



General Certificate of Education (A-level)
June 2012

Mathematics

MFP3

(Specification 6360)

Further Pure 3

Mark Scheme

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
ff or ft or F	follow through from previous incorrect result
CAO	correct answer only CSO
	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
-x EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

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Q	Solution	Marks	Total	Comments
1	$k_1 = 0.25 \times (\sqrt{2 \times 2} + \sqrt{9}) \quad (=1.25)$ $k_2 = 0.25 \text{ f } (2.25, 9 + 1.25)$ $k_2 = 0.25 \times (\sqrt{2 \times 2.25} + \sqrt{9 + 1.25})$ $k_2 = 1.33(072...)$ $y(2.25) = y(2) + \frac{1}{2}[k_1 + k_2]$ $= 9 + 0.5 [1.25 + 1.33(072...)]$ $= 9 + 0.5 \times 2.58(072...)$ $y(2.25) = 10.29036... = 10.29 \text{ (to 2 dp)}$	M1 M1 A1 m1 A1	5	PI. May see within given formula Either $k_2 = 0.25 \text{ f } (2.25, 10.25)$ stated/used or $k_2 = 0.25 \times (\sqrt{2 \times 2.25} + \sqrt{9 + \text{c's } k_1})$ PI. May see within given formula $k_2 = 1.33(072...)$ 2 dp or better PI by later work Dep on previous two Ms and $y(2) = 9$ and numerical values for k 's CAO Must be 10.29
	Total		5	
2(a)	$\sin 2x = 2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} \dots$ $= 2x - \frac{4}{3}x^3 + \frac{4}{15}x^5$	B1	1	Accept ACF even if unsimplified
(b)	$\lim_{x \rightarrow 0} \left[\frac{2x - \sin 2x}{x^2 \ln(1 + kx)} \right]$ $= \lim_{x \rightarrow 0} \frac{2x - (2x - \frac{4}{3}x^3 + \frac{4}{15}x^5 \dots)}{x^2 \left(kx - \frac{(kx)^2}{2} + \dots \right)}$ $= \lim_{x \rightarrow 0} \left[\frac{\frac{4}{3}x^3 - \frac{4}{15}x^5 + \dots}{kx^3 - \frac{k^2}{2}x^4} \right]$ $= \lim_{x \rightarrow 0} \left[\frac{\frac{4}{3} - O(x^2)}{\frac{3}{k} - O(x)} \right]$ $\frac{4}{3k} = 16 \Rightarrow k = \frac{1}{12}$	M1 B1 m1 A1	4	Using series expansions. Expansion of $\ln(1 + kx) = kx - \dots$ Dividing numerator and denominator by x^3 to get constant term in each. Must be at least a total of 3 terms divided by x^3 OE exact value. Dep on numerator being of form $4/3(\text{OE}) + \lambda x^2 \dots (\lambda \neq 0)$ and denominator being of form $k + \mu x \dots (\mu \neq 0)$ before limit taken
	Total		5	

