

General Certificate of Education

Mathematics 6360

MM2B Mechanics 2B

Mark Scheme

2010 examination - January series

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Key to mark scheme and abbreviations used in marking

| M | mark is for method | | | | | |
|------------|--|-----|----------------------------|--|--|--|
| m or dM | mark is dependent on one or more M marks and is for method | | | | | |
| A | mark is dependent on M or m marks and is for accuracy | | | | | |
| В | mark is independent of M or m marks and is for method and accuracy | | | | | |
| E | mark is for explanation | | | | | |
| | | | | | | |
| or ft or F | follow through from previous | | | | | |
| | incorrect result | MC | mis-copy | | | |
| CAO | correct answer only | MR | mis-read | | | |
| CSO | correct solution only RA required accuracy | | | | | |
| AWFW | anything which falls within FW further work | | | | | |
| AWRT | anything which rounds to | ISW | ignore subsequent work | | | |
| ACF | any correct form | FIW | from incorrect work | | | |
| AG | answer given | BOD | given benefit of doubt | | | |
| SC | special case | WR | work replaced by candidate | | | |
| OE | or equivalent | FB | formulae book | | | |
| A2,1 | 2 or 1 (or 0) accuracy marks | NOS | not on scheme | | | |
| –x EE | deduct x marks for each error | G | graph | | | |
| NMS | no method shown | c | candidate | | | |
| PI | possibly implied | sf | significant figure(s) | | | |
| SCA | substantially correct approach | dp | decimal place(s) | | | |

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MM2B

| MM2B | 0.1.4 | 34 1 | TD 4 3 | |
|------|--|------------|--------|--|
| Q | Solution | Marks | Total | Comments |
| 1 | Work done = $Fs \cos \theta$ | M1 | | Accept $Fs \sin \theta$ for M1 |
| | $=40\times5\times\cos30$ | A1 | | |
| | = 173 J | A1 | 3 | |
| | Total | | 3 | |
| 2 | $\overline{X} = \frac{3 \times 15 + 1 \times 7 + 6 \times 8 + 10 \times 12}{3 + 1 + 6 + 10}$ | M1A1 | | M1 for at least 3 multiplication & addition |
| | $=\frac{220}{20}$ or 11 | A1 | | |
| | $\overline{Y} = \frac{3 \times 6 + 1 \times 14 + 6 \times 7 + 10 \times 9}{20}$ | M1A1 | | |
| | $=\frac{164}{20}$ or 8.2 | A1 | 6 | SC 4 (10, 7.4) [omit lamina] ie: B2, B2 |
| | ∴ Centre of mass is at (11, 8.2) | | | |
| | Total | | 6 | |
| 3(a) | R_A $20g$ B $30g$ | B2 | 2 | B1 for four forces B2 for two different reactions and 30g and 20g marked |
| (b) | Taking moments about A: $3.2 \times 30g = R_B \times 5$ $R_B = 19.2g$ | M1B1 A1 | 3 | B1 for 3.2 AG |
| (c) | Resolve vertically: $R_A + R_B = 50g$ $R_A = 30.8g$ or 302 N | M1 A1 | 2 | Can be awarded in (b) |
| (d) | Gravitational force acts through mid-point of the rod | E1 | 1 | |
| | Total | | 8 | |

