4}\right)$	M1		May use coords of $M$ instead of $A$ or $B$ -
	5-1 (4)	1411		condone one slip
	= -2	A1	2	CSO Answer must be simplified to -2
	1			
(c) (i)	Gradient of perpendicular = $\frac{1}{2}$	B1√		ft "their" -1/gradient AB
	$\Rightarrow y-2=\frac{1}{2}(x-3)$	M1		attempt at perp to $AB$ ; ft their $M$ coords
	2		2	
	$\Rightarrow 2y - 4 = x - 3 \Rightarrow x - 2y + 1 = 0 \text{ AG}$	A1	3	CSO Must write down the printed answer
(ii)	$k-2(k+5)+1=0$ or $\frac{(k+5)-2}{k-3}=\frac{1}{2}$			Sub into given line equation or correct
	$k-2(k+3)+1=0$ or $\frac{k-3}{k-3}=\frac{2}{2}$	M1		expression involving gradients Condone omission of brackets or use of <i>x</i>
				Condone offission of brackets of use of x
	$\Rightarrow k = -9$	A1	2	Condone $x = -9$
				(Full marks for correct answer without working)
	Total		9	0/
2(a)	(x-1)(2x-3)	B1	1	(1-x)(3-2x) or $2(x-1)(x-1.5)$ etc
_()	(x-1)(2x-3)			
a)	_			
(b)	Critical values are 1, $1\frac{1}{2}$	B1√		Correct or ft their factors from (a)
	Sign diagram or sketch	M1		+   -   +
	$\Rightarrow 1 < x < 1\frac{1}{2}$	A1	3	1 $1\frac{1}{2}$
				Full marks for correct inequality without working
				Working
	Total		4	
	7   5 2 5			$3-\sqrt{5}$ $\sqrt{5}-3$
3(a)	$\frac{7+\sqrt{5}}{3+\sqrt{5}} \times \frac{3-\sqrt{5}}{3-\sqrt{5}}$	M1		Multiply by $\frac{3-\sqrt{5}}{3-\sqrt{5}}$ or $\frac{\sqrt{5}-3}{\sqrt{5}-3}$
	31 43 3-43			
	Numerator = $21 + 3\sqrt{5} - 7\sqrt{5} - (\sqrt{5})^2$	m1		Condone one slip $16-4\sqrt{5}$
	Denominator = $9 - 5 = 4$	B1		( Or $5 - 9 = -4$ from other conjugate)
				, , , , , , , , , , , , , , , , , , , ,
	$Answer = 4 - \sqrt{5}$	A1	4	CSO
(b)	$\sqrt{45} = 3\sqrt{5}$	B1		
	V TJ — JV J	D1		
	$\frac{20}{\sqrt{5}} = \frac{20\sqrt{5}}{5}$	N # 1		May score if combined as one expression
	$\frac{1}{\sqrt{5}} = \frac{1}{5}$	M1		Must have 5 in denominator
	$Sum = 7\sqrt{5}$	A1	3	
	m . 1		-	
	Total		7	

MPC1 (cont)

Q Q	Solution	Marks	Total	Comments
4(a)(i)		B1		p=1
	+ 4	В1	2	q = 4
(ii)	$(x+1)^2 \geqslant 0 \Rightarrow (x+1)^2 + 4 > 0$	E1	1	Condone if they say $(x+1)^2$ positive
	$(x+1)^2 \ge 0 \Rightarrow (x+1)^2 + 4 > 0$ (\Rightarrow x^2 + 2x + 5 > 0 for all values of x)			and adding 4 so always positive
	$(\rightarrow x + 2x + 3 > 0)$ for all values of $x$ )			and adding 1 30 arways positive
(b)(i)	x = -1 or $y = 4$	M1		ft their $x = -p$ or $y = q$
	Minimum point is $(-1, 4)$	A1	2	
(ii)	\	B1		Sketch roughly as shown
	5	B1	2	y-intercept 5 or (0, 5) marked or stated
(c)	Translation (not shift, move etc)	E1		and NO other transformation stated
	through $\begin{bmatrix} -1\\4 \end{bmatrix}$ (or 1 left, 4 up etc)	M1		either component correct or ft their $-p$ , $q$
		<b>A</b> 1	3	correct translation M1, A1 independent of E mark
	Total		10	111, 111 independent of 2 mark
5(a)(i)	$\frac{\mathrm{d}x}{\mathrm{d}t} = 2t^3 - 40t + 66$	M1		one term correct
	GI.	<b>A</b> 1		another term correct
	12	A1	3	all correct unsimplified (no $+ c$ etc)
(ii)	$\frac{\mathrm{d}^2 x}{\mathrm{d} t^2} = 6t^2 - 40$	M1		ft one term correct
		A1√	2	ft all "correct", 2 terms equivalent
(b)	$\frac{\mathrm{d}x}{\mathrm{d}t} = 54 - 120 + 66$	M1		substitute $t = 3$ into their $\frac{dx}{dt}$
	$dt = 0 \Rightarrow \text{stationary value}$	A1		CSO dt
	= 0 → stationary value	Al		shown = 0 ( 54 or $2 \times 27$ seen ) and statement
	Substitute $t = 3$ into $\frac{d^2x}{dt^2}$ (= 14)	M1		
	$\frac{d^2x}{dt^2} > 0$ $\Rightarrow$ minimum value	<b>A</b> 1	4	CSO; all values (if stated) must be correct
(c)	Substitute $t = 1$ into their $\frac{dx}{dt}$	M1		must be their $\frac{dx}{dt}$ NOT $\frac{d^2x}{dt^2}$ etc
	$\frac{\mathrm{d}x}{\mathrm{d}t} = 28$	A1√	2	ft their $\frac{dx}{dt}$ when $t = 1$
(d)	Substitute $t = 2$ into their $\frac{dx}{dt}$	M1		must be their $\frac{dx}{dt}$ NOT $\frac{d^2x}{dt^2}$ or $x$
	=16-80+66=2  (>0)			Interpreting their value of $\frac{dx}{dt}$
	$\Rightarrow$ increasing when $t = 2$	E1√	2	Allow decreasing if their $\frac{dx}{dt} < 0$
	Total		13	

