

General Certificate of Education (A-level) June 2012

Mathematics

MM2B

(Specification 6360)

Mechanics 2B

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
В	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
−x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Q	Solution	Marks	Total	Comments
1(a)	$KE = \frac{1}{2} \times 76 \times 28^2$	M1		All terms correct
	= 29 792 J = 29 800 J	A1	2	
(b)	Change in PE: $mgh = 76 \times 9.8 \times 31$ J = 23 088.8 J = 23 100 J	M1 A1	2	All terms correct
(c)(i)	KE when touches down on ground = 29 792 + 23 088.8J = 52 881 J	M1		Their values, one correct
	= 52900 J	A1	2	CAO
(ii)	Speed of Alan is $\sqrt{\frac{52881}{\frac{1}{2} \times 76}}$	M1		
	$= 37.304 \text{ m s}^{-1}$ = 37.3 m s^{-1}	A1	2	CAO
	- 37.3 m s	Al	2 8	CAO
2(a)(i)	$a = \frac{dv}{dt}$		<u> </u>	
	$dt = 12t + 8e^{-4t} \mathrm{m s}^{-2}$	M1A1	2	M1 for either term correct
(ii)	When $t = 0.5$, $a = 6 + 8 \times e^{-2}$ = 7.08 m s ⁻²	m1 A1	2	Condone 7.07 SC1 for 7.1 with no working
(b)	Using $F = ma$:			
	$F = 4 \times 7.08$ = 28.3 N	B1ft	1	Ft from value awarded A1
(c)	$r = \int v \mathrm{d}t$	M1		At least two terms correct
	$=2t^3 + \frac{1}{2}e^{-4t} + 8t + c$	A1		Does not need $+c$
	When $t = 0, r = 0 \rightarrow c = -\frac{1}{2}$	m1		Does not need $c = -\frac{1}{2}$
	$r = 2t^3 + \frac{1}{2}e^{-4t} + 8t - \frac{1}{2}$	A1	4	Need r , s (or words)
	Total		9	

MM2B O	Solution	Marks	Total	Comments
3(a)(i)	Moments about AB:	Maiks	Total	Comments
3(a)(1)	$1.6 \times 4 + 0.4 \times 8 = 2 \times x$	M1A1		M1 for 2 terms correct
	x = 4.8	1411711		THE TOT 2 CONTROL
	Distance is 4.8 cm	A1	3	
(ii)	Moments about <i>AD</i> :			
	$1.6 \times 6 + 0.4 \times 12 = 2 \times y$	M1A1		M1 for 2 terms correct
	y = 7.2	A 1	2	SC2+SC2 f ()(') 1()('') 1
	Distance is 7.2 cm	A1	3	SC2+SC2 for (a)(i) and (a)(ii) reversed
(b)	Moments about A:			
	$1.6g \times 6 + 0.4 g \times 12 = 12 \times T_B$	M1A1		M1 for 1 side of equation
	3 3			Or using above: moments about A
				$12 \times T_B = 7.2 \times 2g$ (ft for M marks)
	$T_B = 1.2g = 11.8 \text{ N}$	A1		
	Resolve vertically: $T_A + T_B = 2g$	M1	_	
	$T_A = 0.8g = 7.84 \text{ N}$	A1	5	1.2 and 0.8 is zero marks
				If 11.8 and 7.8 as final answer, must lose 1 mark somewhere
	Total		11	1 mark somewhere
4(a)	Distance of particle from the origin is			
	$\{(4\cos 3t)^2 + (4\sin 3t)^2\}^{\frac{1}{2}}$	M1		
	$\{(4\cos 3i) + (4\sin 3i)\}^2$	1711		
	= 4 which is a constant	A1	2	
	∴ particle is moving in a circle centre the	Al	2	
	origin			
	S			
(b)	$\mathbf{v} = \frac{d\mathbf{r}}{d\mathbf{r}}$			
(b)	$\mathrm{d}t$			
	$\mathbf{v} = -12\sin 3t\mathbf{i} - 12\cos 3t\mathbf{j}$	M1A1	2	M1 for either term correct
(c)	$\mathbf{a} = \frac{\mathrm{d}\mathbf{v}}{\mathbf{c}}$			
	$\mathrm{d}t$	3.61.4.1	2	MIC 31
	$\mathbf{a} = -36\cos 3t\mathbf{i} + 36\sin 3t\mathbf{j}$	M1A1	2	M1 for either term correct
(4)	$\mathbf{a} = -9 (4 \cos 3t \mathbf{i} - 4 \sin 3t \mathbf{j})$			
(u)	$\mathbf{a} = -9 \left(4 \cos 3t 1 - 4 \sin 3t \mathbf{j} \right)$ $= -9 \mathbf{r}$			
	k = -9	B2	2	B1 for 9
(e)	Acceleration is towards centre of circle	E1	1	
	(or origin)			
	Total		9	