Q	Solution	Marks	Total	Comments
3	$\text{Area} = \frac{1}{2} \int (2\sqrt{1+\tan\theta})^2 (d\theta)$ $= \frac{1}{2} \int_{-\frac{\pi}{4}}^0 4(1+\tan\theta) d\theta$ $= 2 \left[\theta + \ln \sec\theta \right]_{-\frac{\pi}{4}}^0$ $= 2 \left\{ 0 - \left[-\frac{\pi}{4} + \ln \sec\left(-\frac{\pi}{4}\right) \right] \right\}$ $= 2 \left(\frac{\pi}{4} - \ln \sqrt{2} \right) = \frac{\pi}{2} - 2 \ln \sqrt{2} = \frac{\pi}{2} - \ln 2$	M1 B1 B1 A1	4	Use of $\frac{1}{2} \int r^2 (d\theta)$ Correct limits. If any contradiction use the limits at the substitution stage $\int k(1+\tan\theta)(d\theta) = k(\theta + \ln \sec\theta)$ ACF ft on c's k CSO AG
Total			4	
4(a)	IF is $e^{\int \frac{4}{2x+1} dx}$ $e^{2 \ln(2x+1) (+c)} = e^{\ln(2x+1)^2 (+c)}$ $= (A)(2x+1)^2$ $(2x+1)^2 \frac{dy}{dx} + 4(2x+1)y = 4(2x+1)^7$ $\frac{d}{dx}[(2x+1)^2 y] = 4(2x+1)^7$ $(2x+1)^2 y = \int 4(2x+1)^7 dx$ $(2x+1)^2 y = \frac{1}{4}(2x+1)^8 (+c)$ (GS): $y = \frac{1}{4}(2x+1)^6 + c(2x+1)^{-2}$	M1 A1 A1F M1 A1 B1F A1	7	PI Either O.E. Condone missing '+ c' Ft on earlier $e^{\lambda \ln(2x+1)}$, condone missing 'A' LHS as d/dx ($y \times c$'s IF) PI and also RHS of form $p(2x+1)^q$ Correct integration of $p(2x+1)^q$ to $\frac{p(2x+1)^{q+1}}{2(q+1)} (+c)$ ft for $q \neq -2$ only Must be in the form $y = f(x)$, where $f(x)$ is ACF
(b)	$y = \frac{1}{4}(2x+1)^6 + c(2x+1)^{-2}$ When $x = 0$, $\frac{dy}{dx} = 0$ $\Rightarrow y = 1 \left[\frac{dy}{dx} = 3(2x+1)^5 - 4c(2x+1)^{-3} \right]$ $\Rightarrow c = \frac{3}{4}$ so $y = \frac{1}{4}(2x+1)^6 + \frac{3}{4}(2x+1)^{-2}$	M1 B1 A1	3	Using boundary condition $x = 0$, $\frac{dy}{dx} = 0$ and c's GS in (a) towards obtaining a value for c Either $y = 1$ or correct expression for dy/dx in terms of x only CSO
Total			10	

Q	Solution	Marks	Total	Comments
5(a)	$\int x^2 e^{-x} dx = -x^2 e^{-x} - \int -2xe^{-x} dx$ $= -x^2 e^{-x} + 2\{-xe^{-x} - \int -e^{-x} dx\}$ $= -x^2 e^{-x} - 2xe^{-x} - 2e^{-x} (+c)$	M1 A1 m1 A1	4	$kx^2 e^{-x} - \int 2kx e^{-x} (dx)$ for $k = \pm 1$ $\int x e^{-x} dx = \lambda x e^{-x} - \int \lambda e^{-x} (dx)$ for $\lambda = \pm 1$ in 2nd application of integration by parts Condone absence of $+c$
(b)	$I = \int_0^{\infty} x^2 e^{-x} dx = \lim_{a \rightarrow \infty} \int_0^a x^2 e^{-x} dx$ $\lim_{a \rightarrow \infty} \{-a^2 e^{-a} - 2ae^{-a} - 2e^{-a}\} - [-2]$ $\lim_{a \rightarrow \infty} a^k e^{-a} = 0, \quad (k > 0)$ $\int_0^{\infty} x^2 e^{-x} dx = 2$	M1 E1 A1	2	$F(a) - F(0)$ with an indication of limit ' $a \rightarrow \infty$ ' and $F(x)$ containing at least one $x^n e^{-x}, n > 0$ term For general statement or specific statement for either $k = 1$ or $k = 2$ CSO
Total				
6(a)	$y = \ln(1 + \sin x), \quad \frac{dy}{dx} = \frac{1}{1 + \sin x} \times (\cos x)$	M1 A1	2	Chain rule OE ACF eg $e^{-y} \cos x$
(b)	$\left(\frac{d^2 y}{dx^2}\right) = \frac{(1 + \sin x)(-\sin x) - \cos x(\cos x)}{(1 + \sin x)^2}$ $\frac{d^2 y}{dx^2} = \frac{-\sin x - 1}{(1 + \sin x)^2} = \frac{-1}{1 + \sin x} = \frac{-1}{e^y} = -e^{-y}$	M1 A1 A1	3	Quotient rule OE, with u and v non constant ACF CSO AG Completion must be convincing
(c)	$\frac{d^3 y}{dx^3} = e^{-y} \frac{dy}{dx}$ $\frac{d^4 y}{dx^4} = -e^{-y} \left(\frac{dy}{dx}\right)^2 + e^{-y} \frac{d^2 y}{dx^2}$ $\frac{d^4 y}{dx^4} = -e^{-y} \left(\frac{dy}{dx}\right)^2 - (e^{-y})^2$	B1 M1 A1	3	ACF for $\frac{d^3 y}{dx^3}$ Product rule OE and chain rule OE in terms of e^{-y} and $\frac{dy}{dx}$ only
(d)	$y(0) = 0; y'(0) = 1; y''(0) = -1;$ $y(x) \approx$ $y(0) + xy'(0) + \frac{x^2}{2} y''(0) + \frac{x^3}{3!} y'''(0) + \frac{x^4}{4!} y^{(iv)}(0)$ $y'''(0) = 1; y^{(iv)}(0) = -2$ $\ln(1 + \sin x) \approx x - \frac{1}{2}x^2 + \frac{1}{6}x^3 - \frac{1}{12}x^4 \dots$	B1F M1 A1	3	Ft only for $y'(0)$; other two values must be correct Maclaurin's theorem applied with numerical values for $y'(0), y''(0), y'''(0)$ and $y^{(iv)}(0)$. M0 if missing an expression for any one of the 1 st , 3 rd or 4 th derivatives A0 if FIW
Total			11	