MPC1 (cont)

Q Q	Solution	Marks	Total	Comments
6(a)(i)	p(2) = 8 + 2 - 10	M1		Must find p(2) NOT long division
	$\Rightarrow$ p(2) = 0 $\Rightarrow$ (x-2) is factor	A1	2	Shown = 0 plus a statement
(ii)	Attempt at long division (generous)	M1		Obtaining a quotient $x^2 + cx + d$ or equating coefficients (full method)
	$p(x) = (x-2)(x^2 + 2x + 5)$	<b>A</b> 1	2	a = 2, $b = 5$ by inspection B1, B1
(b)(i)	$\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 1$	M1 A1		One term correct All correct – no + <i>c</i> etc
	When $x = 2 \frac{dy}{dx} = 3 \times 4 + 1$	m1		Sub $x = 2$ into their $\frac{dy}{dx}$
	Therefore gradient at $Q$ is 13	A1	4	CSO
(ii)	y = 13(x-2)	M1		Tangent (NOT normal) attempted ft their gradient answer from (b)(i)
		A1	2	CSO; correct in any form
(iii)	$\int \dots dx = \frac{x^4}{4} + \frac{x^2}{2} - 10x (+c)$	M1 A1 A1	3	one term correct second term correct all correct (condone no +c)
(iv)	[4+2-20]-[0] = -14	M1		F(2) attempted and possibly F(0) Must have earned M1 in (b)(iii)
	Area of shaded region = 14	A1	2	CSO; separate statement following correct evaluation of limits
	Total		15	

MPC1 (cont)

MPC1 (cont	Solution	Marks	Total	Comments
~	~VAWAVA	I LOSE IND		Commence
7(a)(i)	$(x-3)^2 + (y+5)^2$	B1		One term correct
		B1		LHS correct with + and squares
	$=25-9+9=25  (=5^2)$	B1	3	Condone RHS = 25
(b)(i)	C(3,-5)	B1√		
(ii)	Radius = 5	B1√	2	Correct or ft their RHS provided > 0
				1
(c)(i)	$(7-3)^2 + (-2+5)^2 = 16+9=25$			Or sub'n of $(7, -2)$ in original equation
(0)(1)	$\Rightarrow D \text{ lies on circle}$			, , ,
	Must see statement	B1	1	$7^2 + (-2)^2 - 42 - 20 + 9 = 0$
				Or sub $x=7$ into eqn & showing $y = -2$ etc
(ii)	Attempt at gradient of CD as normal	M1		withhold if subsequently uses $m_1 m_2 = -1$
	grad $CD = \frac{-2 - (-5)}{7 - 3} = \frac{3}{4}$			$\frac{\Delta y}{\Delta x}$ (condone one slip) FT their centre C
	7 3 1			$\Delta x$ (condone one sup) 1.1 then centre $C$
	$y+2 = \frac{3}{4}(x-7)$ or $y+5 = \frac{3}{4}(x-3)$	A1		Correct equation in any form $y = \frac{3}{4}x - \frac{29}{4}$
	$\Rightarrow 3x - 4y = 29$	A1	3	CSO Integer coefficients
	7 6.1	711	3	Condone $4y - 3x + 29 = 0$ etc
(d)(i)	y = kx sub'd into original circle equation	M1		or using their completed square form and
(-)()	$x^{2} + (kx)^{2} - 6x + 10kx + 9 = 0$			multiplying out
	$\Rightarrow (k^2 + 1)x^2 + 2(5k - 3)x + 9 = 0$ AG	A1	2	CSO
	, ()(		_	must see at least previous line for A1
				any error such as $kx^2 = = k^2x^2$ gets A0
(ii)	$4(5k-3)^2 - 36(k^2+1)$	M1		Discriminant in $k$ (can be seen in quad
(11)	4(3k-3) - 36(k+1)	1711		formula)
				Condone one slip
	$=64k^2-120k$	A1		or $8k^2 - 15k = 0$ OE
	Equal roots: $4(5k-3)^2 - 36(k^2+1) = 0$	B1		$b^2 - 4ac = 0$ clearly stated or evident by an equation in k with at most 2 slips.
				an equation in a with at most 2 sups.
	$8k^2 - 15k = 0$			
		m1		Attempt to solve <i>their</i> quadratic or linear equation if <i>k</i> has been cancelled
	$\Rightarrow k = 0,  k = \frac{15}{8}$	A1	5	OE but must have $k=0$
	$\rightarrow \kappa = 0,  \kappa = \frac{1}{8}$	Al	3	
				If "=0" is not seen but correct values of <i>k</i> are found, candidate will lose B1 mark but
				may earn all other marks
(iii)	(Line is a) <b>tangent</b> (to the circle)	E1	1	Line <b>touches</b> circle at one point
(111)	(Eine is a) tangent (to the chere)		1 	Ente touches enere at one point
	Total		17	
	TOTAL		75	