5(a) For particle B, tension in string = 2.1 g N B1 Resolve horizontally for particle A: M1 Or $m_1\omega^2 r = m_2 g$ or $\frac{m_1v^2}{r} = m_2 g$ (condone lack of 1 and 2) $1.4\omega^2 \times 0.3 = 2.1 g$ $\omega^2 = 49$ A1 4 Angular velocity is 7 rad/sec A1 4 (b) Using $v = r\omega$: speed = 0.3×7 = 2.1 m s^{-1} M1 2 Part (b) marks can be awarded in (a) (c) Time taken is $2\pi/\omega$ M1 Or $\frac{2\pi r}{2.1}$ Accept $\frac{2\pi}{7}$ (0.895 M1A0) 6(a) Using conservation of energy: $\frac{1}{2}mv^2 = mgh$ M1 M1 for 2 or 3 terms, 1 KE and 1 or 2 PE $\frac{1}{2}mv^2 = mg 2.4(1 - \cos 18)$ = 2.302 v = 1.52 m s ⁻¹ A1 4 Condone 1.51 (b) Resolving vertically: T = $mg + \frac{mv^2}{a}$ A1 A1 4 Correct 3 terms Correct signs $\frac{1}{2}mv^2 = \frac{22x}{2.302}$ = $\frac{22x}{2.4}$ = $\frac{22x}{2.4}$ = $\frac{236.7 \times N}{2.4}$ A1 3	Q	Solution	Marks	Total	Comments
Resolve horizontally for particle A : $m\omega^2 r = T$ M1 $1.4\omega^2 \times 0.3 = 2.1g$ $\omega^2 = 49$ Angular velocity is 7 rad/sec A1 (b) Using $v = r\omega$: speed = 0.3×7 = 2.1 m s^{-1} A1 (c) Time taken is $2\pi/\omega$ $= \frac{2\pi}{7} = 0.898 \text{ sec}$ M1 Gauge on $2\pi r$ $= 2.898 \text{ sec}$ M1 Or $2\pi r$ $= 2.1 \text{ m s}^{-1}$ A1 Or $2\pi r$ $= 2.1 \text{ m s}^{-1}$ A1 Or $2\pi r$ $= 2.1 \text{ m s}^{-1}$ A1 Or $2\pi r$ $= 2.1 \text{ m s}^{-1}$ A1 Or $2\pi r$ $= 2.1 \text{ m s}^{-1}$ A1 Or $2\pi r$ $= 0.895 \text{ M1A0}$ Total 8 6(a) Using conservation of energy: $\frac{1}{2}mv^2 = mgh$ $\frac{1}{2}mv^2 = mg 2.4(1-\cos 18)$ $= 2.302$ $v = 1.52 \text{ m s}^{-1}$ A1 A1 A1 Condone 1.51 Correct 3 terms Correct 3 terms Correct 3 terms Correct signs	5(a)	For particle <i>B</i> ,			
$m\omega^{2}r = T \qquad M1 \qquad Or \ m_{i}\omega^{2}r = m_{2}g \text{ or } \frac{m_{i}v^{2}}{r} = m_{2}g $ $1.4\omega^{2} \times 0.3 = 2.1g \qquad A1 \qquad (condone lack of 1 and 2)$ $\omega^{2} = 49 \qquad Angular velocity is 7 rad/sec \qquad A1 \qquad 4$ $(b) Using \ v = r \omega: \text{ speed } = 0.3 \times 7 \qquad M1 \qquad 2 \qquad \text{Part (b) marks can be awarded in (a)}$ $(c) Time taken is 2\pi/\omega \qquad M1 \qquad Or \frac{2\pi r}{2.1} \qquad Accept \frac{2\pi}{7} = 0.898 \text{ sec} \qquad A1 \qquad 2 \qquad Accept \frac{2\pi}{7} \qquad (0.895 \text{ M1A0}) \frac{1}{2}mv^{2} = mgh \qquad M1 \qquad M1 \qquad M1 \text{ for 2 or 3 terms, 1 KE and 1 or 2 PE} \frac{1}{2}mv^{2} = mg2.4(1 - \cos 18) \qquad m1A1 \qquad m1A1 \text{ for finding } h v^{2} = 4.8g(1 - \cos 18) \qquad m1A1 \qquad m1A1 \text{ for finding } h v^{3} = 4.8g(1 - \cos 18) \qquad m1A1 \qquad Condone 1.51 (b) \text{Resolving vertically:} \qquad M1 \qquad A1 \qquad 4 \qquad Condone 1.51 (c) \text{Correct 3 terms} \qquad Correct 3 \text{ terms} \qquad Correct $		tension in string = $2.1g \text{ N}$	B1		
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		Resolve horizontally for particle <i>A</i> :			2