Q	Solution	Marks	Total	Comments
7(a)	$\frac{dx}{dt} \frac{dy}{dx} = \frac{dy}{dt}$ $e^t \frac{dy}{dx} = \frac{dy}{dt} \Rightarrow x \frac{dy}{dx} = \frac{dy}{dt}$ $\frac{d}{dt} \left(x \frac{dy}{dx} \right) = \frac{d^2 y}{dt^2}; \frac{dx}{dt} \frac{d}{dx} \left(x \frac{dy}{dx} \right) = \frac{d^2 y}{dt^2}$ $\frac{dx}{dt} \left(\frac{dy}{dx} + x \frac{d^2 y}{dx^2} \right) = \frac{d^2 y}{dt^2}$ $x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} = \frac{d^2 y}{dt^2}$ $x^2 \frac{d^2 y}{dx^2} - 4x \frac{dy}{dx} + 6y = 3 + 20 \sin(\ln x)$ becomes $\frac{d^2 y}{dt^2} - x \frac{dy}{dx} - 4x \frac{dy}{dx} + 6y = 3 + 20 \sin(\ln x)$ $\Rightarrow \frac{d^2 y}{dt^2} - 5 \frac{dy}{dt} + 6y = 3 + 20 \sin(\ln e^t)$ $\Rightarrow \frac{d^2 y}{dt^2} - 5 \frac{dy}{dt} + 6y = 3 + 20 \sin t$	M1 A1 M1 m1 A1 m1 A1	7	OE Relevant chain rule eg $\frac{dy}{dx} = \frac{dt}{dx} \frac{dy}{dt}$ OE eg $\frac{dy}{dx} = e^{-t} \frac{dy}{dt}$ OE. Valid 1st stage to differentiate $x y'(x)$ oe with respect to t or to differentiate $x^{-1} y'(t)$ oe with respect to x. Product rule (dep on previous M) OE eg $\frac{d^2 y}{dx^2} = e^{-t} \left[-e^{-t} \frac{dy}{dt} + e^{-t} \frac{d^2 y}{dx^2} \right]$ {Note: e^{-t} could be replaced by $1/x$} Substitution to reach a 'one-step away' stage for LHS. Dep on previous M M m CSO AG
(b)	Auxl eqn $m^2 - 5m + 6 = 0$ $(m - 2)(m - 3) = 0, m = 2, 3$ CF: $(y_c) = Ae^{2t} + Be^{3t}$ P.Int. Try $(y_p) = a + b \sin t + c \cos t$ $(y'(t)) = b \cos t - c \sin t$ $(y''(t)) = -b \sin t - c \cos t$ Substitute into DE gives $a = 0.5$ $5c + 5b = 20$ and $5c - 5b = 0$ $b = c = 2$ GS $(y) = Ae^{2t} + Be^{3t} + 2 \sin t + 2 \cos t + \frac{1}{2}$	M1 A1 A1F M1 A1 A1F M1 B1 A1 A1 B1F	11	PI Ft wrong values of m provided 2 real roots, and 2 arb. constants in CF. Condone x for t here Condone 'a' missing here ft can be consistent sign error(s) Substitution and comparing coefficients at least once OE Ft on c's CF + PI, provided PI is non-zero and CF has two arbitrary constants and RHS is fn of t only
(c)	$y = Ax^2 + Bx^3 + 2 \sin(\ln x) + 2 \cos(\ln x) + 0.5$	B1	1	CAO
Total			19	