| VIMIZB (con | Solution | Marks | Total | Comments |
|-------------|---|------------|-------|---|
| 4(a) | $\mathbf{r} = \int \mathbf{v} \mathrm{d}t$ | M1 | | M1 for at least one term correct |
| | $= (t^4 - 6t^2 + 3t)\mathbf{i} + 5t\mathbf{j} + 4t^2\mathbf{k} + \mathbf{c}$ | A1m1 | | $m1 \text{ for } + \mathbf{c}$ |
| | When $t=0$, $\mathbf{r}=-5\mathbf{i}+6\mathbf{k}$:: $\mathbf{c}=-5\mathbf{i}+6\mathbf{k}$ | | | |
| | $\therefore \mathbf{r} = (t^4 - 6t^2 + 3t - 5)\mathbf{i} + 5t\mathbf{j} + (6 + 4t^2)\mathbf{k}$ | A1 | 4 | |
| (b) | $\mathbf{a} = (12t^2 - 12)\mathbf{i} + 8\mathbf{k}$ | M1A1 | 2 | M1 for either component |
| (c) | Magnitude is $\{(12t^2 - 12)^2 + 64\}^{\frac{1}{2}}$ | M1 A1F | 2 | |
| (d) | Magnitude is a minimum when $12t^2 - 12$ | M1 | | M1 for correct differentiation of correct |
| | is zero ie when $t = 1$ | A 1 | 2 | expression in (c) |
| (e) | Minimum acceleration is 8 | | | |
| | Using $F = ma$, | M1 | | a could be a vector |
| | $F = 7 \times 8 = 56$ | A1 | 2 | CAO |
| | Total | | 12 | |

| Q Q | Solution | Marks | Total | Comments |
|------|--|-------------|----------|--|
| 5(a) | Using $F = ma$, | 1.2002 22.5 | 2 0 0001 | |
| | $-0.2mv^{\frac{1}{2}} = m\frac{dv}{dt}$ $\therefore \frac{dv}{dt} = -0.2v^{\frac{1}{2}}$ | B1 | 1 | AG Must see equ'n containing m |
| (b) | $\int \frac{\mathrm{d}v}{v^{\frac{1}{2}}} = -\int 0.2 \mathrm{d}t$ $2v^{\frac{1}{2}} = -0.2t + c$ | M1 | | |
| | $2v^{\frac{1}{2}} = -0.2t + c$ | Alml | | $m1 	ext{ for } + c$ |
| | When $t=0, v=16 :: C = 8$ | A1 | | |
| | $2v^{\frac{1}{2}} = -0.2t + 8$ $v = (4 - 0.1t)^{2}$ | A1 | 5 | AG |
| (c) | When $v = 1$, $1 = (4 - 0.1t)^2$ $4 - 0.1t = \pm 1$ | M1 | | |
| | t = 30 or 50 | A1 | | if use $2v^{\frac{1}{2}} = 8 - 0.2t$ no need to see 50 |
| | t=30 | A 1 | 3 | $t \neq 50$ as ball stops when $t = 40$ |
| (d) | Integrating $v = (4 - 0.1t)^2$: $v = 16 - 0.8t + 0.01t^2$ | | | |
| | $x = 16t - 0.4t^2 + \frac{0.01}{3}t^3 + d$ | M1 | | M1 for first 3 terms or $-\frac{10}{3} (4-01t)^3$ |
| | When $t=0$, $x=0 \Rightarrow d=0$ | A1 | | |
| | $x = 16t - 0.4t^2 + \frac{0.01}{3}t^3$ | | | |
| | When speed is 1ms^{-1} , $t = 30$ | | | |
| | x = 480 - 360 + 90 | m1 | 1 | dep on M1 above |
| | = 210 Total | A1 | 13 | [No 'd', 3 marks only] |