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$		$m\omega^2 r = T$	M1		
Angular velocity is 7 rad/sec (b) Using $v = r \omega$: speed = 0.3×7 = 2.1 m s^{-1} (c) Time taken is $2\pi / \omega$ $= \frac{2\pi}{7} = 0.898 \text{ sec}$ M1 Or $\frac{2\pi r}{2.1}$ $= \frac{2\pi}{7} = 0.898 \text{ sec}$ A1 Or $\frac{2\pi r}{2.1}$ $= \frac{2\pi}{7} = 0.898 \text{ sec}$ A1 Or $\frac{2\pi r}{2.1}$ $= \frac{2\pi}{7} = 0.898 \text{ sec}$ A1 Or $\frac{2\pi r}{2.1}$ $= \frac{2\pi}{7} = 0.898 \text{ sec}$ A1 Or $\frac{2\pi r}{2.1}$ $= \frac{2\pi}{7} = 0.898 \text{ sec}$ A1 Or $\frac{2\pi r}{2.1}$ $= \frac{2\pi}{7} = 0.898 \text{ sec}$ A1 Or $\frac{2\pi r}{2.1}$ $= \frac{2\pi}{7} = 0.898 \text{ sec}$ A1 Or $\frac{2\pi r}{2.1}$ $= \frac{\pi}{7} = 0.898 \text{ sec}$ A1 Or $\frac{2\pi r}{7} = 0.898 \text{ sec}$ A1			A1		(condone lack of 1 and 2)
			A1	4	
(c) Time taken is $2\pi / \omega$	(b)	speed = 0.3×7		2	Part (b) marks can be awarded in (a)
	(c)	Time taken is $2\pi / \omega$	M1		
G(a) Using conservation of energy: M1 M1 for 2 or 3 terms, 1 KE and 1 or 2 PE $\frac{1}{2}mv^2 = mgh$ m1A1 m1A1 for finding h $v^2 = 4.8g(1 - \cos 18)$ m1A1 m1A1 for finding h $v^2 = 4.8g(1 - \cos 18)$ and an example of the second		$=\frac{2\pi}{7}=0.898 \text{ sec}$	A1	2	Accept $\frac{2\pi}{7}$
6(a) Using conservation of energy: $\frac{1}{2}mv^2 = mgh$ M1 M1 for 2 or 3 terms, 1 KE and 1 or 2 PE $\frac{1}{2}mv^2 = mg2.4(1-\cos 18)$ m1A1 m1A1 for finding h w1 and 1 or 2 PE $\frac{1}{2}mv^2 = 4.8g(1-\cos 18)$ m1A1 for finding h Condone 1.51 (b) Resolving vertically: $T = mg + \frac{mv^2}{a}$ A1 Correct 3 terms $\frac{1}{2}mv^2 = \frac{1}{2}mv^2 = \frac{1}{2$		Total		8	(0.050 1411110)
$ \frac{1}{2}mv^{2} = mgh $ $ \frac{1}{2}mv^{2} = mg \cdot 2.4(1 - \cos 18) $ $ v^{2} = 4.8g(1 - \cos 18) $ $ = 2.302 $ $ v = 1.52 \text{ m s}^{-1} $ (b) Resolving vertically: $T = mg + \frac{mv^{2}}{a} $ $ = 22g + \frac{22 \times 2.302}{2.4} $ M1 for 2 or 3 terms, 1 KE and 1 or 2 PE m1A1 M1 for 2 or 3 terms, 1 KE and 1 or 2 PE m1A1 Condone 1.51 Correct 3 terms Correct 3 terms Correct signs	6(a)				
$v^{2} = 4.8g(1 - \cos 18)$ $= 2.302$ $v = 1.52 \text{ m s}^{-1}$ A1 $V = mg + \frac{mv^{2}}{a}$ $= 22g + \frac{22 \times 2.302}{2.4}$ A1 $V = mg + \frac{mv^{2}}{a}$ $= 22g + \frac{22 \times 2.302}{2.4}$ Condone 1.51 $V = mg + \frac{mv^{2}}{a}$ $V = mg + m$		4	M1		M1 for 2 or 3 terms, 1 KE and 1 or 2 PE
		$\frac{1}{2}mv^2 = mg 2.4(1-\cos 18)$	m1A1		m1A1 for finding h
$T = mg + \frac{mv^2}{a}$ $= 22g + \frac{22 \times 2.302}{2.4}$ M1 A1 Correct 3 terms Correct signs		= 2.302	A1	4	Condone 1.51
$T = mg + \frac{mv^2}{a}$ $= 22g + \frac{22 \times 2.302}{2.4}$ M1 A1 Correct 3 terms Correct signs	(b)	Resolving vertically:			
		$T = mg + \frac{mv^2}{a}$			
		$= 22g + \frac{22 \times 2.302}{2.4}$			
= 237 N Accept 236 N		= 236.7 N	A1	3	Accept 236 N
Total 7				7	1100pt 250 11