Q	Solution	Marks	Total	Comments
8(a)	$xy = 8 \Rightarrow r \cos \theta \ r \sin \theta = 8$	M1	3	Use of $\sin 2\theta = 2 \sin \theta \cos \theta$ AG Completion
	$\frac{1}{2} r^2 \sin 2\theta = 8$	m1		
	$r^2 = \frac{16}{\sin 2\theta} = 16 \operatorname{cosec} 2\theta$	A1		
	(b)(i) (At N, r is a minimum $\Rightarrow \sin 2\theta = 1$)	B1B1	2	B1 for each correct coordinate.
	$N\left(4, \frac{\pi}{4}\right)$			
	(ii) At pts of intersection,	M1	4	PI by $\operatorname{cosec} 2\theta = 2$ and a correct exact or 3SF value for 2θ or θ PI OE exact values Both required, written in correct order
	$(4\sqrt{2})^2 = 16 \operatorname{cosec} 2\theta$	A1		
	$\sin 2\theta = \frac{1}{2}$	A1		
	$2\theta = \frac{\pi}{6}, \frac{5\pi}{6}$	A1		
	$\left(4\sqrt{2}, \frac{\pi}{12}\right) \left(4\sqrt{2}, \frac{5\pi}{12}\right)$	A1		
	(iii) $\angle POQ = \frac{5\pi}{12} - \frac{\pi}{12} = \frac{\pi}{3}$	B1F		Ft on c's $\theta_P, \theta_Q, \theta_N$ as appropriate OE
	or $\angle PON = \frac{\pi}{6} (= \angle QON)$			
	$PN^2 = (4\sqrt{2})^2 + (r_N)^2 - 2(4\sqrt{2}) r_N \cos\left(\frac{1}{2}POQ\right)$	M1		Finding the lengths of two unequal sides of $\triangle PNQ$ or $\triangle PNT$ or $\triangle QNT$, where T is the point at which ON produced meets PQ . Any valid equivalent methods eg finding $\tan \angle OPN$ or finding $\sin \angle ONP$.
	or $PT = 4\sqrt{2} \sin\left(\frac{1}{2}POQ\right)$			
	or $PT = \frac{1}{2} \times 4\sqrt{2}$	A1		Two correct unequal lengths of sides of $\triangle PNQ$ or $\triangle PNT$ or $\triangle QNT$ PI OE eg $\tan \angle OPN = 1/(2\sqrt{2} - \sqrt{3})$ or $\sin \angle ONP = 2\sqrt{2}/(\sqrt{48 - 16\sqrt{6}})$
	or $NT = 4\sqrt{2} \cos\left(\frac{1}{2}POQ\right) - r_N$			
	$PN = \sqrt{(48 - 16\sqrt{6})} [=2.96(7855...)] = NQ$	A1		Valid method to reach an eqn in α (or in $\frac{\alpha}{2}$) only; dep on prev M but not on prev A. Alternative choosing eg obtuse ONP then $\frac{\alpha}{2} = \pi - 1.87(85...)$
	or $PT = 2\sqrt{2} [=2.82(8427...)]$			
	or $PQ = 4\sqrt{2}$	m1		2.53... Condone >3sf.
	or $NT = 2\sqrt{6} - 4 [=0.898(979...)]$			
	$\tan \frac{\alpha}{2} = \frac{PT}{NT} = \frac{2\sqrt{2}}{2\sqrt{6} - 4} [=3.14626...] \text{ OE}$	A1	5	
	or $\frac{\alpha}{2} = \frac{\pi}{2} - \left[\frac{\pi}{3} - \tan^{-1}\left(\frac{1}{2\sqrt{2} - \sqrt{3}}\right) \right] \text{ or}$			
	$32 = 2PN^2(1 - \cos \alpha) \Rightarrow 1 - \cos \alpha = \frac{1}{3 - \sqrt{6}}$	A1		
	$\frac{\alpha}{2} = 1.263056... ; \alpha = 2.5261...2.53 \text{ to 3sf}$			
Total			14	
TOTAL			75	