| Q Q | Solution | Marks | Total | Comments |
|------|---|-------|-------|--|
| 6(a) | $r = 1.2 \sin \theta$ | B1 | 1 | $1.2 \cos \theta $ 0 marks |
| (b) | Resolve horiz: $T \sin \theta = m\omega^2 r$ | M1A1 | | $T\cos\theta = m\omega^2 r \text{ etc M1 (+ second M1)}$ |
| | $T\sin\theta = 4 \times 5^2 \times 1.2\sin\theta$ $T = 120$ | A1 | | |
| | Resolve vert: $T\cos\theta = 4g$ | M1A1 | | |
| | $\cos\theta = 0.32666$ | | | M1 for $\tan \theta = \frac{30 \sin \theta}{g}$ |
| | $\theta = 70.9^{\circ} \text{ or } 1.24^{\circ}$ | A1 | 6 | |
| | Total | | 7 | |
| 7(a) | Using conservation of energy: | | | NU C O O O O O O O O O O O O O O O O O O |
| | $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 - mgh$ | M1A1 | | M1 for 3 terms, 2 KE and PE or 4 terms, 2 KE and 2 PE |
| | $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 - mga(1 - \cos\theta)$ | M1A1 | | M1A1 for finding <i>h</i> |
| | $v^2 = u^2 + 2ga(1 - \cos\theta)$ | | | |
| | $v^{2} = u^{2} + 2ga(1 - \cos\theta)$ $v = (u^{2} + 2ga[1 - \cos\theta])^{\frac{1}{2}}$ | A1 | 5 | AG |
| (b) | Using $F = ma$ radially, | | | |
| | $mg\cos\theta - N = \frac{mv^2}{a}$ | M1A1 | | M1 Correct 3 terms A1 Correct signs (-N or +N) |
| | Particle leaves surface of hemisphere when $N = 0$ | B1 | | |
| | $mg\cos\theta = \frac{m}{a}(u^2 + 2ga[1 - \cos\theta])$ | M1 | | |
| | $\cos\theta = \frac{u^2}{ga} + 2 - 2\cos\theta$ | | | |
| | $\cos\theta = \frac{1}{3} \left(\frac{u^2}{ga} + 2 \right)$ | A1 | 5 | |
| | Total | | 10 | |

| Q Q | Solution | Marks | Total | Comments |
|--------|---|------------|-------|---|
| 8(a) | When $x \ge 22$, KE is $\frac{1}{2} \times 49 \times v^2$ | | | |
| | EPE is $\frac{1078(x-22)^2}{2\times 22}$ Change in PE is $49 \times g \times x$ | M1A1 | | M1 for any $\frac{1078p^2}{2\times22}$ |
| | Conservation of energy: $\frac{1}{2} \times 49 \times v^2 + \frac{1078(x-22)^2}{2 \times 22} = 49 \times g \times x$ $\frac{49}{2} v^2 + \frac{49}{2} (x-22)^2 = 49gx$ | M1A1 A1 | | M1 3 terms (KE, PE, EPE) A1 2 terms correct A1 all 3 terms correct |
| | $v^{2} + (x - 22)^{2} = 19.6x$ | | | SC3 $\frac{49}{2}v^2 + \frac{49}{2}e^2 = 49g(e+22)$ [could use x for e] |
| | $5v^2 = 318x - 5x^2 - 2420$ | A 1 | 6 | AG |
| (b) | If x is not greater than 22, cord is not stretched | В1 | 1 | |
| (c) | At maximum value of x , $v = 0$ $\therefore 5x^2 - 318x + 2420 = 0$ | M1 | | |
| | $x = \frac{318 \pm \sqrt{318^2 - 4 \times 5 \times 2420}}{2 \times 5}$ | m1 | | dep on M1 above |
| | x = 54.76 or 8.84 = 54.8 | A1 E1 | 4 | A1 for either solution Needs to give a reason for deletion of second root. Both roots must be positive: one above 22, one below 22 |
| (d)(i) | When speed is a maximum, $a = 0$ tension = gravitational force | M1 | | $\frac{d(5v^2)}{dx} = 318 - 10x$ |
| | $\frac{1078(x-22)}{22} = 49g$ | A1 | | $= 0$ at maximum speed $\Rightarrow 318 - 10x = 0$ |
| | x - 22 = 9.8 x = 31.8 | A 1 | 3 | AG |
| (ii) | From part (a), $v^2 = 19.6 \times 31.8 - 9.8^2$ v = 22.96 Maximum speed is $23.0 \mathrm{ms}^{-1}$ | M1 A1 | 2 | |
| | Total | | 16 | |
| | TOTAL | | 75 | |