Q	Solution	Marks	Total	Comments
7(a)	Using $F = ma$:			
	$m\frac{dv}{dt} = 49 - 9.8v$ or $5g - 9.8v$	M1		Need to see $m \frac{dv}{dt}$ or $5 \frac{dv}{dt}$ or $a = \frac{49 - 9.81}{5}$
	$\therefore \frac{\mathrm{d}v}{\mathrm{d}t} = -1.96 (v-5)$	A1	2	Must see m terms (not $a =$)
(b)	$\int \frac{dv}{v-5} = -1.96 \int dt$ $\ln(v-5) = -1.96t + c$ When $t = 0$, $v = 7 \implies c = \ln 2$	M1		And one side integrated
	$\ln{(v-5)} = -1.96t + c$	A1A1		Need $+ c$, A1 each side
	When $t = 0$, $v = 7 \implies c = \ln 2$	A 1		OE
	$ \ln \frac{v-5}{2} = -1.96t $			
	$\ln \frac{v-5}{2} = -1.96t$ $\frac{v-5}{2} = e^{-1.96t}$ $v = 5 + 2e^{-1.96t}$			
	$v = 5 + 2e^{-1.96t}$	A1	5	CAO
	Total		7	

Q	Solution	Marks	Total	Comments
8(a)	Initial EPE = $\frac{\lambda x^2}{2I}$			
	21	2.64		
	$=\frac{120\times(0.5)^2}{2\times5}$	M1		M1 for formula with extension 0.5
	= 3 J	A1		
	Initial KE is $\frac{1}{2} \times 0.4 \times 9^2 = 16.2 \text{ J}$			
	When block is at <i>A</i> , $\frac{1}{2}mv^2 = 3 + 16.2$	M1		
	$v^2 = 19.2 \div 0.2 = 96$	1711		
	$V = 19.2 \cdot 0.2 = 90$ Speed is 9.80 m s ⁻¹	A1	4	Accept $4\sqrt{6}$; condone 9.79
	Speed 10 3100 III 2		·	The first of the second of the
(b)(i)	Normal reaction is $mg = 0.4g$	M1 A1		
	Frictional force is $0.4\mu g$ N	Aı		
	Work done by frictional force is	1		
	$5.5 \times (0.4 \mu g)$ or $2.2 \mu g$	m1		
	C of Energy, when at A, gives	M1		Three terms as initial energy in (a)
	$19.2 - 5.5 \times (0.4\mu g) = \frac{1}{2} \times 0.4 \times v^2$	IVI 1		Three terms, eg initial energy in (a) (=3 or 19.2); work done; KE at A.
	$10.2 2.2 -0.2 \cdot ^{2}$	A1		Fully correct
	$19.2 - 2.2 \mu g = 0.2v^2$ $v = \sqrt{96 - 11\mu g}$	A 1	6	Ft $v = \sqrt{(v^2 \operatorname{in}(a)) - 11\mu g}$
	γ γ/ο 11μ8	A1	U	$\int V = \sqrt{(V \operatorname{III}(a))} = 11 \mu g$
(ii)	Speed when rebounding is $\frac{1}{2}\sqrt{96-11\mu g}$	B1ft		
	Block is stationary at B			
	$\frac{1}{2} \times 0.4 \times \frac{1}{4} (96 - 11 \mu g) - 2.2 \mu g$	M1		Three terms
	$=\frac{120\times(0.5)^2}{2.5}$	A1		Two terms correct with sign
	$=\frac{2\times5}{2\times5}$	A1		Third term correct with sign
	$\frac{1}{2} \times 0.1(96 - 11\mu g) - 2.2\mu g = 3$			
	$4.8 - 2.75 \mu g = 3$	A1		Or $4.8 - 0.55 \mu g - 2.2 \mu g = 3$
	$\mu = 0.0668$ Total	A1	6 16	
	TOTAL		75	